

Control of Small Scale Rotary Wing Unmanned Aerial Vehicle (R-UAV) Using Backstepping.

Somasundar Kannan, Bao- Hua Lian, Tae-Won Hwang ,Hyo- Choong Bang

Department of Aerospace Engineering,
Korea Advanced Institute of Science and Technology
Daejeon ,Korea
(Tel : +82-42-869-5758; E-mail: soma@fdcl.kaist.ac.kr)

Abstract: A Nonlinear approach to control of Small Scale Rotary Wing Unmanned Aerial Vehicle (R-UAV) is presented. Using Backstepping, a globally stabilizing control law is derived. We derive backstepping control law for angle of attack and sideslip control. The inherent nonlinear nature of the system are considered here which helps in naturally stabilizing without extensive external effort .Thus, the resulting control law is much simpler than if the feedback linearization had been used.

Keywords: R-UAV, Backstepping Control, Lyapunov technique, Angle of Attack and Sideslip Control

1. INTRODUCTION

Many conventional flight control designs assume the aircraft dynamics to be linear about some nominal flight condition. But this paper deals with the problem of controlling an aircraft explicitly considering its nonlinear dynamics. Our main tool will be backstepping [4], a Lyapunov based design method that has received a lot of attention in the recent years. Compared to other nonlinear techniques like feedback linearization, backstepping offers a more flexible way of dealing the nonlinearities. Using Lyapunov functions, their influence on the system can be analyzed and stabilized, and thus in a sense, useful nonlinearities can be cancelled or dominated by the control signal. Not having to cancel all nonlinearities means that the resulting control law may be much simpler than if feedback linearization had been used.

In this paper, the application is angle of attack and sideslip control. Using inherent characteristics, we will show that despite its nonlinear dynamics the required state of the R-UAV is stabilized around the desired trajectory, regardless of the initial condition.

The remainder of the paper is organized as follows- In Section 2, a nonlinear model of the Rotary wing Unmanned Aerial Vehicle (R-UAV) is described and the dynamic equations are transformed into pure-feedback form which is crucial to the backstepping design. In Section 3, the backstepping control law is derived in detail, and in Section 4 the control law is implemented using a generic simulation model of R-UAV.

2. AIRCRAFT MODEL

The aircraft considered here is a small scale Rotary-Wing Unmanned Aerial Vehicle(R-UAV) [1] which has a mass m of 8.2 Kg. The body fixed axes nonlinear equations of motion for an aircraft are given by [8]

$$\begin{aligned} \dot{V}_T &= \frac{1}{m}(-D + mg_1) \\ \dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta + \frac{1}{mV_T \cos \beta}(-L + mg_2) \quad (1) \\ \dot{\beta} &= p \sin \alpha - r \cos \alpha + \frac{1}{mV_T}(Y + mg_3) \end{aligned}$$

The gravity components are given by

$$\begin{aligned} g_1 &= g(-\cos \alpha \cos \beta \sin \theta + \sin \beta \cos \theta \sin \phi + \sin \alpha \cos \beta \cos \theta \cos \phi) \\ g_2 &= g(\cos \alpha \cos \theta \cos \phi + \sin \alpha \sin \theta) \\ g_3 &= g(\cos \beta \cos \theta \sin \phi + \sin \beta \cos \alpha \sin \theta - \sin \alpha \sin \beta \cos \theta \cos \phi) \end{aligned}$$

Where V_T, α and β are the velocity, angle of attack and sideslip respectively. D, L and Y are the drag, lift and side force given by

$$\begin{aligned} D &= -X \cos \alpha \cos \beta - y \sin \beta - Z \sin \alpha \cos \beta \\ L &= X \sin \alpha - Z \cos \alpha \\ Y &= -X \cos \alpha \sin \beta + y \cos \beta - Z \sin \alpha \sin \beta \end{aligned}$$

The objective of the backstepping approach here is that the aircraft follows a reference angle of attack α^{ref} and roll rate p^{ref} and the sideslip β should be maintained zero at all time. In order to apply the backstepping algorithm to the above given system the state equations should be transformed into a strict feedback form as given below

$$\begin{aligned} \dot{p} &= u_1 \\ \dot{\alpha} &= q - p \tan \beta + \frac{1}{mV_T \cos \beta}(-L(\alpha) + mg_2) \\ \dot{q} &= u_2 \\ \dot{\beta} &= -r + \frac{1}{mV_T}(Y(\beta) + mg_3) \\ \dot{r} &= u_3 \end{aligned} \quad (2)$$

Where u_1, u_2 and u_3 are the control inputs.

3. CONTROL DESIGN

The controller for the above mentioned objectives is obtained by a generic backstepping approach [2, 3] which obviously involves the derivation of the Lyapunov equations.

3.1 Backstepping approach

Let us consider a generic system which can be described by the following equations

$$\begin{aligned} \dot{x}_1 &= f(x_1, y) + x_2 \\ \dot{x}_2 &= u \end{aligned} \quad (3)$$

Considering the above system let us determine a control law that globally stabilizes the system at $x_1 = r$. To have further insight, we introduce the deviation from the steady state, $\xi_1 = x_1 - r$, $\xi_2 = x_2 + f(r, y)$, and $\varphi(\xi_1) = f(\xi_1 + r, y) - f(r, y)$. This gives us

$$\dot{\xi}_1 = \varphi(\xi_1) + \xi_2 \quad (4)$$

$$\dot{\xi}_2 = u \quad (5)$$

We now assume that there exists a maximum slope

$$a = \max_{\substack{\xi_1 \in \mathbb{R} \\ r \in \Omega_r \\ y \in \Omega_y}} \frac{\varphi(\xi_1)}{\xi_1} \leq \max_{\substack{n \in \mathbb{R} \\ y \in \Omega_y}} \frac{\partial f(x_1, y)}{\partial x_1} \quad (6)$$

Equality holds if r is not restricted, i.e., when $\Omega_r = \mathbb{R}$. To use this property in our backstepping application we can rewrite it as

$$\xi_1 \varphi(\xi_1) \leq a \xi_1^2 \quad (7)$$

We now design the actual control law in two steps.

Step 1:

Let us consider ξ_2 as the control input of Equation (4) and find a desired stabilizing virtual control law ξ_2^{des} using the control lyapunov function (clf)

$$V_1 = \frac{1}{2} \xi_1^2$$

Differentiating with respect to time we get

$$\dot{V}_1 = \xi_1 (\varphi(\xi_1) + \xi_2) \leq \xi_1 (a \xi_1 + \xi_2)$$

Using (7) \dot{V}_1 is made negative definite by selecting

$$\xi_2^{des} = -k_1 \xi_1, \quad k_1 > a$$

The resulting ξ_1 dynamics, $\varphi(\xi_1) - k_1 \xi_1$, lie in the second and the fourth quadrants and hence ξ_1 is stabilized.

Step 2:

Continue by introducing the residual

$$\tilde{\xi}_2 = \xi_2 - \xi_2^{des} = \xi_2 + k_1 \xi_1$$

And rewrite the system dynamics in terms of ξ_1 and $\tilde{\xi}_2$

$$\dot{\xi}_1 = \varphi(\xi_1) - k_1 \xi_1 + \tilde{\xi}_2 \quad (8)$$

$$\dot{\tilde{\xi}}_2 = u + k_1 (\varphi(\xi_1) - k_1 \xi_1 + \tilde{\xi}_2) \quad (9)$$

Proceeding in the usual backstepping manner, by adding a $\tilde{\xi}_2$ term to the clf would lead to a control law that cancels these components. The control law would thereby require exact knowledge of φ and consequently f not only at the

equilibrium, $x_1 = r$. Add $F(\xi_1)$ as an extra degree of freedom. This extension of backstepping is due to [4]. Thus

$$V_2 = \frac{k_0}{2} \xi_1^2 + F(\xi_1) + \frac{1}{2} \tilde{\xi}_2^2$$

Where $F(\xi_1)$ is a positive definite, radially unbounded function, satisfying

$$F'(\xi_1) \xi_1 > 0, \quad \xi_1 \neq 0 \quad (10)$$

Where $F'(\xi_1) = \partial F(\xi_1) / \partial \xi_1$

We now aim at finding u that will make \dot{V}_2 negative definite.

$$\begin{aligned} \dot{V}_2 = & k_0 \xi_1 (\varphi(\xi_1) - k_1 \xi_1 + \tilde{\xi}_2) \\ & + F'(\xi_1) (\varphi(\xi_1) - k_1 \xi_1 + \tilde{\xi}_2) \\ & + \tilde{\xi}_2 (u + k_1 (\varphi(\xi_1) - k_1 \xi_1 + \tilde{\xi}_2)) \end{aligned}$$

At this stage it is rewarding to make the split

$$\varphi(\xi_1) = \varphi - (\xi_1) + a \xi_1$$

where $\varphi - (\xi_1)$ is guaranteed to just stay inside the second and fourth quadrants. i.e.,

$$\xi_1 \varphi - (\xi_1) \leq 0$$

We note that $\varphi(\xi_1) - k_1 \xi_1$ is also restricted to the second and fourth quadrants. Combining this with Equation (10) we have that

$$F'(\xi_1) (\varphi(\xi_1) - k_1 \xi_1) \leq 0$$

also holds. Using these relationships we get

$$\begin{aligned} \dot{V}_2 \leq & -k_0 (k_1 - a) \xi_1^2 + \tilde{\xi}_2 (k_0 \xi_1 + F'(\xi_1) \\ & + u + k_1 (\varphi - (\xi_1) + (a - k_1) \xi_1 + \tilde{\xi}_2)) \end{aligned}$$

We can simplify this expression further

$$\begin{aligned} k_0 &= k_1 (k_1 - a) \\ F'(\xi_1) &= -k_1 \varphi - (\xi_1), \quad k_1 > 0, F(0) = 0 \end{aligned}$$

And the final expression

$$\dot{V}_2 \leq -k_1 (k_1 - a)^2 \xi_1^2 + \tilde{\xi}_2 (u + k_1 \tilde{\xi}_2)$$

To make the right hand side negative definite, and the closed loop system globally stable, we select the control law

$$u = -k_2 \tilde{\xi}_2, \quad k_2 > k_1 \quad (11)$$

Summary: Let us summarize our results. Despite the nonlinear nature of the system (4-5), the linear control law (11) is globally stabilizing. In terms of the original state variables from (3) the control law becomes

$$u = -k_2(x_2 + k_1(x_1 - r) + f(r, y)), \quad (12)$$

k_1 and k_2 are design parameters restricted by $k_2 > k_1 > \max(a, 0)$ with a from (6)

3.2 Application to α and β control

The control law (12) can be applied to angle of attack control by substituting

$$x_1 = \alpha, x_2 = q, u = u_2$$

Therefore we get

$$u_2 = -k_2(q + k_1(\alpha - \alpha_{ref}) + f_\alpha)$$

Where $f_\alpha = -p \tan \beta + \frac{1}{mV_T}(-L(\alpha) + mg_2)$ is a nonlinear term involving lift. Similarly the control law (12) can be applied to sideslip control by substituting

$$x_1 = \beta, x_2 = -r \text{ and } u = -u_3$$

$$\text{we get } u_3 = k_2\left(-r_s + k_1\beta + \frac{1}{V_T} g \cos \theta \sin \phi\right)$$

where $f_\beta = \frac{1}{mV_T}(Y(\beta) + mg_3)$ is the nonlinear term involving the sideforce. Note that u_3 does not depend explicitly on the side force.

3.3 Roll control

Controlling the roll rate is straight forward.

$$\dot{p} = u_1$$

From equation (2) we can assign

$$u_1 = k(p^{ref} - p)$$

Where k is an arbitrary tuning parameter.

4. SIMULATION

The derived control law was evaluated on a small scale R-UAV such as X-Cell 60 [1]. The flight condition was assumed to be a level flight with constant forward velocity of 20 m/sec. The simulation response are shown in Fig.1

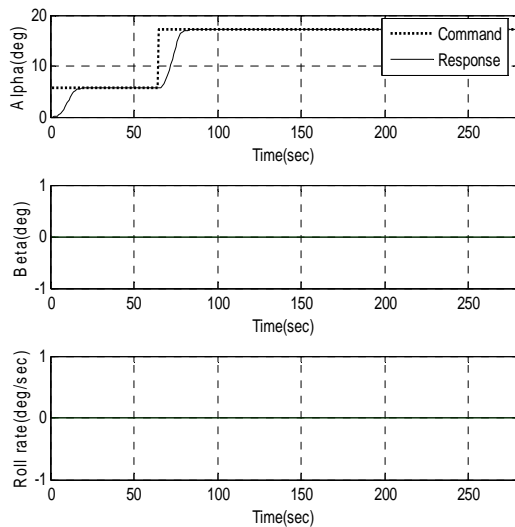


Fig.1 Angle of attack, Sideslip and Rollrate response for reference Angle of attack command.

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