

## Development of Mandibular Movements Measuring System Using Double Stereo-Cameras

Soonyong Park\*, Sung-Kee Park\*, Changhyun Cho\*\*, Munsang Kim\* and Mignon Park\*\*\*

\*Intelligent Robotics Research Center, KIST, 39-1 Hawolgok-dong, Sungbuk-gu, Seoul, Korea  
(Tel : +82-2-958-6186; E-mail: {scipio77, skee, munsang}@kist.re.kr)

\*\*Center for Intelligent Robotics 21C Frontier Program, 39-1 Hawolgok-dong, Sungbuk-gu, Seoul, Korea  
(Tel : +82-2-958-6979; E-mail: chcho@kist.re.kr)

\*\*\*Department of Electrical & Electronic Engineering, Yonsei University, 134 Sinchon-dong, Sedaemun-gu, Seoul, Korea  
(Tel : +82-2-2123-2868; E-mail: mignpark@yonsei.ac.kr)

**Abstract:** In this paper, we propose a 3D automated measuring system which measures the mandibular movements and the reference plane of the jaw movements. In diagnosis and treatment of the malocclusions, it is necessary to estimate the mandibular movements and the reference plane of the jaw movements. The proposed system is configured with double stereo-cameras, PC, two moving pattern plates(MPPs), two fixed pattern plates(FPPs) and one orbital marker. The virtual pattern plate is applied to calculate the homogeneous transformation matrices which describe the coordinates systems of the FPP and MPP with respect to the world coordinates system. To estimate the parameters of the hinge axis, the Euler's theorem is applied. The hinge axis points are intersections between the FPPs and the hinge axis. The coordinates of a hinge axis point with respect to the MPP coordinates system are set up to fixed value. And then, the paths of the jaw movement can be calculated by applying the homogeneous transformation matrix to fixed hinge axis point. To examine the accuracy of the measurements, experiments of measuring the hinge axis points and floating paths of them are performed using the jaw motion simulator. As results, the measurement errors of the hinge axis points are within reasonable boundary, and the floating paths are very similar to the simulator's moving path.

**Keywords:** Mandibular movements, Reference plane, Malocclusion, Double stereo-cameras, Virtual pattern plate, Euler's theorem, Hinge axis points, Homogeneous transformation matrix

### 1. Introduction

Teeth that do not mesh properly are called malocclusions. Untreated malocclusions can lead to teeth that wear poorly with time. Also, poorly fitting teeth can contribute to stress and strain on the muscles that support the jaws. This can lead to muscle pain in the chewing muscles [1].

In diagnosis and treatment of the malocclusions, the articulator which reconstructs the jaw movements is used. To reconstruct the mandibular movements, the patient's jaw movements and the reference plane of the jaw movements should be measured by using a measuring apparatus, and then, the measured reference plane should be transferred to the articulator. The reference plane is composed of one anterior reference point and two posterior reference points. The articulator's property for the jaw movement reconstruction is not affected by a position of the anterior reference point. On the other hand, the positions of two posterior reference points affect the reconstructive property.

In general, the hinge axis point, one of the condylar points, is a practical posterior reference point for the reconstruction of the jaw movements in the articulator[2]. The hinge axis point is a point on the hinge axis. The hinge axis is the center of terminal hinge movement of lower jaw. Terminal hinge movement is an opening movement of a distance of up to 20 mm between the incisors with both condyles remaining in their most posterior positions[3]. Furthermore, the anterior reference point, as it represents the condyle, is used for analyzing and measurement of the mandibular motions.

Conventional hand-operated measuring apparatus[4] has defects of long measuring time, which conflicts both patient and doctor all together, and also the degree of reproducibility was unreliable. In addition, nowadays many researches about 3D simulation system for malocclusion diagnosis and treatment using CAD/CAM technologies are in progress actively[5]. However, hand-operated measuring apparatus is incapable of producing the adequate data for the 3D

simulation system. Hence, a measuring apparatus is necessary, which is able to interface with PC and is capable of presenting the digitized data for the 3D simulation system.

Many previous works for 3D automated measuring apparatus have been carried out. However, almost all these devices measure only mandibular movements, and do not consider the transfer of the reference plane[8-12]. Therefore, these apparatuses do not measure the anterior reference point and the hinge axis point. Virtually, if 3D simulation system is applied in general dental clinic, the articulator might be useless. However, 3D simulation system has been embodied in laboratory level, and still has not been used generally in dental clinic.

In this paper, we propose a 3D automated measuring system which measures the hinge axis point, the anterior reference point and the mandibular movements. The proposed system is configured with double-stereo cameras, PC, two moving pattern plates(MPPs), two fixed pattern plates(FPPs) and one orbital marker. The MPPs are attached to holding fixture mounted on the lower jaw. The FPPs are attached to the left and right side of patient's face. The orbital marker is attached to the eye rims for measuring the anterior reference point.

To reduce the estimating errors which are affected by various noises, we propose a virtual pattern plate. Since our methods for computing the hinge axis point and movements of them are based on the homogeneous transformation matrices which describe the coordinates systems of the FPP and MPP with respect to the world coordinates system, it is important to compute the homogeneous transformation matrix accurately. So we applied the virtual pattern plate to calculate the homogeneous transformation matrix.

The Euler's theorem, which described a rigid body motion with one point fixed rotation axis, is applied to estimate the parameters of the hinge axis. The hinge axis points are intersections between the FPPs and the hinge axis. To

compute the floating paths of a hinge axis point, the coordinates of the hinge axis point with respect to the MPP coordinates system are set up to fixed value. And then, the paths of the jaw movement can be calculated by applying the homogeneous transformation matrix, which describes the MPP's coordinates system in terms of the reference plane's coordinates system, to fixed hinge axis point.

This paper is organized as follows. After introducing the measuring references in the following section, we will present the configuration of proposed system in section 3. In section 4 we will present a method for calculating the homogeneous transformation matrix. A procedure for determining the parameters of the hinge axis is followed in section 5. In section 6, we will present a method for computing the reference plane and the mandibular motions. And we will show experimental results in section 7, and conclusion in section 8.

### 2. Measuring References

An example for the measuring references is shown in Fig. 1. The reference plane is the horizontal reference for analyzing the mandibular movements in 3D space, and is composed of the left hinge axis point, the right hinge axis point and the anterior reference point.

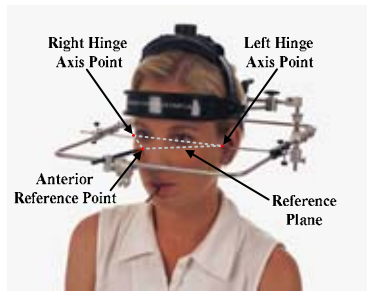


Fig. 1 Measuring references

The left and right hinge points are some points on the hinge axis, and are the reference points for measuring the jaw motions. The anterior reference point can be any point nearby the eye rims because the jaw movement reconstructibility of the articulator is not affected by a position of the anterior reference point.

### 3. System Configuration

The measuring system is configured with double stereo-cameras, PC, two MPPs, two FPPs and one orbital marker. The system configuration is shown in Fig. 2.

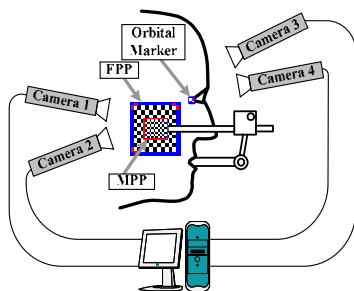


Fig. 2 System configuration

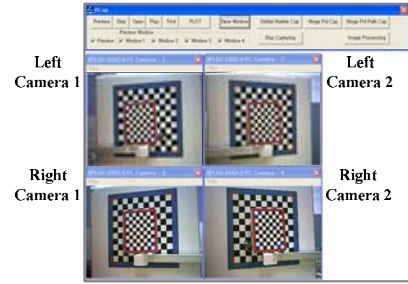


Fig. 3 Image Capturing Software

The pair of FPPs is attached to both sides of the patient's face by using a head frame which is fixed on the head. The MPPs are attached to holding fixture mounted on the lower jaw, and are disposed to face the FPPs in a spaced distance. The MPPs move in unison with the movements of the lower jaw. Each stereo-camera records the relative movements of the MPP in relation to FPP, as lower jaw moves. The PC receives and processes the image signals fed from connected cameras.

The FPP and the MPP have a shape of checkerboard which is formed by the corner points. Each corner point is identified as the point where corners of quadrangles meet each other. The corner points which comprise the FPP and the MPP are used as data points for establishing the local coordinates system of the FPP and the MPP respectively.

The orbital marker which is formed by one corner point is attached to a position around the eye rims. Therefore the corner point is the anterior reference point.

The image capturing software is shown in Fig. 3. While monitoring the camera images, this software can capture the image frames of each camera.

### 4. Coordinates System

In all the image frames captured by four cameras while the patient's lower jaw makes a motion, all the data points of the FPP and the MPP are extracted. Then, the three-dimensional coordinates of each data point are calculated by applying the 3D reconstruction techniques of stereo-vision[13]. The three-dimensional coordinates of the data points are defined with respect to the world coordinates system. In every image frame, the respective local coordinates systems of the FPP and the MPP are computed by using the three-dimensional coordinates of the data points. The local coordinates systems of the FPP and the MPP are used for estimating the hinge axis and the jaw movements. In the follows of this section, a method for computing the local coordinates system of the MPP is described. The FPP's local coordinates system is computed by applying the same procedure as the MPP.

#### 4.1 Virtual pattern plate

The estimating noises cause errors in calculating the three-dimensional coordinates of the data points. Therefore, the computed local coordinates system might be inaccurate. So we applied a virtual pattern plate(VPP) in order to improve the accuracy of the local coordinates system. By fitting the real measured data points to the VPP, the local coordinates system can be computed more accurately.

The VPP has same size as the real MPP. As shown in Fig. 4, the three-dimensional coordinates vector  ${}^w\mathbf{v}_i$  of the VPP's  $i$ th corner point is defined with respect to the world coordinates system  $\{W\}$ . For convenience, all the Z coordinates are set to be zero.

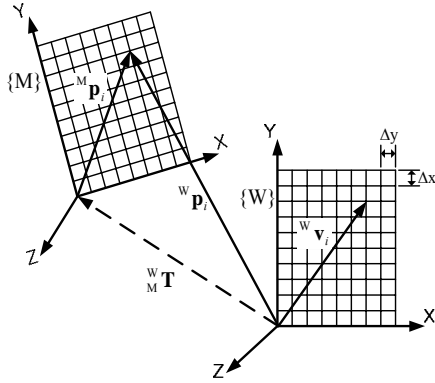


Fig. 4 Computing the local coordinates system of the MPP

The unit increment values in X axis and Y axis are  $\Delta x$  and  $\Delta y$  respectively. The  $\Delta x$  and  $\Delta y$  are coincided with the real MPP's.

#### 4.2 Homogeneous transformation matrix

The coordinates vector  ${}^M\mathbf{p}_i$  is defined with respect to the MPP's local coordinates system  $\{M\}$ . Ideally, the  ${}^M\mathbf{p}_i$  of the real MPP's corner point should be coincided with the  ${}^W\mathbf{v}_i$ . So we compute the homogeneous transformation matrix  ${}^W\mathbf{T}_M$  which transforms the  ${}^W\mathbf{v}_i$  into the  ${}^M\mathbf{p}_i$ , and describes  $\{M\}$  in terms of  $\{W\}$  [14]. The coordinates vector  ${}^W\mathbf{p}_i$  of the real MPP's  $i$ th corner point is measured in world coordinates system  $\{W\}$ .

We determine the parameters(rotation and translation) of the homogeneous transformation matrix by applying the work of John H. Challis[15].

The procedure, which requires the coordinates of three or more non-collinear points, is based around the singular value decomposition(SVD), and provides a least-squares estimate of the rigid body transformation parameters. The brief summary of the procedure is described in the following paragraphs.

The coordinates vectors  ${}^W\mathbf{v}_i$  and  ${}^W\mathbf{p}_i$  are defined as

$${}^W\mathbf{v}_i = [v_{xi} \ v_{yi} \ v_{zi}]^T, \quad {}^W\mathbf{p}_i = [p_{xi} \ p_{yi} \ p_{zi}]^T, \quad (1)$$

where  $i$  is  $1 \sim n$ , and  $n$  is the number of data points. And then, the mean vectors ( ${}^W\bar{\mathbf{v}}$  and  ${}^W\bar{\mathbf{p}}$ ) are computed

$${}^W\bar{\mathbf{v}} = \begin{bmatrix} \bar{v}_x \\ \bar{v}_y \\ \bar{v}_z \end{bmatrix}, \quad \bar{v}_x = \frac{1}{n} \sum_{i=1}^n v_{xi}, \quad \bar{v}_y = \frac{1}{n} \sum_{i=1}^n v_{yi}, \quad \bar{v}_z = \frac{1}{n} \sum_{i=1}^n v_{zi}, \quad (2)$$

$${}^W\bar{\mathbf{p}} = \begin{bmatrix} \bar{p}_x \\ \bar{p}_y \\ \bar{p}_z \end{bmatrix}, \quad \bar{p}_x = \frac{1}{n} \sum_{i=1}^n p_{xi}, \quad \bar{p}_y = \frac{1}{n} \sum_{i=1}^n p_{yi}, \quad \bar{p}_z = \frac{1}{n} \sum_{i=1}^n p_{zi}. \quad (3)$$

The relative position vectors of the  ${}^W\mathbf{v}_i$  and  ${}^W\mathbf{p}_i$  can be determined from the mean vectors

$${}^W\mathbf{v}'_i = \begin{bmatrix} v_{xi} - \bar{v}_x \\ v_{yi} - \bar{v}_y \\ v_{zi} - \bar{v}_z \end{bmatrix}, \quad {}^W\mathbf{p}'_i = \begin{bmatrix} p_{xi} - \bar{p}_x \\ p_{yi} - \bar{p}_y \\ p_{zi} - \bar{p}_z \end{bmatrix}. \quad (4)$$

Using the relative position vectors, the correlation matrix  $\mathbf{C}$  is computed from

$$\mathbf{C} = \frac{1}{n} \sum_{i=1}^n {}^W\mathbf{v}'_i {}^W\mathbf{p}'_i{}^T. \quad (5)$$

The SVD of  $\mathbf{C}$  is computed

$$\mathbf{C} = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T. \quad (6)$$

where  $\mathbf{U}$  and  $\mathbf{V}$  are orthogonal matrices, and  $\mathbf{W}$  is a diagonal matrix which contains the singular values of matrix  $\mathbf{C}$ , the number of non-zero singular values indicating the rank of  $\mathbf{C}$ .

And then, the rotation matrix  ${}^W\mathbf{R}_M$  and translation vector  ${}^W\mathbf{t}_M$  can be computed

$${}^W\mathbf{R}_M = \mathbf{U} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(\mathbf{U} \cdot \mathbf{V}^T) \end{bmatrix} \cdot \mathbf{V}^T, \quad (7)$$

$${}^W\mathbf{t}_M = {}^W\bar{\mathbf{p}} - {}^W\mathbf{R}_M {}^W\bar{\mathbf{v}}. \quad (8)$$

Therefore, the homogeneous transformation matrix which describes  $\{M\}$  in terms of  $\{W\}$  is determined as follow

$${}^W\mathbf{T}_M = \begin{bmatrix} {}^W\mathbf{R}_M & {}^W\mathbf{t}_M \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (9)$$

#### 5. Computing Hinge Axis

The hinge axis is an axis around which the condyles rotate purely during the terminal hinge movement. Since the terminal hinge movement is remarkably reproducible and most readily produced passively, the hinge axis has the advantage that its location on the mandible is relatively easy to find[2]. The movements of the MPPs, which move in unison with the terminal hinge movement, are performed by a combination of rotation and translation. So the Euler's theorem can be applied to determine the parameters(position vector and direction vector) of the hinge axis. Euler's theorem states that the general displacement of a rigid body with one point fixed is a rotation about some axis. This unique axis of rotation is called the screw axis[14, 16]. Therefore, the screw axis is the hinge axis. In what follows, we describe a computing procedure for estimating the parameters of the hinge axis with respect to the right FPP's local coordinates system  $\{F_R\}$ . The parameters defined with respect to the  $\{F_L\}$  can be computed by applying the same procedure as the right side one.

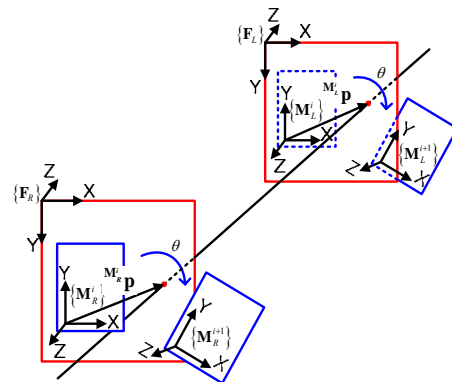


Fig. 5 Determination of the hinge axis

As illustrated in Fig. 5, the MPPs of the left and right sides move in unison with the terminal hinge movement from image frame  $i$  to  $i+1$ . The local coordinates systems of the right FPP and right MPP,  $\{F_R^i\}$ ,  $\{M_R^i\}$  and  $\{M_R^{i+1}\}$ , which are defined with respect to the world coordinates system can be computed by applying the procedure described in section 4.

The coordinates system  $\{M_R^{i+1}\}$  is described as follows : initially coincident with  $\{M_R^i\}$  we rotate  $\{M_R^{i+1}\}$  about the hinge axis by an angle  $\theta$ . The hinge axis is define as the position vector  ${}^{M_R^i}\mathbf{p}$  and direction vector  ${}^{M_R^i}\mathbf{n}$  which are described with respect to the  $\{M_R^i\}$ . The position vector and direction vector are given by

$${}^{M_R^i}\mathbf{p} = [p_x \quad p_y \quad p_z]^T, \quad {}^{M_R^i}\mathbf{n} = [n_x \quad n_y \quad n_z]^T. \quad (10)$$

Since we know the homogeneous transformation matrices which describe  $\{M_R^i\}$  and  $\{M_R^{i+1}\}$  in terms of the world coordinate system  $\{W\}$ , the homogeneous transformation matrix  ${}_{M_R^i}^{M_R^{i+1}}\mathbf{T}$  which describes  $\{M_R^{i+1}\}$  in terms of  $\{M_R^i\}$  can be computed readily. So:

$${}_{M_R^i}^{M_R^{i+1}}\mathbf{T} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

Applying the Euler's theorem,  ${}_{M_R^i}^{M_R^{i+1}}\mathbf{T}$  can be described in terms of the position vector and direction vector

$$\begin{aligned} {}_{M_R^i}^{M_R^{i+1}}\mathbf{T} &= \begin{bmatrix} r_{11} & r_{12} & r_{13} & (1-r_{11})p_x - r_{12}p_y - r_{13}p_z \\ r_{21} & r_{22} & r_{23} & -r_{21}p_x + (1-r_{22})p_y - r_{23}p_z \\ r_{31} & r_{32} & r_{33} & -r_{31}p_x - r_{32}p_y + (1-r_{33})p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} n_x n_x v \theta + c \theta & n_x n_y v \theta - n_z s \theta & n_x n_z v \theta + n_y s \theta & t_x \\ n_x n_y v \theta + n_z s \theta & n_y n_y v \theta + c \theta & n_y n_z v \theta - n_x s \theta & t_y \\ n_x n_z v \theta - n_y s \theta & n_y n_z v \theta + n_x s \theta & n_z n_z v \theta + c \theta & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ & \quad (c \theta = \cos \theta, \quad s \theta = \sin \theta, \quad v \theta = 1 - \cos \theta). \end{aligned} \quad (12)$$

The angle of rotation about the hinge axis is computed in diagonal terms of the rotation matrix

$$\theta = \cos^{-1} \left( \frac{r_{11} + r_{22} + r_{33} - 1}{2} \right). \quad (13)$$

From equation (12), the translation vector is given as follow

$$\begin{aligned} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} &= \begin{bmatrix} (1-r_{11})p_x - r_{12}p_y - r_{13}p_z \\ -r_{21}p_x + (1-r_{22})p_y - r_{23}p_z \\ -r_{31}p_x - r_{32}p_y + (1-r_{33})p_z \end{bmatrix} \\ &= \left( \mathbf{I} - {}_{M_R^i}^{M_R^{i+1}}\mathbf{R} \right) \cdot {}^{M_R^i}\mathbf{p} \\ &= \mathbf{Q} \cdot {}^{M_R^i}\mathbf{p}. \end{aligned} \quad (14)$$

So the position vector is determined from equation (14)

$${}^{M_R^i}\mathbf{p} = \mathbf{Q}^{-1} \cdot \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}. \quad (15)$$

The direction vector is the eigenvector corresponding to the smallest eigenvalue of the matrix  $\mathbf{Q}$ . Finally, the position vector and direction vector are redefined with respect to the FPP's local coordinates system

$${}^{F_R^i}\mathbf{p} = {}_{M_R^i}^{F_R^i}\mathbf{T} \cdot {}^{M_R^i}\mathbf{p}, \quad (16)$$

$${}^{F_R^i}\mathbf{n} = {}_{M_R^i}^{F_R^i}\mathbf{R} \cdot {}^{M_R^i}\mathbf{n}. \quad (17)$$

where  ${}_{M_R^i}^{F_R^i}\mathbf{T}$  is the homogeneous transformation matrix which describes  $\{M_R^i\}$  in terms of  $\{F_R^i\}$ , and  ${}_{M_R^i}^{F_R^i}\mathbf{R}$  is the rotation matrix.

## 6. Computing Reference Plane and Jaw Motions

### 6.1 Reference plane

As shown in Fig. 6, the reference plane is composed of right hinge axis point and left hinge axis point, anterior reference point. These points are defined with respect to the world coordinates system  $\{W\}$ . The right and left hinge axis points are intersections where the hinge axis goes through the right and left FPPs. The anterior reference point is the corner point of the orbital marker.

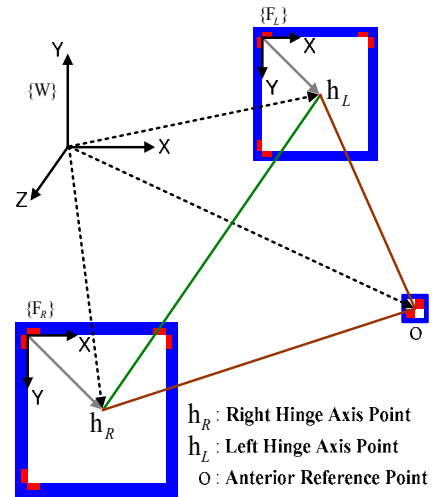


Fig. 6 Reference plane

### 6.2 Mandibular movements

As shown in Fig. 7, the mandibular movements are computed with respect to the reference plane. In first place, the coordinates of the hinge axis point with respect to the MPP coordinates system are set up to fixed vector  ${}^M\mathbf{h}_R$ . The fixed vector is termed as "virtual rotation center". In every image frame which is captured for estimating the jaw motion, the homogeneous transformation matrix  ${}_{M^i}^O\mathbf{T}$  is computed. Where  $i$  is a image frame number, and  $\{O\}$  is the coordinates system of the reference plane. And then, the floating paths of

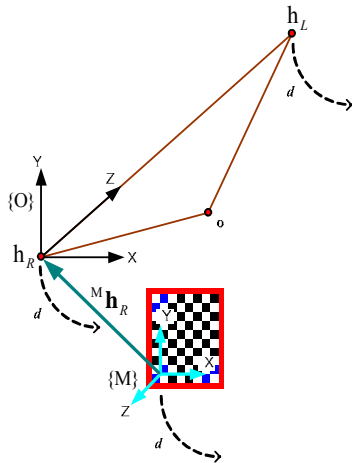


Fig. 7 Floating paths of hinge axis points

the hinge axis point,  ${}^o\mathbf{h}_R^i$ , can be calculated by applying  ${}^o\mathbf{T}_M$  to the virtual rotation center

$${}^o\mathbf{h}_R^i = {}^o\mathbf{T}_M \cdot {}^M\mathbf{h}_R \quad (18)$$

### 7. Experimental Results

We used four USB 2.0 PC cameras(MU2-130, Aplus) with resolution of  $640 \times 480$  pixels. To examine the accuracy of the measurements, experiments of measuring the hinge axis points and floating paths of them are performed using the jaw motion simulator. Fig. 8 shows the jaw motion simulator.

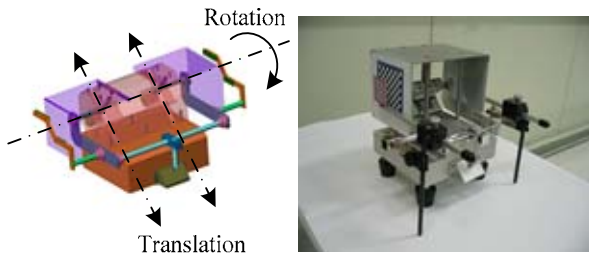


Fig. 8 Jaw motion simulator

Moving frame, which is a part of the jaw motion simulator, can rotational and translational motions, and performs a role as holding fixture mounted on the lower jaw.

#### 7.1 Hinge axis point

To analysis the experimental results, we determined the absolute coordinates of the left and right hinge axis points

Table 1 Absolute coordinates of the left and right hinge axis points

Test Num.	Left hinge axis point		Right hinge axis point	
	X	Y	X	Y
1	24.6089	44.4101	27.4689	45.5873
2	24.6754	44.2834	27.3814	45.7694
3	24.6073	44.2963	27.3786	45.4497
4	24.6136	44.1035	27.3851	45.4612
5	24.6267	44.0362	27.4043	45.6629
Mean	24.6263	44.2259	27.4036	45.5861
Max.-Min.	0.0681	0.3739	0.0903	0.3197

absolute coordinates of the left and right hinge axis points. Virtually, the human's lower jaw can be rotated about 7 degrees maximumly. So we performed 5 tests which rotate the moving frame more than 30 degrees, and then we fixed mean values of the calculated coordinates as the absolute coordinates (Table 1).

We performed 25 experiments which rotate the moving frame up to 7 degrees. The experimental results are shown in Fig. 9 and Fig. 10.

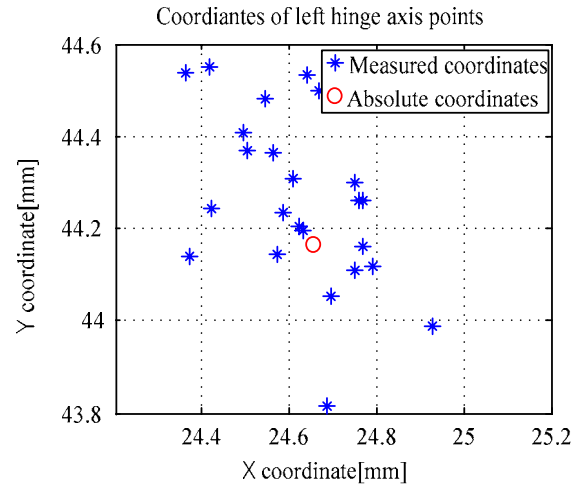


Fig. 9 Experimental results of left hinge axis point

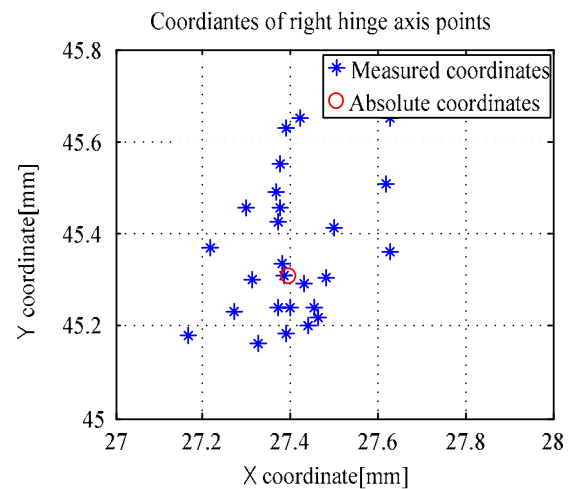


Fig. 10 Experimental results of right hinge axis point

The left and right maximum error distance between absolute coordinates and measured coordinates are 0.4143mm and 0.4685mm respectively. These are reasonable error boundaries, and are small enough for dental clinic.

#### 7.2 Protrusive motion

The moving frame can simulate the protrusive motion of the lower jaw. Fig. 11 shows a protrusive motion. We made the moving frame moves down linearly. The angle of inclination is 45 degrees. As shown in Fig. 12, the floating paths of the right hinge axis point are very similar to the moving frame's path.



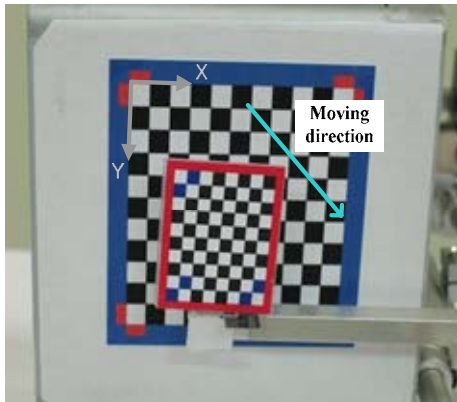


Fig. 11 Protrusive motion

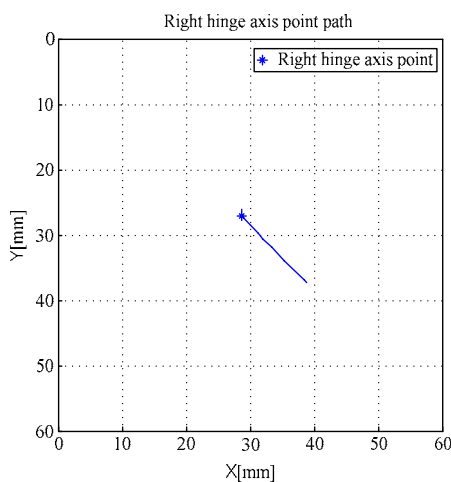


Fig. 12 Experimental result of a protrusive path of the right hinge axis point

**8. Conclusion**

In this paper, we proposed a 3D automated measuring system which measures the hinge axis point, the anterior reference point and the mandibular movements. The virtual pattern plate is applied to calculate the coordinates systems of the moving pattern plate and fixed pattern plate accurately. And the Euler's theorem is applied to estimate the parameters of the hinge axis. By using the jaw motion simulator, we showed the measurement errors of the hinge axis points are reasonable, and the floating paths of the hinge axis point are very similar to the jaw motion simulator's moving path. Therefore, the proposed system can be useful for diagnosis in general dental clinics.

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