# Motion Compensation Based on Signal Processing Method for Airborne SAR

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Abstract: In the synthetic aperture radar (SAR) system, the motion error is the main phase error sources and the motion compensation is very important. The phase gradient autofocus (PGA) is a state of art technique for phase error correction of SAR. It exploits the redundancy of the phase-error information among range bins by selecting the strongest scatter for each range bin and synthesizes them. The motivation of this paper is based on the observation that the redundancy of phase error is also among the cross-range direction. Moreover, the proposed method applies the weighting function to better utilize the phase error information. The validity of the proposed scheme for PGA is tested with some numerical simulation.

Keywords: Synthetic Aperture Radar (SAR), Motion Compensation, Phase Gradient Autofocus (PGA)

### 1. Introduction

In the synthetic aperture radar (SAR) system, the platform motion deviations from ideal motion are the main phase error sources and the motion compensation is very important. The motion error is defined as the error between the actual flight path and the nominal path. For airborne SAR, it is assumed that the aircraft travels along a straight line with a constant velocity and the imaged area is flat. However, in practice, the true trajectory has small deviations from the ideal straight line and the imaged region may have terrain variations resulting in a phase error in echo [2].

The airborne SAR is equipped with inertial measurement unit or global positioning system to remove the phase error caused by the antenna phase center motion. The high resolution SAR systems require very precise measurement sensors for motion compensation. The motion errors are considerably high due to atmospheric turbulence and aircraft properties. There is velocity mismatch or along-track acceleration, when the SAR emits pulse at a constant temporal pulse repetition frequency (PRF), it will place the azimuth sample at different part of the ground, thus cause rarefactions of the image relative to a map of the imaged region. After having determined the motion errors of the aircraft, motion compensation can be realized by adjusting the PRF, applying a range-dependent phase shift to each received pulse and delaying it.

If the inertial data is not available, autofocus technique is applied to the phase error correction cause defocus. The phase errors are estimated from the collected data itself. Autofocus methods generally follow the idea of finding points with a certain property and using the property to estimate phase error [3].

Phase gradient autofocus (PGA) algorithm is a state of art autofocus method for its robustness [1]. It exploits the redundancy of the phase-error information contained in the degraded image, independent of the underlying scene content. By a procedure of center shifting, windowing, phase gradient estimation and iterative correction, the point spread function (PSF) of the azimuth image due to the phase error is isolated and estimated to deblur the image. The PGA algorithm can be viewed as a kind of prominent scatters method. Different from the generic prominent scatters methods which find the prominent scatters from the collected data directly, the PGA algorithm constructs the prominent scatters itself by windowing in the image domain. In this paper, we propose the improved PGA algorithm to better utilize the phase error information contained in the image.

#### 2. Phase Gradient Autofocus

Consider the range-compressed phase history domain data as follows [1]:

$$F_n(u) = |F_n(u)| \exp\{j[\phi_n(u) + \phi_e(u)]\}$$
 (1)

where n means n-th range bin, u is the relative position along the cross-range,  $|F_n(u)|$  and  $\phi_n(u)$  are the magnitude and phase, respectively.  $\phi_e(u)$  is the phase error and it is independent of n. The above equation is processed with azimuth Fourier transformation to reconstruct the image. Each range bin of the image data consists of a sum,

$$\mathcal{F}\{F_n(u)\} = \sum_m h(x) * a_{m,n} s(x - x_{m,n})$$
 (2)

where  $h(x) = \mathcal{F}\{\exp\{j\phi_e(u)\}\}\$  is the transform of the complex aperture phase error function, the  $a_{m,n}s(x-x_{m,n})$  are the scatter-induced impulse response, and \* denotes convolution.

The procedure of the PGA can be summarized as [1]: A. Circular shifting

The first step in PGA is to select for each range bin n the strongest scatterer  $a_n$ , and shift it to the origin (center of the image), to remove the frequency offset due to the Doppler of the scatterer. If SAR images consisted solely of isolated targets, the circular shifting would indeed attempt to align them for each range bin. However, highly defocused images rarely contain isolated scatterers (nonoverlapping point spread functions). In addition, many SAR images consist solely of clutter-like objects, such as trees, grass, dirt roads,

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rocky and brushy fields, etc. It is important to be able to focus scenes like these as well.

## B. Windowing

Windowing has the effect of preserving the width of the dominant blur for each range bin while discarding data that cannot contribute to the phase-error estimation. This allows the phase-error estimation to proceed using input data having the highest signal-to-noise ratio.

### C. Phase gradient estimation

After the image data is circularly shifted and windowed, the phase gradient is estimated. Let us denote the shifted and windowed image data as  $g_n(x)$  and  $G_n(u) = |G_n(u)| \exp\{j(\phi_e(u) + \theta_n(u))\}$  be the inverse Fourier transform.  $\theta_n(u)$  is the scatterer-dependent phase function for each range bin. It has been shown that a linear unbiased minimum variance (LUMV) estimate of the gradient of the phase error,  $\phi_e(u)$ , is given by

$$\hat{\dot{\phi}}_e(u) = \frac{\sum_n \text{Im}\{G_n^*(u)\dot{G}_n(u)\}}{\sum_n |G_n(u)|^2}$$
(3)

Thus, this LUMV estimate yields the gradient of the true phase error, plus an error term that has been made as small as possible (at this step) by the circular shifting and windowing operations.

## D. Iterative phase correction

The estimated phase gradient,  $\dot{\phi}_e(u)$  is integrated to obtain  $\hat{\phi}_e(u)$ , and any bias and linear trend is removed prior to correction. Phase correction is imposed by complex multiplication of the range-compressed phase history domain data by  $\exp\{-j\hat{\phi}_e(u)\}$ . The estimation can correction process is repeated iteratively. This new image is the start point of a new iteration.

## 3. Proposed Methods

## 3.1. Scatter Selection

From the description of the PGA algorithm in the previous section we can see that prominent scatters are selected during step A. For each range bin, the strongest scatter is selected. Mathematically,  $\max_n(a_{m,n})$  is used as prominent scatters. The underlying objective of this selection method is to maximize the signal to clutter and noise rate. Such selection method exploits the redundancy of phase-error information among different range bins.

In the proposed method, we choose a number of scatters that have largest amplitudes among the whole image domain instead of each range bin. The distance between two selected scatters in the same range bin must be larger than W, the window size. Note that the distance means a circular distance. The window size is to be estimated in each iteration and will decrease for subsequent iterations because the image is becoming more focused. Moreover, we can choose the window size dependent on each chosen scatter. Therefore, we use following scheme:

A. Select Scatter and Circular shifting

In this operation, we select a n-th strongest scatter  $a_n$  among the whole image domain, and shift it to the origin. Note that  $1 \le n \le N$  where N is the number of selected scatters.

### B. Windowing

In the corresponding range bin, we decide the window size. For example, we can choose  $W_n$  by thresholding the shifted imagery signal at the point 10 dB down from its peak.

C. Repeat Step A and B

We repeat this operation till we select N scatters.

D. Phase Gradient Estimation and Iteration

These operation is similar to the existing PGA algorithm.

$$\hat{\dot{\phi}}_e(u) = \frac{\sum_n \text{Im}\{G_n^*(u)\dot{G}_n(u)\}}{\sum_n |G_n(u)|^2}$$
(4)

where  $g_n(x)$  is the shifted and windowed image data of the range bin corresponding to n-th selected scatter and  $G_n(u)$  is the inverse Fourier transformed signal. Of course, the estimation has the iterative process.

## 3.2. Weighted Compensation

In the PGA algorithm, the estimated phase errors from all range bin are same weight and the finally estimation is the mean of estimated phase errors. However, the range bins having weak scatters may induce a wrong phase error. Thus, the weighted compensation is needed.

In the above subsection, we select N scatters. For the range bin including the scatter  $a_n$ , we define  $q_n$  as follows:

$$q_n = \frac{|a_n|}{\sum_m |a_m|} \tag{5}$$

and normalize the  $q_n$ . Then, in the phase gradient estimation step, the algorithm has following form:

$$\hat{\dot{\phi}}_e(u) = \frac{\sum_n q_n \text{Im}\{G_n^*(u)\dot{G}_n(u)\}}{\sum_n q_n |G_n(u)|^2}$$
(6)

### 4. Simulation Results

In order to test the validity and effectiveness of the our improved PGA method, we simulate the radar imaging of point targets. In each received pulse, there's an observation phase error that is invariant along the range. This phase error term is mainly due to the difference between the ideal flight path and the real flight path.

The radar parameters are set as follows: the carrier frequency is 10GHz, the transmitted waveform is a chirp signal with 233.5MHz bandwidth, the pulse repetition frequency is 1KHz, the pulse duration is  $1\mu$ sec, the platform speed is 100m/s, and the sampling frequency is 485MHz. The synthetic aperture length is 150m, distance between aperture center and scene center is 2000m, and platform height is 1000m. The phase error is modelled as polynomial phase signal of up to fourth order. The coefficients of polynomial are  $3\times 10^{-4}$ ,  $2\times 10^{-6}$ , and  $1\times 10^{-6}$ , respectively.

We consider 49 point targets with same reflectivity. Figure 1 shows that the image reconstruction result with and without phase error. The reconstructed image with phase error is blurred along the cross range direction. We show the original PGA result and proposed PGA result in Fig. 2. For fair judgement, we consider 3 iterations. In practice, the original PGA algorithm reaches convergence in 4-5 iterations, while the proposed method converges in 2-3 iterations. In order to

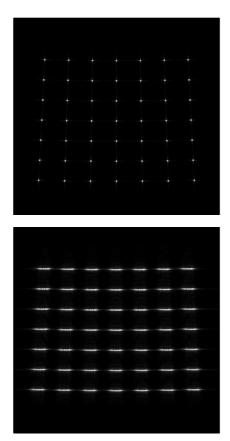


Fig. 1. (Top) Reconstructed image with no error; (Bottom) Reconstructed image with error

show the redundancy of phase error information among the cross range direction in the image domain, a range bin that contains several point scatters is shown in Fig. 3. We know that the PSF is sharpened after the phase error compensation.

## 5. Conclusions and Further Works

The motivation of this paper is based on the observation that the redundancy of phase error is also among the cross-range direction. Moreover, the proposed method applies the weighting function to better utilize the phase error information. The effectiveness of our proposed PGA algorithm is theoretically analyzed and tested with some numerical simulation. It is shown that with our new method, more effective scatters are utilized and the convergence of the algorithm can be speed up.

Further work is to test the proposed algorithm with real radar data. Theoretically, since the new method exploits the phase redundancy in both range and azimuth cells, the degree of improvement to the original algorithm depends on the distribution of strong scatters in the imagery. If strong scatters don't tend to locate in all different range cells, the result will be better.

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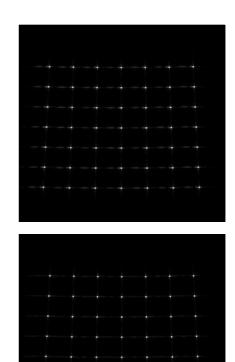


Fig. 2. (Top) Reconstructed image by PGA with 3 iterations; (Bottom) Reconstructed image by proposed PGA with 3 iterations

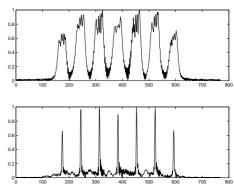


Fig. 3. Normalized azimuth image of a particular range bin without/with phase error estimation

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