Hybrid Controller of Neural Network and Linear Regulator for Multi-trailer Systems Optimized by Genetic Algorithms

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Abstract: A hybrid control scheme is proposed for the stabilization of backward movement along simple paths for a vehicle composed of a truck and six trailers. The hybrid comprises the combination of a linear quadratic regulator (LQR) and a neurocontroller (NC) that is trained by a genetic algorithm (GA). Acting singly, either the NC or the LQR are unable to perform satisfactorily over the entire range of the operation required, but the proposed hybrid is shown to be capable of providing good overall system performance. The evaluation function of the NC in the hybrid design has been modified from the conventional type to incorporate both the squared errors and the running steps errors. The reverse movement of the trailer-truck system can be modeled as an unstable nonlinear system, with the control problem focusing on the steering angle. Achieving good backward movement is difficult because of the restraints of physical angular limitations. Due to these constraints the system is impossible to globally stabilize with standard smooth control techniques, since some initial states necessarily lead to jack-knife locks. This paper demonstrates that a hybrid of neural networks and LQR can be used effectively for the control of nonlinear dynamical systems. Results from simulated trials are reported.

Keywords: Hybrid controller, LQR, Neurocontroller, Genetic algorithm, Trailer backward movement control.

1. Introduction

The control problem for the trailer truck system is known to be one of the highly non-linear, multi-variable and unstable control problems. Difficult in control intensifies with increase in the number of trailers. The system presents saturations on the steering angle and on the relative angles between the trailers. These constraints present a challenging problem of unstable nonlinear dynamics. The control task is to back up the vehicle to a desired trajectory without getting out of control and without expriencing jack-knife locks.

Soft computing techniques, notably Fuzzy technology and Neural computing have found major applications in engineering design and manufacturing [1]. Unlike fuzzy methods that use 'vague' data sets, neural networks are well adapted to nonlinear systems due to their ability to learn [2]. They are well suited for the trailer backward control problem as learning is enhanced by the availability of quantifiable training data. Further, suitable genetic algorithms (GAs) can be modeled to offer a powerful training base [3].

Many control systems for the problem have been proposed. Earlier works by Tanaka *et al.* [4,5] have shown that fuzzy control exhibits good control performance. With regard to the neurocontrol system, Nguyen and Widrow [6] reported the pioneering work for the neurocontrol application on the trailer-truck system. They successfully designed a controller utilizing a back-propagation (BP) algorithm. Jenkins and Yuhas [7] have presented a small-sized neurocontroller (NC), also based on BP. In previous studies, Kinjo *et al.* proposed various control methods for a single trailer-truck combination, for a truck connected to two trailers and for a five trailer-truck combination, using NCs evolved by GA [8-10]. Though evolution of the NCs for the designs were successful, the methods are computationally expensive.

In this study, we present the concept of a hybrid control system comprising the linear quadratic regulator (LQR) for the linear part and the NC for the nonlinear part. Our main motivation derives from the fact that the aforementioned methods fall short of globally stabilizing the control object. The LQR alone can not offer the full control performance due to the nonlinear characteristics of the control object. Likewise, the NC on its own is not very efficient as the method takes many training times to evolve the NCs and sometimes the evolution fails. To tackle these problems our proposed hybrid utilizes a synthesis of both control outputs of the LQR and NC. Our prime focus is twofold: to control the steering angle such that the system does not yield the out of control states, and minimizing the physical limitations of the angular differences thereby avoiding the jack-knife phenomenon.

The remainder of this paper is organized as follows. The model of the trailer-truck system is presented in Section 2. Section 3 explains the hybrid concept that incorporates the LQR and NCs. Simulation results are shown in Section 4 while a discussion on performance of the scheme follows in Section 5. We draw some general conclusions and our perspectives in Section 6.

2. Modeling and Problem Formulation

2.1. The Trailer-truck Model

Figure 1 details the geometry of our control object: the six trailer-truck model with an actuated front steering, and its orientation in a coordinate base. Table 1 explains the terminology used for the trailer-truck system parameters. The steering angle $u(t)$, is the input to the system and is determined by the state variables $x_1 - x_{13}$. The kinematics of the system are described by the set of equations (1) - (15).

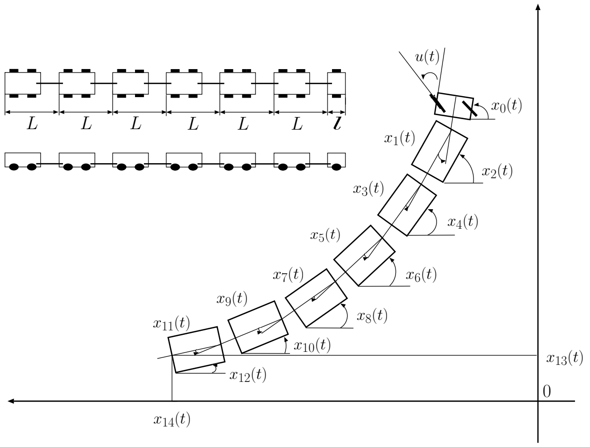


Fig. 1. Schematic model of the trailer-truck

Table 1. Parameters of the trailer-truck system

Quantity	Description
l	Truck length
L	Trailer length
Δt	Sampling time
v	Speed of system
$u(t)$	Steering angle
$x_0, x_2, x_4, x_6,$ x_8, x_{10} and x_{12}	Angles of truck, 1st, 2nd, 3rd, 4th, 5th and 6th trailers
	Angular Differences, between:
x_1	Truck and 1st trailer
x_3	1st and 2nd trailer
x_5	2nd and 3rd trailer
x_7	3rd and 4th trailer
x_9	4th and 5th trailer
x_{11}	5th and 6th trailer
x_{13}	Vertical position of 6th trailer
x_{14}	Horizontal position of 6th trailer

$$x_0(t+1) = x_0 + \tfrac{v\Delta t}{l}\tan[u(t)] \tag{1}$$

$$x_1(t) = x_0(t) - x_2(t) \tag{2}$$

$$x_2(t+1) = x_2(t) + \tfrac{v\Delta t}{L}\sin[x_1(t)] \tag{3}$$

$$x_3(t) = x_2(t) - x_4(t) \tag{4}$$

$$x_4(t+1) = x_4(t) + \tfrac{v\Delta t}{L}\sin[x_3(t)] \tag{5}$$

$$x_5(t) = x_4(t) - x_6(t) \tag{6}$$

$$x_6(t+1) = x_6(t) + \tfrac{v\Delta t}{L}\sin[x_5(t)] \tag{7}$$

$$x_7(t) = x_6(t) - x_8(t) \tag{8}$$

$$x_8(t+1) = x_8(t) + \tfrac{v\Delta t}{L}\sin[x_7(t)] \tag{9}$$

$$x_9(t) = x_8(t) - x_{10}(t) \tag{10}$$

$$x_{10}(t+1) = x_{10}(t) + \tfrac{v\Delta t}{L}\sin[x_9(t)] \tag{11}$$

$$x_{11}(t) = x_{10}(t) - x_{12}(t) \tag{12}$$

$$x_{12}(t+1) = x_{12}(t) + \tfrac{v\Delta t}{L}\sin[x_{11}(t)] \tag{13}$$

$$x_{13}(t+1) = x_{13}(t) + v\Delta t \times \cos[x_{11}(t)] \times \sin[\tfrac{x_{12}(t+1)+x_{12}(t)}{2}] \tag{14}$$

$$x_{14}(t+1) = x_{14}(t) + v\Delta t \times \cos[x_{11}(t)] \times \cos[\tfrac{x_{12}(t+1)+x_{12}(t)}{2}] \tag{15}$$

2.2. Problem Formulation

The steering angle $u(t)$ is to be controlled such that the system is asymptotically stabilized along a desired trajectory, for our case, along the straight line $x_{13}(t)$=0. This requires that the relative angles, angle of last trailer and the vertical position are kept to a minimum, thus

$$X(t) = [x_1(t),\ \ x_3(t),\ \ x_5(t),\ \ x_7(t),\ \ x_9(t),\ \ x_{11}(t),\ \ x_{12}(t),\ \ x_{13}(t)]^T \to 0.$$

The expressions (1) - (15) have the form of time-dependent state equations and by setting the initial conditions, ongoing nominal values of the position and orientation of the vehicle are produced. This restricts the controlled statement to only the truck steering angle $u(t)$.

3. Control System

3.1. The Hybrid Controller

Figure 2 shows the Hybrid control system. The stabilizing controller of the hybrid is based on both the LQR, which covers the linear part, and the NC that covers the nonlinear part of the system. X^{ref} is the reference for the state variables while GA denotes the genetic algorithm procedure.

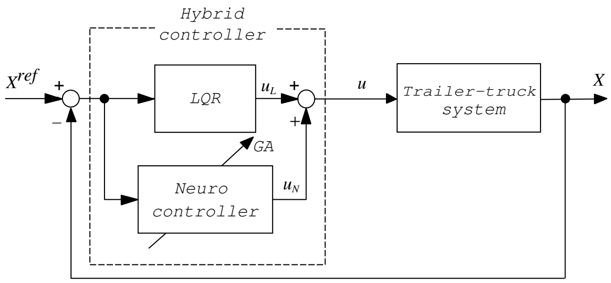


Fig. 2. Hybrid control system

The LQR and NC receive the error of angles $x_1, x_3, x_5, x_7, x_9, x_{11}, x_{12}$ and position x_{13} as inputs and output the steering angles $u_L(t)$ and $u_N(t)$, respectively. The input to the system is the hybrid steering angle $u(t)$, comprising the outputs from the LQR and the NC respectively,

$$u(t){=}u_L(t){+}u_N(t) \tag{16}$$

The system outputs the state vector X. Considering that this is a regulator problem, necessarily $X^{ref} = 0$.

3.2. LQR Controller

We used the linear quadratic regulator from optimal control theory [11,12] to solve the linear part of the design problem in which the state is accessible. The stochastic formulation of the LQR design problem for this system is linearized and described by

$$X(t+1) = AX(t) + Bu_L(t) \tag{17}$$

where A and B are linearized parameters of the nonlinear system described by Eqs.(1) - (15). A and B are obtained from the assumption that the angular differences and angle of last trailer have magnitudes

$$[x_1, x_3, x_5, x_7, x_9, x_{11}] \ll 1 \ \text{ and } \ x_{12} \ll 1 \tag{18}$$

respectively.

The LQR cost function is the sum of the steady-state mean-square weighted state X, and the steady-state mean-square weighted actuator signal $u_L(t)$:

$$J = \sum_{t=0}^{\infty} \{X(t)WX(t) + wu_L(t)^2\} \tag{19}$$

where W and w are positive semidefinite weight matrices; the first term penalizes deviations of $X(t)$ from zero, and the second represents the cost of using the actuator signal. One of the methods of LQR design is by use of the system gain to control the system errors. For our case the control gain G is obtained by the Riccati equation. Thus, linear output of the discrete system is:

$$u_L(t) = -GX(t) \tag{20}$$

3.3. Construction of the NC

The NC covers the nonlinear part of the system. We used a 3-layered, 8-5-1 configuration neural network with nonlinear activity functions for both the hidden and output layers. For the hidden layer we utilized the Sigmoid function:

$$f(x) = \frac{1}{1-e^{-x}} \tag{21}$$

while the activity function of the output layer is a cubic,

$$f(x) = ax^3 \tag{22}$$

With this function, when the magnitude of the state variable X is small, the control object tends to be linear and it is easy to output $u_N = 0$.

3.4. Training of the NNs with Real-coded GA

Training the neural network involved setting the position and orientation of the system, initializing the backward control and evaluating the errors. Table 2 gives the initial configurations of the starting positions chosen by us. The number of patterns used, P=9.

Table 2. Initial starting configurations

Pattern No.	x_0, x_2, x_4, x_6, x_8 x_{10} and x_{12}[rad]	x_{13} [m]
1	0	0.0
2	$\pi/4$	
3	$\pi/2$	
4	0	3.0
5	$\pi/4$	
6	$\pi/2$	
7	0	6.0
8	$\pi/4$	
9	$\pi/2$	

The methodology of training the NCs is as follows. First we randomly set the connecting weights of the neural network. The trailer-truck is set to an initial configuration, such as pattern no.1 in Table 2. The truck backs up using the NC, undergoing several individual cycles of backing up until it stops on attaining a preset number of running steps, or gets out of control. The final error of the trailer-truck system is recorded. This error is a function of the state variables and the running steps. Next, the trailer-truck is placed in another initial configuration, say, pattern no.2 and let to back up until it stops. On completion of control trials from all the nine configurations $P = 9$, the control performance of the NC is evaluated, as explained in subsection 3.5. All the NCs in the population are evaluated in the same fashion.

We used a real-coded GA [13] to train the neural network and obtain the best individual from among the evolved NCs. This involved use of the GA to evaluate and adjust the connecting weights appropriately. Our GA procedure relies on the Blend crossover (BLX) method. The BLX tool utilizes interval schemata and has been shown to have good training characteristics for neural networks by real-coded GAs. We employed a real-coded GA for two reasons: the range of the connecting weights' values is unbounded and quantization errors normally associated with bit-string GAs are non-existent.

3.5. Evaluation Function of the NC

The evaluation function E for the NC training by the GA consists of the squared errors E_S, and the cumulative running steps error E_T when system is out of control, and is computed from the relation:

$$E = \alpha E_S + \beta E_T \tag{23}$$

where α and β are weights of the squared errors and running steps errors respectively. The first term gives the squared errors accumulated for all the patterns, P

$$E_S = \sum_{p=1}^{P} E_p \tag{24}$$

and the squared error per pattern E_p, is evaluated from the following expression

$$\begin{aligned} E_p = {} & q_1(x_1^{ref} - x_{1p}^{end})^2 + q_3(x_3^{ref} - x_{3p}^{end})^2 \\ & + q_5(x_5^{ref} - x_{5p}^{end})^2 + q_7(x_7^{ref} - x_{7p}^{end})^2 \\ & + q_9(x_9^{ref} - x_{9p}^{end})^2 + q_{11}(x_{11}^{ref} - x_{11p}^{end})^2 \\ & + q_{12}(x_{12}^{ref} - x_{12p}^{end})^2 + q_{13}(x_{13}^{ref} - x_{13p}^{end})^2 \end{aligned} \tag{25}$$

where x^{ref} is the reference variable and x_p^{end} is the final value of the state variable which starts from any initial configuration of p. The factor q is the weight of the squared error function and adjusts the importance of the control variables.

The second term E_T, in Eq. (23) refers to the cumulative running steps when the system is in the out of control states

$$E_T = \sum_{p=1}^{P} (t_{max} - t_p) \tag{26}$$

where t_{max} is the maximum steps set for the design and t_p refers to the running steps from initial configuration per pattern p, prior to the out of control state. Ideally, $E_T = 0$ when no pattern exhibits the out of control state.

A neurocontroller is considered highly evolved when E_S is very small and a large quantity of running steps t_p on the desired trajectory is realized.

4. Simulations and Results

4.1. Parameters

4.1.1 The 6 Trailer-truck Model

The modeling of the six trailer-truck system is based on the quantities in Table 3.

Table 3. Parameter values for the trailer-truck system

Truck length, l	0.3m
Trailer length, L	1.0m
Velocity of system, v	-0.2m/s
Sampling time, Δt	0.25s

For the state and input constraints of the system we set the following limit for the steering angle:

$$|u| \leq \pi/2 \text{ rad} \tag{27a}$$

and for the relative angles:

$$|x_1, x_3, x_5, x_7, x_9, x_{11}| \leq \pi/2 \text{ rad} \tag{27b}$$

A consequence of the latter constraints in Eq. (27b) is the appearance of the jack-knife configurations, corresponding to at least one of the relative angles reaching its saturation value, $\pi/2$ rad. Then the truck is not able to push the trailer backwards anymore.

4.1.2 Design of LQR and NC

In solving the Riccati equation, the diagonal of the matrix W in Eq. (19) may be set variously, to achieve the optimum gain. In our case W is weighted such that W=diag[1,1,1,1,1,1,1,100], and w=100. With $u_L(t)$ as given in Eq. (20), we used the following values of the gain

$$\begin{aligned} G = [&-4.01, \quad 22.87, \quad -71.72, \quad 132.04, \\ &-138.49, \quad 68.86, \quad -3.76, \quad 0.72 \ ] \end{aligned}$$

Notice that some of the G term values may be large thus by Eq. (20) the output $u_L(t)$, the steering angle, may exceed the physical limitations of the system leading to the trailer-truck getting out of control.

For the construction of the NCs, we determined and set $a = 0.1$ in Eq. (22) by trial and error. The output neuron function was tried for $f(x) = ax^i$, i=1,2,3 and $f(x) = ax^3$ had the best performance.

4.1.3 GA Parameters and Evaluation function

A real-coded GA was employed in the NN training, with properties as shown in Table 4. The range factor for the interval in the BLX scheme is 0.8.

Table 4. Constant Parameters of the GA

Parameter	Value/method
Population	50
No. of offspring	30
Selection scheme	Roulette wheel
Crossover	BLX

For the evaluation function of the NCs used in the GA as given in Eq. (23), we set $\alpha = 1.0$ and $\beta = 1.0$.

In Eq. (25) we set $\{q_1, q_3, q_5, q_7, q_9, q_{11}, q_{12}\}$=1.0 for the associated variables x_1, x_3, x_5, x_7, x_9, x_{11}, x_{12}, and q_{13}=0.1 for x_{13}.

We set the running steps limit in Eq. (26), $t_{max} = 600$.

4.2. Design Performance

In Table 5, results of our investigation on trucks with 4, 5 and 6 trailers and the success rate of the evolution of the controllers are presented. The criterion for successful evolution is the percentage of NCs in 3000 generations with error $E \leq 0.001$.

Firstly, we observe that optimal performance is degraded in either controller as the number of connected trailers increases. Secondly, controller design with the NC only is not successful with $\beta = 0$; though the NC improves with $\beta = 1.0$, it does not yield a suitable controller for the six trailer case.

Table 5. Success rate of training with either Hybrid or NC controller, on 4, 5, and 6 trailers. ($\alpha = 1.0$).

No. of trailers	Success Rate, [%]			
	Hybrid controller		Only NC	
	$\beta = 1.0$	$\beta = 0.0$	$\beta = 1.0$	$\beta = 0.0$
4	90	69	78	0
5	81	31	6	0
6	20	0	0	0

Generally, the conventional neurocontroller that utilizes the NC only with $\beta = 0$ fails to evolve a controller while the hybrid controller offers the best performance. The tabulated results show that modifying the evaluation function to include the running steps error term improves the design for either the hybrid controller or the NC acting singly. It is clear that the hybrid controller with $\beta = 1.0$ is the most suitable design for the six trailer-truck configuration.

Figure 3 shows one of the successful evolutions of the hybrid controller when $\alpha = 1.0$ and $\beta = 1.0$. The system is designed for 3000 generations. The squared error E_S forms part of the evaluation function E of the hybrid controller. It is observed that it is relatively small throughout the evolution. At 300 generations the running steps error $E_T = 0$, thus the hybrid is able to control all the state variables of the six trailer-truck system for the maximum running steps from all the patterns. Beyond 300 generations only the squared errors E_S are minimized.

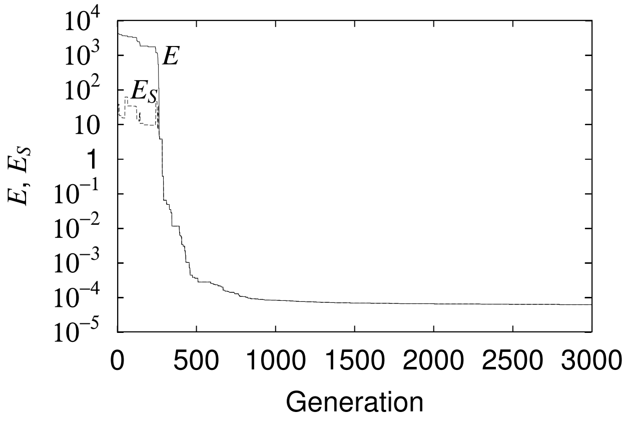


Fig. 3. Effect of generational training of the NCs on system errors E_S, E. ($\alpha = 1.0$, $\beta = 1.0$).

4.3. Control Performance

Figure 4 is an example of the controlled results, for pattern no. 9, where the initial orientation and starting vertical position are set to 90 degrees and 6.0m respectively. It is observed that the hybrid controller is able to control the trailer-truck within the training area [-5.0, 10.0]m along x_{13} successfully.

Figure 5 shows that the hybrid controller optimally controls the state variables. Fig. 5 (a) shows variation of the relative angles ($x_1, x_3, x_5, x_7, x_9, x_{11}$) and angle of last trailer x_{12} with running steps. We see that the angles are controlled

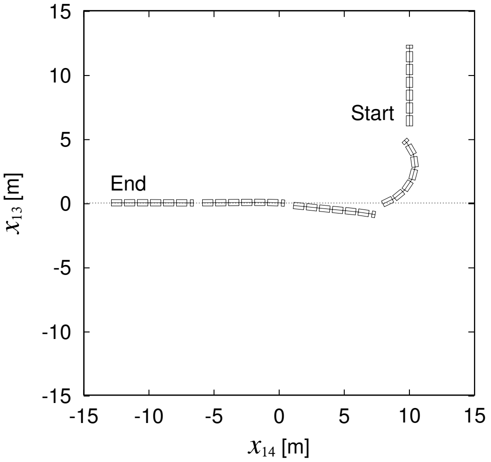
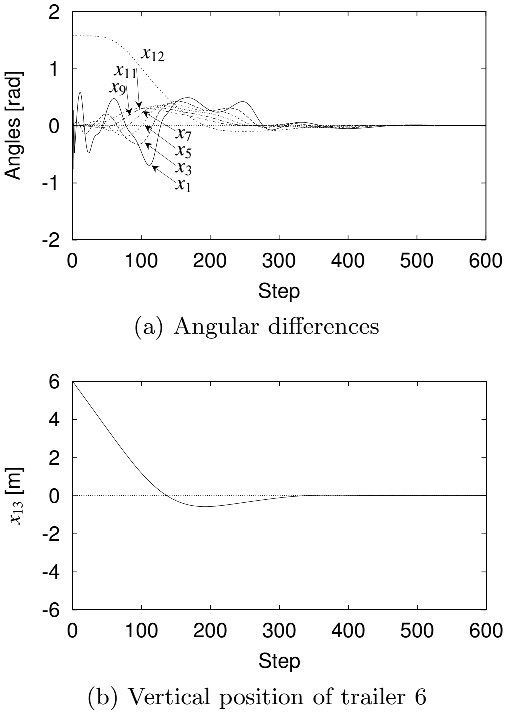


Fig. 4. Trajectory for pattern no. 9. Display interval: 140 time steps.

successfully by the hybrid controller design to within the set range $[-\pi/2,\ \pi/2]$ rad. Fig. 5(b) gives the vertical position x_{13} of the last trailer. It is well within the training range. Generally, at 300 steps all the state variables are very small and the system eventually realigns and stabilizes at the desired path x_{13}=0.

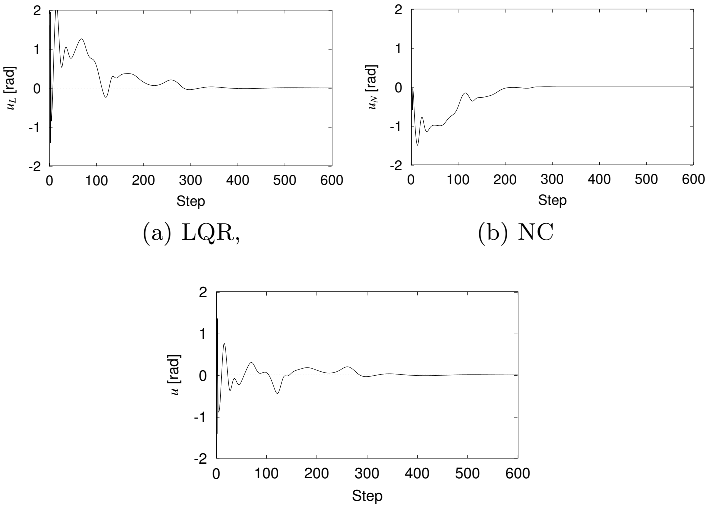


(a) Angular differences

(b) Vertical position of trailer 6

Fig. 5. Variation with running steps

Figures 6(a), (b) and (c) show, respectively, the variation of the LQR, NC and hybrid controller outputs $u_L(t)$, $u_N(t)$ and $u(t)$, with running steps. We observe that the LQR output is initially out of range but stabilizes after a while. At about 200 running steps $u_N(t)$ is practically zero and only the LQR is effective. The hybrid output $u(t)$ does not exceed the set range $[-\pi/2, \pi/2]$ of the steering angle since the effect of out of range $u_L(t)$ is regulated by the NC.



(a) LQR,

(b) NC

(c) Hybrid

Fig. 6. Controller outputs

4.4. Control performance for the initial conditions

We investigated the workable ranges for both the orientation angle and the starting vertical position for the hybrid controller as depicted in Figures 7 and 8. The 3 vertical lines denote the trained initial angles or positions while the Squared error value, 10^4, refers to the out of control state. In Figure 7, starting vertical position is 3.0m and training values of the angles are 0, $\pi/4$, and $\pi/2$ rad. It is observed that there is a wider range of untrained start angles $[-3\pi/16, \pi/2]$. Figure 8 shows that the hybrid offers a wider operating range [-3.0, 9.0]m, for the starting vertical position. Orientation is $\pi/4$ rad and training values are 0.0, 3.0 and 6.0m.

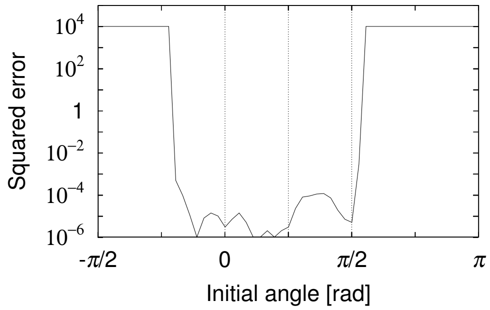


Fig. 7. Control performance for initial angles

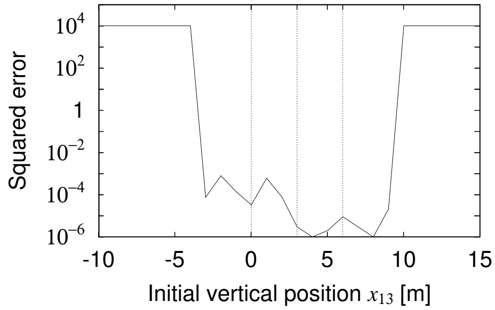


Fig. 8. Control performance for initial vertical position

5. Discussion

Table 5 forms the basis of our research. We observe the difficult in designing a controller as the number of trailers increases. Notably, using only the NC fails to control the 6 trailer-truck configuration. Our proposed hybrid controller, comprising the LQR to cater for the linear part and the NC for the nonlinearity, is suitable for the model. The evaluation function of the NC is modified to include both squared errors E_S and those due to out of control running steps E_T.

For the hybrid system, we designed the LQR and NC controllers separately. We optimally chose the best gain factor generated by use of the Riccati equation for the LQR, while for the NC our criterion was based on evaluating both E_S and E_T. We used trial and error methods to get the best combination of the weight matrix W in Eq. (19) for the LQR and the weight q for the NC in Eq. (25). The respective values of weight, 100 for the position magnitude in W and $q_{13} = 0.1$ gave the best results in our simulations. Figure 3 shows the sequence of evolution of the hybrid controller when $\alpha = 1.0$ and $\beta = 1.0$. First, E_T is successfully minimized, then E is evaluated by minimizing the squared errors. In case α and β are changed then the evaluation strategy changes.

Figure 4 is representative of the effectiveness of the hybrid controller in steering the trailer-truck along the desired path from the various starting configurations while backing up. We note from Figure 5(a) and 6 that at 200 steps, all the state variables are relatively small and the NC output gradually falls to zero. From this point onward only the LQR is active, thus $u(t) = u_L(t)$. The problems of out of control steering and jack-knife locks are eliminated since all the relative angles are kept to within $[-\pi/2, \pi/2]$rad as per Eqs. (27a) and (27b). We note the superiority of the hybrid controller in Figure 6(c), where the output steering angle $u(t)$ oscillates within the desired range $[-\pi/2, \pi/2]$rad.

The neural network property of adaptive learning is significant in Figures 7 and 8, where extrapolated ranges of operation for hybrid are obtained from the three-point training values. Thus even for untrained starting orientation and position, after a while the relative angles are small and thus the system is realigned on the desired trajectory.

The effectiveness of the hybrid concept is demonstrated by the fact that though the complexity of the control problem increases with the number of connected trailers, the combination of the LQR and the NC yield a very powerful controller that operates optimally to execute the trajectory.

6. Conclusion

Our main contribution in this paper is a hybrid control scheme, comprising a LQR and a NC to stabilize the backward motion of a six trailer-truck configuration. Motivation for the study stems from the inability of conventional neurocontrollers to optimally control the system. Difficult in control is rendered by the nonlinear dynamics of the system and the physical limitations such as jack-knife locks. The system is nonlinear hence the linear system methods are not suitable. Design of the hybrid is based on the condition that we have to include a linearized system for the nonlinear problem. Our core objective was to design a high performance controller utilizing neural networks coupled with a linear method. In total the system is able to successfully execute the manoeuvre to a given trajectory from generic initial conditions while in the backward mode. We have mooted a method that has shown very good performance in controlling a six trailer-truck configuration. We strongly believe this method may be applied to other nonlinear dynamical control problems.

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