# A Sequencing Problem in Mixed-Model Assembly Line Including a Painting Line

J.K. Yoo*, T. Moriyama* and Y. Shimizu*

Department of Production Systems Engineering, Toyohashi University of Technology, Toyohashi, 441-8580, Japan
(Tel: +81-532-44-6652; Fax: +81-532-44-6690, Email:yoo@pse.tut.ac.jp)

**Abstract:** In order to keep production balance at a mixed-model assembly line and a painting line, large WIP(Work-In-Process) inventories are required between two lines. To increase the efficiency of line handling through reducing the inventories under this circumstance, this paper concerns with a sequencing problem for a mixed-model assembly line that includes a painting line where the uncertain elements regarding the defective products exist. Then, we formulate a new type of the sequencing problem minimizing the line stoppage time and the idle time with forecasting the supply time of the products from the painting line. Finally, we examine the effectiveness of the proposed sequencing through computer simulations.

**Keywords:** Line handling, Mixed-model assembly line, Painting line, Sequencing problem, Uncertainly, Forecasting method.

## 1. Introduction

Recently, due to rapidly changing markets, global competition, highly diversified customer demands, shorter production cycle time are required in manufacturing environments. In order to cope with this requirements, it is desirable that low inventory level and high operating rate can be maintained through the efficient production management with line handling. From such a viewpoint, sequencing is recognized as an important stage for raising the efficiency of line handling of the assembly line where mixed-models are assembled every constant cycle time. In the mixed-model sequencing problem, one of the two major goals is to level the workload at each workstation on the assembly line against different assembly time per product model (Miltenburg,1989) [1]. Another one is to keep the constant usage rate of every part at the assembly line (Duplag and Bragg,1998) [2]. Concerns about these two goals have been widely discussed in the literature. For examples, the workload-leveling problem was addressed by Okamura and Yamashina (1978) [3]. Yano and Rachamadugu (1991) concerned with the problem that aims to minimize the risk of assembly line stop [4]. Sumichrast and Russell (1990) discussed the parts-usage smoothing problem [5]. Moreover, the problem to attain these two goals simultaneously was discussed by Korkmazel and Meral (2001) [6].

On the other hand, there is a painting line providing products to the mixed-model assembly line in the manufacture equipment. Generally, it is difficult to keep a proper production corresponding to the demands of the mixed-model asswmbly line when the defective products occur in the painting line. In this situation, line stoppages happen because the painting line is unable to promptly fulfill the demands. For this reason, large WIP (Work-In-Process) inventory becomes necessary for the painting line. But, it is one of factors reducing the efficiency of the line handling. Associated with such production of the painting line, Nagamoto et al., (1998) proposed a method to achieve simultaneously lot production for painting line and production smoothing for assembly line by the installation of two painting lines [7]. However, as a drawback, this approach raises the equipment cost in terms of the additional installation. Monden (1991) introduced the individual sequencing method for the painting line and the assembly line to correct the sequence disturbed by wastrels in the painting line [8]. This method requires the storage facility that adjusts the sequence of product injection at the assembly line.

From the above statements, reduction of the WIP inventory is essential to achieve a rational production in the two lines. In this paper, we concern with a new sequencing problem for the mixed-model assembly line in order to reduce the WIP inventory as well to attain the above two major goals, simultaneously. For this purpose, we formulate the sequencing problem as an optimization problem minimizing the weighted sum of the line stoppage time and the idle time by the un-achievement of the above two goals and the production unbalance between the two lines. In formulating this problem, we consider that the line stoppage time and the idle time regarding the two goals occur when leveling of the workload or the parts usage collapses at the mixed-model assembly line. In order to calculate the line stoppage time by the unbalance between the two lines, we must accurately predict the supply time, i.e., the complete time of the product models at the painting line. From this fact, after predicting the supply time including the uncertain correction time of the defective products, we give the line stoppage time regarding the unbalance using the Goal Chasing method [8]. Then, we use SA (Simulated Annealing) [9] to solve the combinational optimization problem involved in the injection sequencing at the mixed-model assembly line. Effectiveness of our sequencing is verified through some numerical experiments.

## 2. Formulation by an Optimization Model

Figure 1 shows an example of the mixed-model assembly line including the painting line where each product is inserted from the body-line every interval of cycle time ($CT$). The painting line is operated in the order of sub-painting, main painting and check processes. The re-painting products pass through main painting process twice. Defective products are to be the buffer after correction. From buffer, a necessary product is subtracted corresponding to an injection sequence of the mixed-model assembly line where $K$ workstations are linked with a conveyor moving at constant speed. In one workstation, one operator (called just $Worker$ hereinafter) is exclusively assigned. Different parts are used to assemble into product models.

We further assume the following conditions.

[Assumptions]

For the painting line.

**1** Defective product about painting color exist, and its' generation time is unknown.

**2** Time required for correction of defective products is not fixed.

For the mixed-mode assembly line.

**3** Total number of product model is $I$.

**4** The maximum number of parts used on the workstation is $M$.

**5** $Worker$s are confined to their workstation ($k = 1, ..., K$) during assembly work, and their working time do not exceed $CT$.

**6** $Worker$s stop the conveyor whenever they fail to complete the assembly works within their workstations.

**7** The line stoppage occurs due to the part shortages, the work delays of $Worker$ and the product shortages as well.

**8** Idle time occurs when the assembly time of $Worker$ is less than $CT$.

For the two lines.

**9** There exists a buffer as the WIP inventory between the painting line and the assembly line.

**10** The two lines are composed of consecutive conveyor lines.

**11** The production lead-time of the painting line is longer than that of the assembly line.

After all, our sequencing problem can be formulated as follows:

$$\min_{\pi \in \Pi} z \quad = \quad \rho_p \times B^t + \rho_a \times \sum_{t=1}^{T} \max_{1 \le k \le K} (P_k^t, A_k^t)$$

$$+ \quad \rho_w \times \sum_{t=1}^{T} \sum_{k=1}^{K} W_k^t \qquad (1)$$

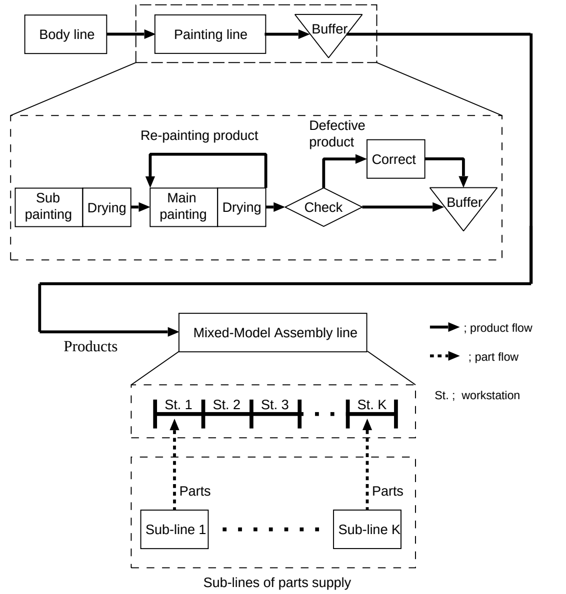


Fig. 1. Scheme of mixed-model assembly line and painting line model

s.t

$$\sum_{i=1}^{I} X_i^t = 1, \qquad t = 1, ..., T \qquad (2)$$

$$\sum_{t=1}^{T} X_i^t = d_i, \qquad i = 1, ..., I \qquad (3)$$

$$g_k^t \ge (t-1) \times CT, \quad k = 1, ..., K, \ t = 1, ..., K \quad (4)$$

The notation is summarized in the following.

$\Pi$ : set of sequence ($\pi \in \Pi$)

$B^t$ : line stoppage time by the product shortage at injection period $t$ (=1,...,T).

$P_k^t$ : line stoppage time by the part shortage in workstation $k$ at injection period $t$.

$A_k^t$ : line stoppage time by the work delay of $Worker$ when the workload exceed $CT$ in workstation $k$ at injection period $t$.

$W_k^t$ : idle time of $Worker$ in workstation $k$ at injection period $t$

$g_k^t$ : work starting time of $Worker$ in workstation $k$ at injection period $t$

$X_t^i$ : 0-1 variable that takes 1 if the product model $i$ is supplied to the assembly line at injection period $t$, otherwise, 0

$d_i$ : total production number of product model $i$ through all injection periods.

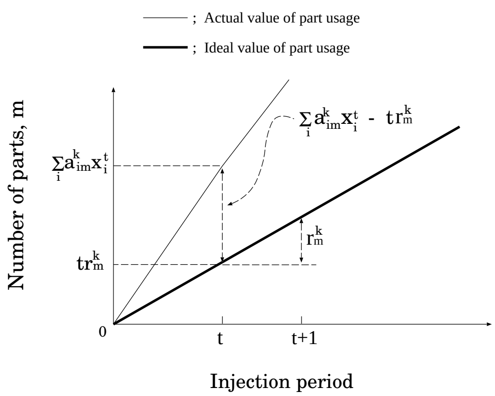


Fig. 2. Calculation of line stoppage time based on goal chasing method

The objective function Eq.(1) is given as the weighted sum of the line stoppage time and the idle time. Where $\rho_p$, $\rho_a$ and $\rho_w$ are weighting factors ($0 < \rho_p, \rho_a, \rho_w < 1$). Sequence $\pi$ is the decision variable of this problem. Among the constraints, Eqs.(2) and (3) are the conditions of that two or more product cannot be assembled simultaneously and that the production number of each product model is guaranteed, respectively. Eq.(4) is regarding the starting time of $Worker$'s assembly work. Moreover, each time will be described below in detail.

Figure 2 illustrates a feature that the part shortage will happen on the workstation $k$ when the quantity of part $m$ used actually ($\sum_i a_{im}^k x_i^t$) exceeds its ideal quantity ($tr_m^k$) at the injection period $t$. In this case, $P_k^t$ is given as Eq.(5):

$$P_k^t = \max(\max_{1 \le m \le M}(\frac{\sum_{i=1}^{I} a_{im}^k x_i^t - tr_m^k}{r_m^k} CT), 0), \qquad (5)$$

where $a_{im}^k$ is the quantity of part $m$ required per model $i$, $x_i^t$ the cumulative amount of production for model $i$ during from injection period 1 to $t$, and $r_m^k$ is the ideal usage rate of part $m$.

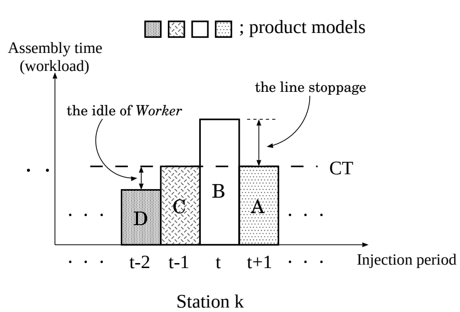


Fig. 3. Line stoppage due to workload

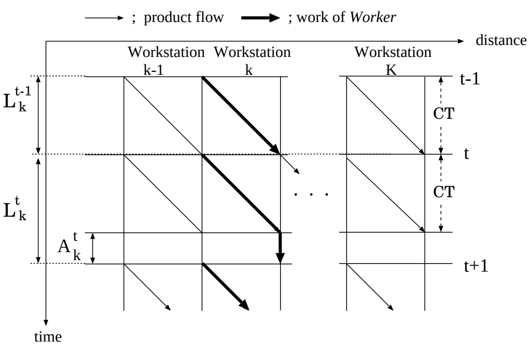


Fig. 4. Line stoppage due to work delay of $Worker$

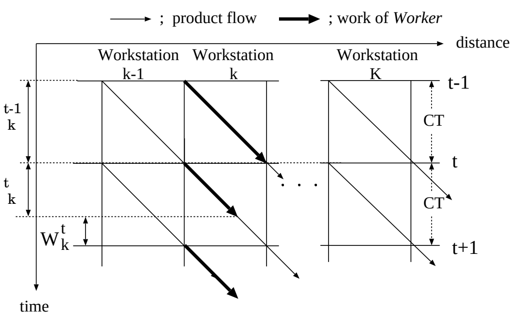


Fig. 5. Idle time of $Worker$

As a simple example of how line stoppage or $Worker$'s idle with regard to workloads occurs, we consider the example (see Figure 3) where the product models (A∼D; workloads differ reciprocally) are put into workstation $k$ every injection period. Since the assembly time by the workload exceeds $CT$ at injection period $t$, the line stoppage occurs, whereas $Worker$'s idle happens at $t-2$. From inspection of this example, the line stoppage time $A_k^t$ and the idle time $W_k^t$ are given by Eqs.(6) and (7) (see Figures 4 and 5).

$$A_k^t \quad = \quad \max(L_k^t - CT, 0), \qquad (6)$$

$$W_k^t = \max(CT - L_k^t, 0), \qquad (7)$$

where $L_k^t$ denotes the working time of $Worker$ on workstation $k$ at injection period $t$.

Considering that the product models from the painting line can handle as the parts from sub-line in the mixed-model assembly line, we give the line stoppage time $B^t$ by these shortages like Eq.(8).

$$B^t = \max(\frac{x_i^t - tr_{pi}}{r_{pi}} CT, 0), \qquad (8)$$
$$t = 1, ..., T, \qquad i = 1, ..., I,$$

where $r_{pi}$ is the supply rate of product model $i$ from the painting line over the whole injection periods, and is a

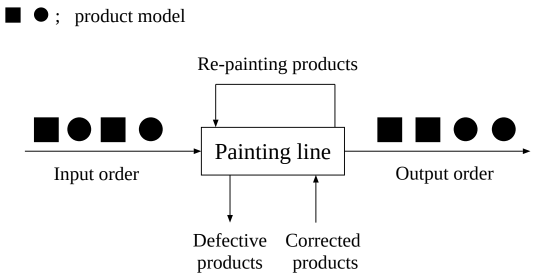


Fig. 6. Variation of input order by painting production

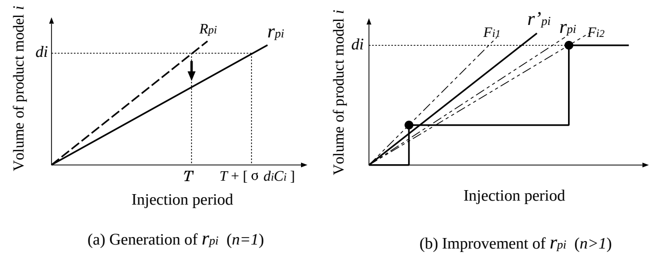


Fig. 7. Forecast scheme of $r_{pi}$ and $r_{pi}^{'}$

value estimated from an input order for the reason of the assumption 11. That is, Eq.(8) means the time gap between the injection time of the product model $i$ on the first workstation of the assembly line and the time which the same model can be supplied from the painting line. As stated above, in the painting line, product models are completed in a different order from an input order, due to the frequency and length of time required to make corrections to defective products (see Figure 6). After sufficiently considering this fact, we give $r_{pi}$ like Eq.(9) based on a prediction that the complete time of defective products is extended by the cause of the correction time.

$$r_{pi} = \frac{d_i}{T + [\sigma d_i C_i]}, \qquad i = 1, ..., I, \tag{9}$$

where $\sigma$ is the defective rate of the products at the painting line, $C_i$ the time required to make corrections against the defective product model $i$, and [ ] is Gauss symbol.

Furthermore, we improve $r_{pi}$ according to the following procedures every production period ($n = 1, ..., N$), in order to raise the accuracy of predictions.

Step1: Forecast $r_{pi}$ from the input order of the painting line at $n = 1$ (see Figure 7(a)).

Step2: After an injection of the production period $n$ is completed, get the quantity and the complete time (called just *delivery information* hereinafter) about product model $i$ which exists in the buffer. And $n = n + 1$.

Step3: Improve $r_{pi}$ based on the *delivery information* of model $i$ acquired at $n - 1$ (see Figure 7(b)).

- Step3-1: Generate the supply rate $F_{ij}$ ($j = 1, 2, ...$) according to the injection period in which model $i$ put into the buffer.
- Step3-2: Let the average of $F_{ij}$ and $r_{pi}$ be the supply rate $r_{pi}^{'}$ of the model $i$ at $n$.

Step4: If $n = N$, stop. Otherwise, go back to Step2.

## 3. Numerical Experiments

Numerical experiments are taken place under the conditions shown in Table 1. As well, we used approximate means like SA (Simulated Annealing ) as an effective solution method in order to cope with the actual sequencing problem. And, the weighting factors $\rho_p$, $\rho_a$ and $\rho_w$ in Eq.(1) are set at 0.5, 0.4 and 0.1, respectively. Moreover, we evaluated the results on the basis of average over 100 data sets generated randomly.

Table 1. Input parameter

| | |
|---|---|
| Cycle time, $CT$[min] | 5 |
| Station number, $K$ | 100 |
| Product model, $I$ | 10 |
| Total production number, $\sum_i d_i$ | 100 |
| Injection period, $T$ | 100 |
| Production period, $N$ | 30 |
| Defective rate | 0.2 |
| Correction time[min] | 15〜25 |

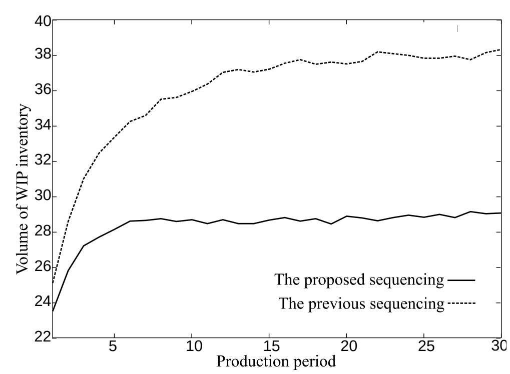


Fig. 8. The change of WIP inventory volume every production period

To examine the effectiveness of the proposed sequencing regarding the value of the WIP inventories between the painting line and the assembly line through computer simulations, we compared our results with those of a previous sequencing in which did not take into consideration the elements in the painting line. From Figure 8, the proposed sequencing is known to realize the efficient line management, i.e. drastic decrease in the WIP inventories. It is also observed that the inventory in the proposed sequencing is maintaining an almost fixed quantity

Table 2. Comparison between proposed method and previous method regarding sequencing strategy

| | WIP inventory volume | Line stoppage time[min] | Idle time[min] |
|---|---|---|---|
| Proposed*1 | 28.7 | 43.7 | 4.2 |
| Previous*2 | 37.5 | 31.2 | 4.1 |

1; the proposed sequencing. 2; the previous sequencing.

in contrast to that in the previous sequencing increasing over production period. These mean that our prediction scheme about the product supply time of the painting line can realize reducing of the WIP inventories.

Table 2 shows the effectiveness of the proposed sequencing regarding the WIP inventory volume, line stoppage time and idle time. Here, the WIP inventory volume means the average value under the condition that the line stoppage by the product shortage must not occur, line stoppage time and idle time mean the values that happen by non-leveling of the parts usage or the workloads at the assembly line. As supposed a priori, regarding the WIP inventory, while the proposed sequencing is superior to the previous sequencing, it is inferior a little in terms of the line stoppage time and the idle time. From the result, we can recognize that the proposed sequencing of aiming at reduction of the WIP inventory between two lines is more effective in raising the efficiency of the whole manufacture lines.

## 4. Conclusion

In this paper, we have considered a new sequencing approach at the mixed-model assembly line in order to increase the efficiency of line handling through reducing the WIP inventory as well to attain the two major goals, i.e. leveling the workload and maintaining constant parts usage rates, simultaneously. For this purpose, we formulated the sequencing problem as a combinatorial optimization problem minimizing the line stoppage time and the idle time through the achievements of the above two goals and the production balance between the assembly line and the painting line. In formulating this problem, we given a forecasting scheme regarding the supply time of product models from the painting line where there are the uncertainty elements, i.e. the frequency and length of time required to make corrections to defective product-models. Through the experiments, we obtained the results that although the case of this approach is slightly inferior compared with the previous sequencing in regards to smoothing of the part usage or the workloads; it is excellent in relation to the WIP inventory. After all, we can confirm that this approach provides a viable method to improve the efficiency of the line handling as well as reduce the WIP inventory between the two lines.

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