Sliding Mode Control for Pneumatic Active Suspension Systems of a One-wheel Car Model

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Abstract: This paper is concerned with the construction of an improved sliding mode control for the active suspension system of a one-wheel car model subject to the excitation from a road profile. The active control is composed of the equivalent and the switching controls where an improved sliding surface is proposed. The active control force is generated by operating a pneumatic actuator due to the control signal that constructed by measuring the state variables of the car model and by estimating the excitation from the road profile using the *VSS* observer. The experimental result indicates that the proposed active suspension system is relatively effective in the vibration suppression of the car model.

Keywords: Sliding mode control, Pneumatic actuator, Active suspension system, One-wheel car model, *VSS* observer

1. INTRODUCTION

Recently, the investigations of active suspension systems for car models are much increasing, and various kinds of the vibration suppression methods have been proposed [1-8].The active suspension systems are more effective to the vibration suppression of the car models and the ride comfort of passengers in high-speed transportation than passive and semi-active suspension systems. Generally, the derivation of the active control for the suspension system has been developed as *LQ* control theory [1-3] assuming that the car model is expressed as a linear system. However, as the car models are denoted as a complicated system including non-negligible non-linearity and uncertainty from the practical viewpoint, the active suspension systems using fuzzy reasoning [7], neural network [5] and sliding mode control theory [4, 6, 8] have been proposed in recent years. The active suspension systems provide more effective performance in the vibration suppression of the car model than the linear active suspension system derived by *LQ* control theory.

This paper presents a sliding mode control for the pneumatic active suspension system of a one-wheel car model subject to the excitation from a road profile [8]. The one-wheel car model is approximately described as a nonlinear two degrees of freedom system subject to the excitation from a road profile. The vibration suppression of the car model due to the active suspension system based on the sliding mode control theory is more improved by using the *VSS* observer to estimate the excitation from the road profile [8]. The sliding mode control is generally relatively simple in structure, and also it guarantees the system stability and robustness. Therefore, the purpose of this paper is to propose the sliding mode control with the *VSS* observer for the active suspension system of the one-wheel car model. Moreover, the degradation of the performance in the vibration suppression of the car model due to the delay of the pneumatic actuator is improved by inserting a compensator after the sensors to measure the time responses of the car model.

2. ONE-W EEL CAR MODEL

The experimental apparatus of a one−wheel car model, vertically confined by two polls, due to the active control force generated by the pneumatic actuator is constructed as shown in Fig. 1. The masses of the car body and the wheel are respectively denoted as m_1 and m_2 whose displacements are respectively corresponding to z_1 and z_2. The restoring force of the suspension part is practically assumed to be nonlinear, and it is constructed by two coil springs with the stiffness k_1 or by four coil springs with the stiffness $k_1 + k_1$, depending on the suspension $(z_1 - z_2)$. Then, the nonlinear restoring force $f(z_1 - z_2)$ is

$$f(z_1 - z_2) = \begin{cases} (k_1 + k_1)(z_1 - z_2) - ak_1 & for \;\; z_1 - z_2 > a \\ k_1(z_1 - z_2) & for \;\; |z_1 - z_2| \le a \\ (k_1 + k_1)(z_1 - z_2) + ak_1 & for \;\; z_1 - z_2 < -a \end{cases} \quad (1)$$

where a is a positive constant. The non-linearity of the restoring force means that the restoring force becomes stronger as the suspension deflection does larger. The gravity mainly due to the masses, m_1 and m_2, is supported by the mass m_3 whose displacement is denoted as z_3, and the coil spring with the stiffness K. The tire part of the wheel is denoted as the stiffness k_2, and the excitation from a road profile is assumed the signal generated by the electric vibrator connecting the signal function generator. The damping force of the suspension part is assumed due to the Coulomb damping caused by contact with two polls and the viscous damping caused by the pneumatic cylinder, and it may be assumed linear with the damping coefficient c as considered relatively small.

Then, the equations of motion for the car model are

$$m_1 \ddot{z}_1 + c(\dot{z}_1 - \dot{z}_2) + f(z_1 - z_2) = u \quad (2)$$

$$m_2 \ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2(z_2 - z_3) = -u \quad (3)$$

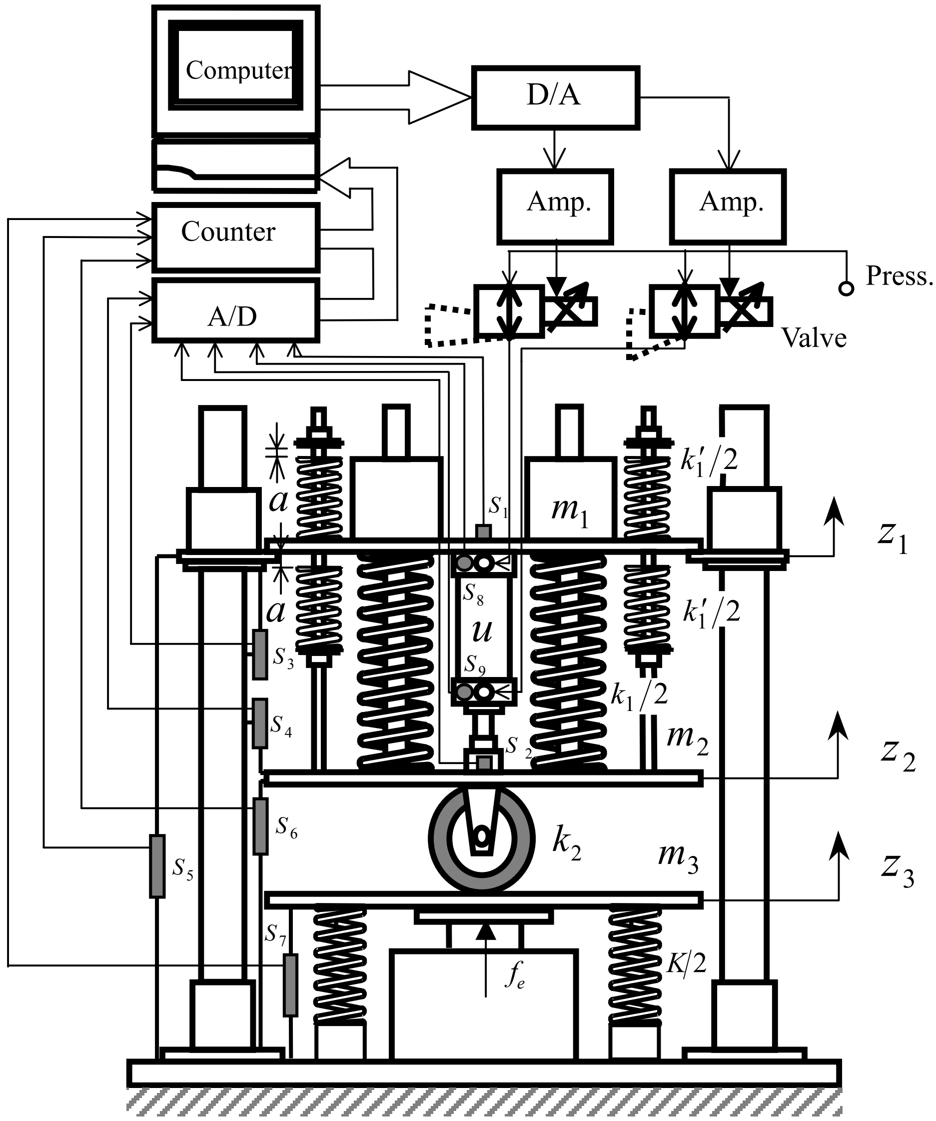

Fig.1 Experimental apparatus of one-wheel car model

$$m_3\ddot{z}_3 - k_2(z_2 - z_3) + Kz_3 = f_e \qquad (4)$$

where u is the active control constructed by actuating a pneumatic actuator, and f_e is the exciting force generated by the electric vibrator. Dividing both sides of Eq. (4) by K, neglecting the first term on the left-hand of Eq. (4) as considered relatively small, defining that

$$k_2 = k_2K\ (k_2 + K\ ), \quad w = f_e\ \ K$$
$$z_2 - w = (k_2\ \ k_2)(z_2 - z_3)$$

and substituting the above definitions into Eq. (3), then the resultant equation becomes

$$m_2\ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2(z_2 - w) = -u \qquad (5)$$

Then, the one-wheel car model described by Eqs.(2) and (5) indicates a nonlinear two degrees of freedom system subject to the excitation from a road profile.

The control part in the one-wheel car model provides the accelerometers (S_1 and S_2), the velocity sensors (S_3 and S_4), and the linear encoders (S_5, S_6 and S_7). The control signal is calculated by using the personal computer based on the measurement data of the state variables through the *A/D* converters and the counters. The control valve of the pneumatic actuator is operated by the control signal through the *D/A* converter and the power amplifier, and the active control u is constructed by actuating the pneumatic actuator as u =77.0 v, experimentally obtained, where v denotes the voltage of the control valve.

3. ACTIVE SUSPENSION S STEM

The active suspension system to be proposed here is constructed as follows. Firstly, a compensator is constructed to improve the degradation of the performance in the vibration suppression of the car model due to the delay of the pneumatic actuator. Secondly, a sliding mode control is proposed as the active control where the excitation from a road profile is estimated by using a *VSS* observer.

3.1 Construction of a compensator

The degradation of the performance in the vibration suppression of the car model due to the delay of the pneumatic actuator is improved by inserting a compensator after the sensors to measure the time responses of the car body and the wheel parts. Measuring the time response of the electro-pneumatic control valve subject to step input, the transfer function for the pneumatic actuator is identified as

$$G_a(s) = \frac{P_u(s)}{K_a V_u(s)} = \frac{900}{s^2 + 30s + 900} e^{-0.035s} \qquad (6)$$

with $K_a = 90kPa/V$. Assuming that the transfer function for the compensator as

$$G_c(s) = \frac{1 + \beta_1 s + \beta_2 s^2}{1 + \alpha_1 s + \alpha_2 s^2} \qquad (7)$$

the parameters characterizing (7) are determined so as to raise the gain and to lead the phase angle in the frequency response function $G_a(j2\pi f)G_c(j2\pi f)$. Modifying these parameters by repeatedly plotting on the Bode diagram for the frequency response function, the parameters are finally obtained as

$$\alpha_1 = 0.025, \ \alpha_2 = 10^{-5}, \ \beta_1 = 0.045, \ \beta_2 = 10^{-5}$$

Then, the Bode diagram for the frequency response functions without and with compensator are shown in Fig. 2.

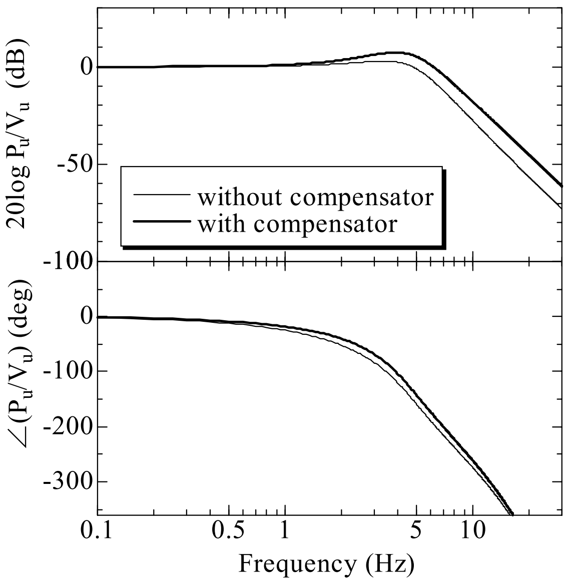

Fig.2 Frequency response functions

3.2 State e uation for the one-wheel car model

Assuming that the excitation from the road profile $w(t)$ is given as $\dot{w}(t) = \xi(t)$, and $w(t)$ and its time derivative $\xi(t)$ are respectively unknown time-varying, but bounded functions, and defining the extended state vector $\mathbf{x}$ as $\dot{z}_1 \; z_1 \; \dot{z}_2 \; z_2 \; w(t) \; ^T$, the state equation can be written as

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \mathbf{b}u + \mathbf{d}\xi(t) \qquad (8)$$

where $\mathbf{F}(\mathbf{x})$ is the vector consisting a nonlinear function of $\mathbf{x}$. The $\mathbf{F}(\mathbf{x})$, $\mathbf{b}$ and $\mathbf{d}$ respectively are denoted as

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} -(c/m_1)(\dot{z}_1 - \dot{z}_2) - (1/m_1) f(z_1 - z_2) \\ \dot{z}_1 \\ (c/m_2)(\dot{z}_1 - \dot{z}_2) + (1/m_2) f(z_1 - z_2) - (k_2/m_2)(z_2 - w) \\ \dot{z}_2 \\ 0 \end{bmatrix}$$

$$\mathbf{b} = [1/m_1 \;\; 0 \;\; -1/m_2 \;\; 0 \;\; 0]^T \quad \mathbf{d} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 1]^T$$

Approximating $f(z_1 - z_2)$ given by (1) as $k_1(z_1 - z_2)$, and neglecting $\xi(t)$, Eq.(8) reduces to

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u \qquad (9)$$

where $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{bmatrix} -c \;\; m_1 & -k_1 \;\; m_1 & c \;\; m_1 & k_1 \;\; m_1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ c \;\; m_2 & k_1 \;\; m_2 & -c \;\; m_2 & -(k_1 + k_2) \;\; m_2 & k_2 \;\; m_2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3.3 Construction of the sliding surface

The sliding surface σ is proposed as

$$\sigma = \mathbf{S}_a \mathbf{x} + \mathbf{S}_b \int_0^t \mathbf{e} \, dt \qquad (10)$$

where $\mathbf{e}$ denotes the estimation error between the measurement $\mathbf{y}$ of the state vector and its estimate $\mathbf{y}$, defined as $\mathbf{y} - \mathbf{y}$, while $\mathbf{S}_a$ is the row vector to be determined by minimizing the given performance index and $\mathbf{S_b}$ is the row vector to be determined by the experiment. It is assumed that the performance index J as

$$J = E \; q_0 \ddot{z}_1^2 + q_1 \dot{z}_1^2 + q_2 (z_1 - z_2)^2 + q_3 \dot{z}_2^2 + q_4 (z_2 - w)^2 + r_1 u^2$$

$$= E\left[\mathbf{x}^T \;\; _1\mathbf{x} + r_1 u^2 + q_0 (u \;\; m_1 + \mathbf{h}^\mathbf{T} \mathbf{x})^2\right] \qquad (11)$$

where $q_0 \sim q_4$ and r_1 are weighting factors, and

$$\mathbf{h}^T = -c \;\; m_1 \;\; -k_1 \;\; m_1 \;\; c \;\; m_1 \;\; k_1 \;\; m_1 \;\; 0$$

$$
{}_1 = \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & -q_2 & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ 0 & -q_2 & 0 & q_2+q_4 & -q_4 \\ 0 & 0 & 0 & -q_4 & q_4 \end{bmatrix}
$$

$$
= \ {}_1 + (q_0 r_1 \ \ r)\mathbf{hh}^T, \mathbf{A}^* = \mathbf{A} - (q_0/m_1 r)\mathbf{bh}^T
$$

$$
r = r_1 + q_0/m_1^2
$$

Then, the optimal control u_c minimizing (11) becomes

$$
u_c = -(\mathbf{b^T P} + q_0 \mathbf{h}^T/m_1)/r \cdot \mathbf{x} \tag{12}
$$

Hence $\mathbf{S}_a$ in (10) is defined as

$$
\mathbf{S}_a = (\mathbf{b^T P} + q_0 \mathbf{h}^T/m_1)/r \tag{13}
$$

where $\mathbf{P}$ is the positive definite matrix derived from

$$
\mathbf{PA}^* + \mathbf{A}^{*T}\mathbf{P} - \mathbf{Pbb}^T\mathbf{P}/r + \quad = \mathbf{0} \tag{14}
$$

3.4 Construction of the sliding mode control

The sliding mode control u_s is proposed as

$$
u_s = u_{eq} + u_{nl} \tag{15}
$$

where the first and second controls on the right-hand of Eq. (15) respectively correspond to the equivalent and switching controls. The first control is derived by differentiating σ with the time t and by setting the result to zero. Thus,

$$
\dot{\sigma} = \mathbf{S}_a \dot{\mathbf{x}} + \mathbf{S_b e} = 0 \tag{16}
$$

Substituting Eq. (8) into Eq. (16), the equivalent control u_{eq} is obtained as

$$
u_{eq} = -(\mathbf{S}_a \mathbf{b})^{-1}[\mathbf{S}_a \mathbf{F}(\mathbf{x}) + \mathbf{S}_a \mathbf{d}\xi(t) + \mathbf{S}_b \mathbf{e}] \tag{17}
$$

While, the switching control u_{nl} is proposed as

$$
u_{nl} = -u_{nl}^* \sigma / |\sigma| \tag{18}
$$

where u_{nl} is a positive constant.

However, as the excitation from a road profile $w(t)$ and its time derivative $\xi(t)$ cannot be practically measured, the time variations are estimated by constructing the *VSS* observer. Replacing $w(t)$ and $\xi(t)$ by the estimates $w(t)$ and $\xi(t)$ respectively, and redefining the augmented state vector $\mathbf{x}$ by $\mathbf{x}$ given as $\dot{z}_1 \ z_1 \ \dot{z}_2 \ z_2 \ w(t) \ ^T$, the equivalent control is modified as

$$
u_{eq} = -(\mathbf{S}_a \mathbf{b})^{-1}|\mathbf{S}_a \mathbf{F}(\mathbf{x}) + \mathbf{S}_a \mathbf{d}\xi(t) + \mathbf{S}_b \mathbf{e}| \tag{19}
$$

and the switching control is done as

$$
u_{nl} = -u_{nl}^* \sigma / (|\sigma| + \varepsilon) \tag{20}
$$

Then, the sliding surface is modified as

$$
\sigma = \mathbf{S}_a \mathbf{x} + \mathbf{S}_b \int_0^t \mathbf{e}\, dt \tag{21}
$$

and ε is small positive constant to compensate the chattering of switching control. Then, the sliding control is modified as

$$
u_s = u_{eq} + u_{nl} \tag{22}
$$

The system becomes stable if the trajectory of the response $\mathbf{x}$ is restricted on the sliding surface σ and slides along the surface to the origin. Therefore, the condition becomes

$$
\frac{d}{dt}(\sigma^2/2) < 0 \tag{23}
$$

Using Eqs. (8), (19), (20) and (22), the $\sigma\dot{\sigma}$ as the time derivative of $\sigma^2/2$ becomes

$$
\begin{aligned}
\sigma\dot{\sigma} &= \sigma(\mathbf{S}_a \dot{\mathbf{x}} + \mathbf{S_b e}) \\
&= \sigma\mathbf{S}_a \mathbf{F}(\mathbf{x}) - \sigma\mathbf{S}_a \mathbf{b}(\mathbf{S}_a \mathbf{b})^{-1}(\mathbf{S}_a \mathbf{F}(\mathbf{x}) + \mathbf{S}_a \mathbf{d}\xi(t) + \mathbf{S_b e}) \\
&\quad + u_{nl}\sigma \ (|\sigma| + \varepsilon) + \sigma\mathbf{S}_a \mathbf{d}\xi(t) + \sigma\mathbf{S_b e} \\
&= \sigma\mathbf{S}_a(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{x})) + \sigma\mathbf{S}_a \mathbf{d}(\xi(t) - \xi(t)) \\
&\quad - \sigma\mathbf{S}_a \mathbf{b} u_{nl}\sigma \ (|\sigma| + \varepsilon) \\
&= \sigma S_{a3}(k_2 \ \ m_2)(w(t) - w(t)) + \sigma S_{a5}(\xi(t) - \xi(t)) \\
&\quad - \sigma\mathbf{S}_a \mathbf{b} u_{nl}\sigma \ (|\sigma| + \varepsilon)
\end{aligned} \tag{24}
$$

where

$$
\mathbf{S_a} = [S_{a1} \ \ S_{a2} \ \ S_{a3} \ \ S_{a4} \ \ S_{a5}]
$$

As $\sigma \ (|\sigma| + \varepsilon)$ is approximated as $\sigma \ |\sigma|$,

$$
\begin{aligned}
\sigma\dot{\sigma} &\le |\sigma||S_{a3}(k_2 \ \ m_2)(w(t) - w(t))| \\
&\quad + |\sigma||S_{a5}(\xi(t) - \xi(t))| - |\sigma|\mathbf{S}_a \mathbf{b} u_{nl} \\
&\le |\sigma| C_w + |\sigma| C_\xi - |\sigma|\mathbf{S}_a \mathbf{b} u_{nl}
\end{aligned} \tag{25}
$$

where C_w and C_ξ are positive constants satisfying that

$$
|S_{a3}(k_2 \ \ m_2)(w(t) - w(t))| \le C_w \ , \ |S_{a5}(\xi(t) - \xi(t))| \le C_\xi
$$

Therefore, $\sigma\dot{\sigma}$ becomes negative if u_{nl} is selected as

$$
u_{nl} > (\mathbf{S}_a \mathbf{b})^{-1}(C_w + C_\xi) \tag{26}
$$

3.5 Construction of the observer

Assuming that ($\dot{z}_1$, z_1, $\dot{z}_2$, z_2) can be measured and $w(t)$ cannot be measured in the state vector $\mathbf{x}$, the *VSS* observer to estimate $w(t)$ is constructed. The new *VSS* observer up-dating the estimates $\dot{z}_1$, $\dot{z}_2$ and $w(t)$ for $\dot{z}_1$, $\dot{z}_2$ and $w(t)$, neglecting those z_1 and z_2 of z_1

and z_2 to reduce the capacity of the *VSS* observer, is proposed as

$$\ddot{\hat{z}}_1 = -(c\ \ m_1)(\dot{\hat{z}}_1 - \dot{\hat{z}}_2) - (1\ \ m_1) f(z_1 - z_2) + (1\ \ m_1)u_s + d_1 \tilde{\dot{z}}_1\ \ (|\tilde{\dot{z}}_1| + \varepsilon_1) \quad (27)$$

$$\ddot{\hat{z}}_2 = (c\ \ m_2)(\dot{\hat{z}}_1 - \dot{\hat{z}}_2) + (1\ \ m_2) f(z_1 - z_2) - (k_2\ \ m_2)(z_2 - \hat{w}(t)) - (1\ \ m_2)u_s + d_2 \tilde{\dot{z}}_2\ \ (|\tilde{\dot{z}}_2| + \varepsilon_2) \quad (28)$$

$$\dot{\hat{w}}(t) = -\alpha \hat{w}(t) + d_3 \tilde{\dot{z}}_1\ \ (|\tilde{\dot{z}}_1| + \varepsilon_1) + d_4 \tilde{\dot{z}}_2\ \ (|\tilde{\dot{z}}_2| + \varepsilon_2) \quad (29)$$

In above relations, $\tilde{\dot{z}}_1$ and $\tilde{\dot{z}}_2$ are respectively defined as the estimation errors, $\dot{z}_1 - \dot{\hat{z}}_1$ and $\dot{z}_2 - \dot{\hat{z}}_2$, α is a positive constant to guarantee the numerical stability of $\hat{w}(t)$, $d_1 \sim d_4$ are positive constants to denote the amplitudes of corrective terms for the estimates, and ε_1 and ε_2 are small positive constants to compensate the chattering of the estimates. It is seen that the up-dating Eqs. (27) and (28) for $\dot{\hat{z}}_1$ and $\dot{\hat{z}}_2$ are respectively derived by adding the corrective terms to Eqs. (2) and (5) where $\dot{z}_1$, $\dot{z}_2$ and $w(t)$ are respectively replaced by $\dot{\hat{z}}_1$, $\dot{\hat{z}}_2$ and $\hat{w}(t)$. Therefore, the sliding mode control u_s is practically composed of Eqs. (19) and (20) with the *VSS* observer given by Eqs. (27), (28) and (29).

4. RESULT AND DISCUSSION

The active control u_s is generated at any sampling instant with the time interval 10ms. The parameters characterizing the car model are respectively given by

$$m_1 = 46.16kg,\ m_2 = 13.4kg,\ m_3 = 1.28kg,$$
$$c = 400Ns\ \ m,\ k_1 = 6.8kN\ \ m, k_1 = 20kN\ \ m,$$
$$k_2 = 100kN\ \ m,\ K = 100kN\ \ m,\ a = 1.4mm$$

where c is determined by the free vibration experiment of the car model. The bandwidth of $w(t)$ is denoted as $5Hz$. The weighting factors of J given by (11) are denoted as

$$q_0 = 3\times 10^{-4},\ q_1 = 1,\ q_2 = 0.1,\ q_3 = 0.1,\ q_4 = 1,\ r_1 = 10^{-6}$$

Therefore, $\mathbf{S}_a$ is calculated as

$$\mathbf{S}_a = [708.95 \quad 413.73 \quad -84.638 \quad -6189.3 \quad 5775.5]$$

and $\mathbf{S}_b$ is determined by the experiment as

$$\mathbf{S}_b = [4000 \quad 1000]$$

The parameters characterizing the proposed VSS observer are respectively determined by the experiment as

$$\alpha = 0.01,\ d_1 = 4,\ d_2 = 4,\ d_3 = 0.14,\ d_4 = 0.08,$$
$$\varepsilon_1 = 0.06,\ \varepsilon_2 = 0.06$$

and u_{nl}^* and ε characterizing the switching control given by Eq. (20) are respectively determined by the experiment and the capacity of the pneumatic actuator as

$$u_{nl}^* = 3,\ \varepsilon = 0.06$$

The following six kinds of suspensions are presented to compare the performance:

Method A: Passive suspension
Method : Linear active suspension based on *LQ* control
Method C: Active suspension without compensator where $(q_0 = 0, \mathbf{S}_b = \mathbf{0})$
Method D: Active suspension with compensator where $(q_0 = 0, \mathbf{S}_b = \mathbf{0})$
Method E: Active suspension with compensator where $(q_0 \neq 0, \mathbf{S}_b = \mathbf{0})$
Method F: Proposed active suspension with compensator where $(q_0 \neq 0, \mathbf{S}_b \neq \mathbf{0})$

Table 1 shows *RMS* (Root mean square) values of the responses of the variables; the acceleration $\ddot{z}_1$, the velocity $\dot{z}_1$ and the displacement z_1 of the car body, the suspension deflection $(z_1 - z_2)$, the tire deflection $(z_2 - w)$ and the active control u obtained by using six kinds of methods. It is seen from the table that (i) Methods C~F more improve the *RMS* values of the variables except for u than Methods A and B, (ii) Method D (with compensator) more improves the *RMS* values of the variables except for u than Method C (without compensator) , (iii) Method E$(q_0 \neq 0)$ more improves the *RMS* values of $\ddot{z}_1$, $\dot{z}_1$ and $(z_2 - w)$ than Method D$(q_0 = 0)$, and (iv) Method F ($\mathbf{S_b} \neq \mathbf{0}$) more improves the *RMS* values of the variables except for u than Method E ($\mathbf{S_b} = \mathbf{0}$).

Fig.3 denotes the effectiveness of the proposed VSS observer because the exact and estimated values of $w(t)$ obtained by Method F have good agreement.

5. CONCLUSION

This paper proposed the construction of a pneumatic active suspension system for a one-wheel car model using the sliding mode control with the *VSS* observer. The proposed compensator effectively improve d the degradation of the performance in the vibration suppression due to the delay of the pneumatic actuator. The proposed sliding surface was constructed by adding the term of the time integral of the estimation error of the state variables to the weighted sum of the augmented state variables. The weighted factors wer e calculated by the optimal control theory based on the performance index including the acceleration of the car body to improve the ride comfort. The time variation of the excitation from the road profile was assumed unknown, and it was effectively estimated by using the proposed *VSS* observer. The experimental result showed that the proposed active suspension system much improved the vibration suppression of the car model.

Table 1 *RMS* values of the time response of the variables

	Method A	*Method B*	*Method C*	*Method D*	*Method E*	*Method*	*Unit*
$\ddot{z}_1$	9.98×10^{-1}	8.54×10^{-1}	6.74×10^{-1}	6.05×10^{-1}	5.97×10^{-1}	5.59×10^{-1}	$m \quad s^2$
$\dot{z}_1$	3.93×10^{-2}	3.35×10^{-2}	2.73×10^{-2}	2.46×10^{-2}	2.43×10^{-2}	2.29×10^{-2}	m/s
z_1	2.32×10^{-3}	1.76×10^{-3}	1.63×10^{-3}	1.55×10^{-3}	1.58×10^{-3}	1.42×10^{-3}	m
$z_1 - z_2$	1.67×10^{-3}	1.16×10^{-3}	1.31×10^{-3}	1.28×10^{-3}	1.34×10^{-3}	1.22×10^{-3}	m
$z_2 - w$	4.89×10^{-4}	4.82×10^{-4}	3.48×10^{-4}	3.23×10^{-4}	3.12×10^{-4}	3.05×10^{-4}	m
u	0	26.12	14.43	18.61	19.33	20.39	N

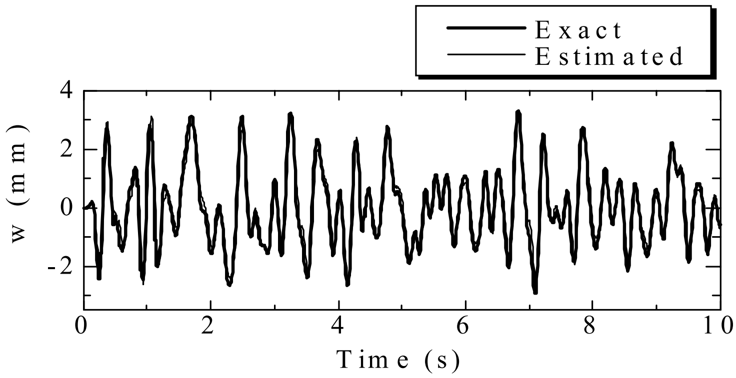

Fig.3 Estimation of the excitation from the road profile

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