Identification and Control for Nonlinear Discrete Time Systems Using an Interconnected Neural Network

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Abstract: A new control method, called a simple model matching, has been recently developed by the author. This is very simple and be applied for linear and nonlinear discrete time systems with/without time lag. Based on this formulation, identification is examined in this paper using an interconnected neural network with the EBP-EWLS learning algorithm. With this result, a control method is also presented for a nonlinear discrete time system.

Keywords: Identification, Simple model matching, Interconnected neural network, Nonlinear discrete time system

1. INTRODUCTION

This paper presents, in the first part, an identification method using an interconnected neural network (INN) with the EBP-EWLS learning algorithm [1] for nonlinear systems. A system considered here is a nonlinear discrete time system with a linear term of a current input variable. In this formulation, it is possible to estimate a coefficient of a linear input term separately with other terms. The estimate of this coefficient can be seen as an equivalent constant value over the interval on which the input is applied.

In the latter half of this paper, it is discussed to use the identification result for control. Control methodology is the simple model matching (SMM) which is recently developed by the author [2]. Only using an identification result obtained by a neural network, it becomes an open loop control and the output response may not be tolerable. But, this defect can be recovered by introducing an integral action. This result is compared with the one used for an adaptive control [3].

There are many papers and books about nonlinear systems for identification, control, and adaptive control. It is the features of this paper that the SMM and the INN are used.

2. IDENTIFICATION

2.1 Nonlinear discrete time system

Consider a scalar nonlinear system in general form

$$y_{k+1} = g(y_k, y_{k-1}, \dots, u_k, u_{k-1}, \dots)\,. \tag{1}$$

If the nonlinear function $g(.)$ is differentiable with respect to the variable u_k, it can be linearized as follows.

$$y_{k+1} = r_1 u_k + f_k(y,u) \tag{2-1}$$
$$f_k(y,u) \equiv f(y_k, y_{k-1}, \dots, u_k,\, u_{k-1}, \dots)\,. \tag{2-2}$$

If the linearization is performed mathematically, the value r_1 is uniquely determined. But, if the system model is obtained by modeling, or identification in a form

$$y_{k+1} = r_1 u_k + f_k(y,u)\,, \tag{3-1}$$
$$f_k(y,u) \equiv f(y_k, y_{k-1}, \dots, u_k,\, u_{k-1}, \dots)\,, \tag{3-2}$$

Then the term $r_1 u_k$ does not necessarily mean the linear term of the right hand side of eq. (3-1). It is the main concern of this paper that what can be obtained by identification for a system model of the form eq. (3-1). An identification method is proposed for this model by using an interconnected neural network (INN). Using INN makes it possible to identify the first and second terms in the right hand side of eq. (3-1) separately. In what follows, the configuration of the INN used here, the EBP-EWLS learning algorithm developed by the author and some simulation studies are explained. The reason why the form of eq. (3-1) is necessary becomes clear in the section 3 where the simple model matching control is performed using the identification result.

2.2 Interconnected neural network

An INN is described by the following equations.

$$y_{k+1} = w_{OO} y_k + \mathbf{w}^T_{OH} \quad_k + w_{OI} u_k \tag{4-1}$$
$$_{k+1} = h(\mathbf{x}_{k+1}) \tag{4-2}$$
$$\mathbf{x}_{k+1} = \mathbf{w}_{HO} y_k + \quad^T_{HH} \quad_k + w_{HI} u_k \tag{4-3}$$

This is a scalar neural network with input u_k and output y_{k+1}. All variables have appropriate dimensions. Bias elements are omitted here for simplicity. If the INN (6) is supposed to be a system model of the system (2), the coefficient w_{OI} which combines the input variable u_k to the output variable y_{k+1} directly should be recognized as the estimate of r_1 and have a same role of r_1 in eq. (3). Here, the indices of eqs. (4-2) and (4-3) are shifted as follows,

$$_k = h(\mathbf{x}_k) \tag{4-4}$$
$$\mathbf{x}_k = \mathbf{w}_{HO} y_{k-1} + \quad^T_{HH} \quad_{k-1} + w_{HI} u_{k-1}\,. \tag{4-5}$$

Then, the INN is equivalently represented by

$$y_{k+1} = \mathbf{w}^T_O \mathbf{v}_k\,, \tag{5-1}$$
$$\mathbf{w}^T_O = (w_{OO}, w^T_{OH}, w_{OI})\,,\quad \mathbf{v}^T_k = (y_k, \quad^T_k, u_k)$$
$$_k = h(\mathbf{x}_k)\,, \tag{5-2}$$
$$\mathbf{x}_k = \mathbf{W}^T_H \mathbf{v}_{k-1}\,, \tag{5-3}$$
$$\mathbf{W}^T_H = (\mathbf{w}^T_{HO}, \mathbf{W}^T_{HH}, \mathbf{w}^T_{HI})$$

This can be regarded as a three-layered NN. The EBP-EWLS learning algorithm can be used for learning a nonlinear system (2) with y_{k+1} as a target signal. This algorithm is explained next.

2.3 Learning algorithm

The algorithm is composed of two steps. The first step is the determination of a fictitious target signals for each output of hidden units. This is done by an error back propagation (EBP) method, which is different from the well known BP method. The following are derived to make the output error zero by the variation $\Delta\mathbf{v}_k$ of $\quad_k$.

$$\Delta \mathbf{v}_k = \mathbf{w}_O (w_O^T w_O)^{-1} (y_{k+1} - y_{k+1}) \qquad (6\text{-}1)$$

$$^{*}_{N,k} = {}_k + \Delta {}_k ,\ \Delta \mathbf{v}_k^T = (\Delta y_k , \Delta {}_k^T , \Delta u_k) \qquad (6\text{-}2)$$

$$_{N,k} = H \ {}^{*}_{N,k} \qquad (6\text{-}3)$$

Here the symbol H represents the operator for a vector $_{N,k}$ to be included in the range space of the function h. The resulting vector $_{N,k}$ has a role of a temporal target function for the output of the hidden unit $_k$. The original learning problem for a three-layered NN is then divided into two problems of two layered NNs each have a respective target signal. Then the second step is an update of neural network weight parameters and this can be solved by the exponentially weighted least squares (EWLS) method, where the inverse function of h is used. For more details, see [1].

2.4 Identification

Using an interconnected neural network with the EBP-EWLS learning algorithm, a nonlinear system can be identified. Especially, as described before, the estimate of the coefficient of linear input term can be obtained numerically. This is confirmed by many simulation results.

In what follows, two types of test input signal are used. D1 is a random number uniformly distributed on the interval [-0.5, 0.5]. D2 is the one on the interval [-0.1, 0.1]. A forgetting factor of the EWLS algorithm is 0.96 for every example.

(Example 1) Consider a system of the form

$$y_{k+1} = 0.5 y_k - 0.06 y_{k-1} + r_1 u_k - 0.2 u_{k-1} + K \sin u_k$$

In this system, the nonlinear term $K \sin u_k$ includes a linear term of the form $K u_k$. It is interesting to know what value of r_1 can be estimated. r_1, $r_1 + K$ or other value Fig.1 shows the result of the case that $r_1 = 0.9$ and $K = 0.1$, where the test input u_k is D1. It can be seen that the estimate r_1 is a good approximation of $r_1 + K$ =1. Fig.2 is the case for $r_1 = 0.1$ and $K = 0.9$ with input D1. There exists an offset obviously because of the strong nonlinearity. This offset decreases if D2 is used for a test input signal which is seen in Fig.3. This is due to the small range of the signal D2. From these figures, it may be concluded that the estimate of the coefficient of the linear input term is an equivalent constant value over the interval on which the input signal is applied.

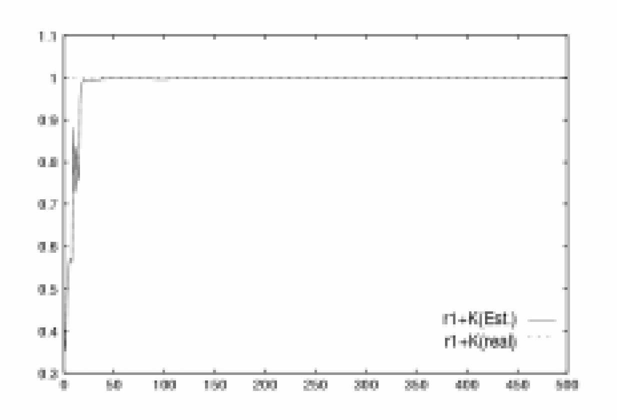

Fig.1 The case: $r_1 = 0.9$, $K = 0.1$ and D1

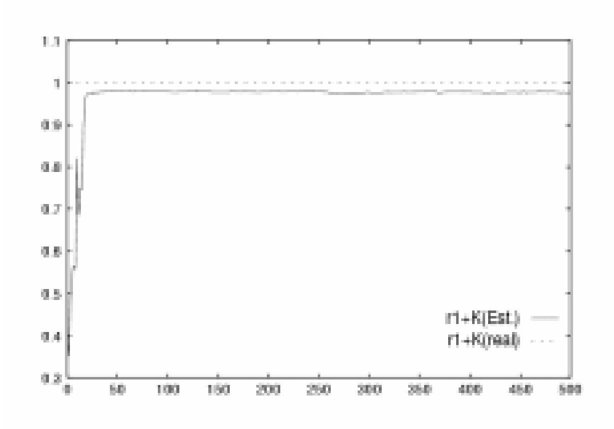

Fig.2 The case: $r_1 = 0.1$, $K = 0.9$ and D1

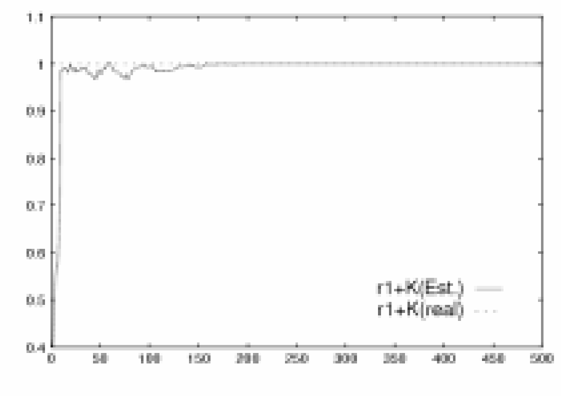

Fig.3 The case: $r_1 = 0.1$, $K = 0.9$ and D2

(Example 2) Consider a system described by

$$y_{k+1} = \frac{u_k}{1 + u_k^2} + 0.5 y_k + 0.06 y_{k-1}$$

This system has a strong nonlinearity with respect to the input variable and also represented by

$$y_{k+1} = u_k - \frac{u_k^3}{1 + u_k^2} + 0.5 y_k + 0.06 y_{k-1}.$$

Fig.4 is the result using a D1 input signal and Fig.5 is for D2. In the Figures 1 to 5, the number of hidden units of the INN is 8. But, Fig.6 is the result using the same condition with Fig.5 but the number of hidden units is 10. It is clear that the identification result depends on the number of hidden units.

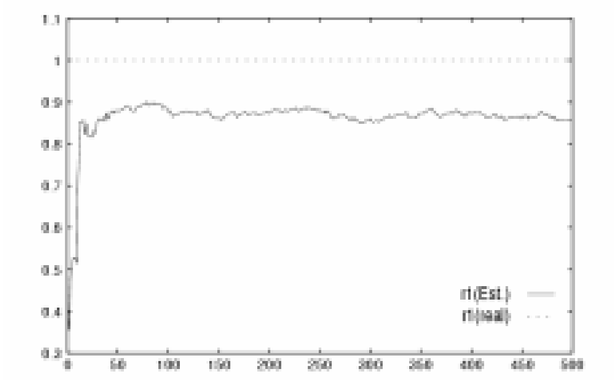

Fig.4 The case 1 of example 2

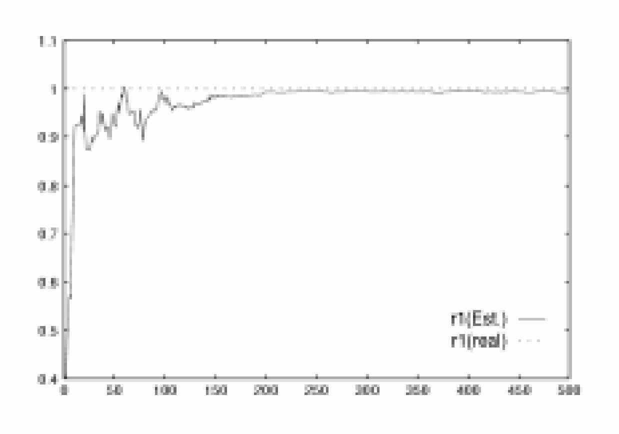

Fig.5 The case 2 of example 2

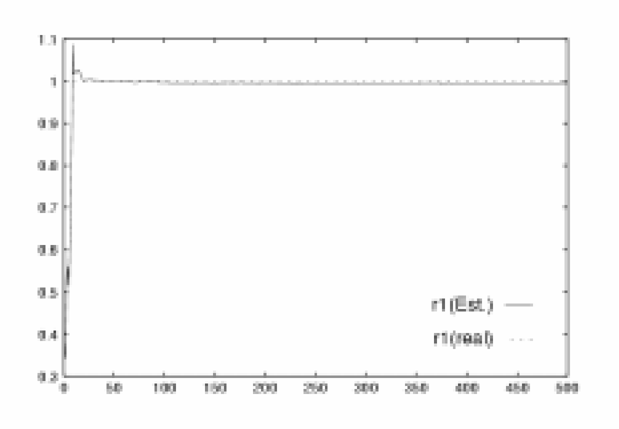

Fig.6 The case 3 of example 2

From these results, it became clear that a coefficient of a linear input term for a nonlinear discrete time system can be identified as an equivalent constant value over the interval on which the input is applied. If an estimate moves oscillatory like in Fig.4, it is considered to use the average over the last few data.

3. CONTROL

In the last section, an identification method for nonlinear discrete time systems was presented. It is presented here to use the identification result discussed above for control. A simple model matching method (SMM) is first explained, which is developed by the author [2]. This method is different from the conventional model matching (model following) method [4] and is applicable for linear/nonlinear discrete time systems with time lag. In this configuration, an identification result is used as a system model with an integral action. This result is compared with an adaptive control where an identification scheme described in a last section is utilized in on line recursive form.

3.1 Simple model matching method (SMM)

Consider a system and a desired system described by

$$y_{k+1} = r_1 u_k + f_k(y,u), \tag{6-1}$$

$$f_k(y,u) \equiv f(y_k, y_{k-1}, ..., u_{k-1}, ...) \tag{6-2}$$

$$y_{d,k+1} = f_d(y_{d,k}, y_{d,k-1}, ..., u_{d,k}, ...), \tag{7}$$

respectively, where y_k and u_k are system output and input,

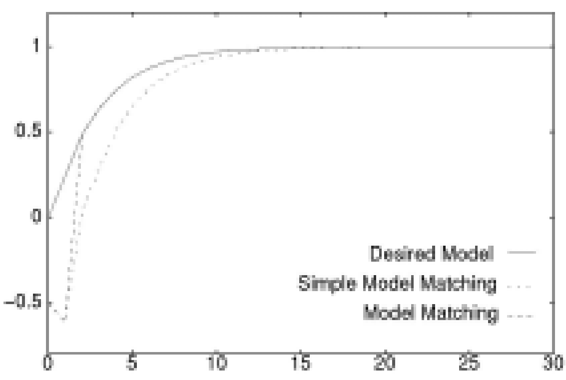

Fig.7 Output Responses

and $y_{d,k}$ and $u_{d,k}$ are desired system output and input (reference input) respectively. Note that the current input variable u_k does not included in the right hand side of eq. (1-1). Then, a control input is determined as

$$u_k = \{y_{d,k+1} - f_k(y,u)\}/r_1 \tag{8}$$

by replacing the output y_{k+1} in (1) with $y_{d,k+1}$. Then, the system output coincides with the output of the desired system in one step. On the other hand, the output of a conventional model matching method converges asymptotically to the desired output. This fact is demonstrated in Fig.7.

In the real situation, eq. (1) can not be used and, instead of it, a system model

$$y_{k+1} = r_1 u_k + f_k(y,u) \tag{9}$$
$$f_k(y,u) \equiv f(y_k, y_{k-1}, ..., u_{k-1}, ...)$$

is utilized. Then, the control input is really executed by

$$u_k = \{y_{d,k+1} - f_k(y,u)\}/r_1. \tag{10}$$

In this method, the value r_1 is of great significance for the stability of control input. Especially, there is a stability problem for a non-minimum phase system. This problem is partly resolved in [4].

3.1.1 Integral action

There exists an offset if the control signal (10) is used since a system and a system model are, in general, different. To erase an offset, introduction of an integral action is useful. Doing this in the SMM, it is enough to add an output error of the system and the system model as follows

$$u_k = \{y_{d,k+1} - f_k(y,u) - y_k + y_k\}/r_1. \tag{11}$$

It is not hard to see that the closed loop system with input (11) has an integral action. For example, it can be seen that the steady state of the system output coincides with that of the desired output for a step response under the assumption that the step response is stable.

3.1.2 Systems with time lag

For a system and a system model with time lag described by

$$y_{k+2} = r_1 u_k + f_k(y_{k+1}; y, u) \tag{12-1}$$

$$f_k(y_{k+1}; y, u) \equiv f(y_{k+1}, y_k, \ldots, u_{k-1}, \ldots), \tag{12-2}$$

control input is determined as

$$\begin{aligned} u_k &= \{y_{d,k+1} - f_k(y_{k+1}; y, u)\} / r_1 \\ &= \{y_{d,k+1} - f_k(r_1 u_{k-1} + f_{k-1}; y, u)\} / r_1 \end{aligned} \tag{13}$$

The same manner can be similarly extended for a system with time lag of multi steps. Integral action can also be introduced.

3.2 Control by using the identification results

For a system (6), the coefficient of the linear term can be identified numerically as explained in the last section. Then, a nonlinear term is seemed to be obtained as

$$f_k(y, u) = y_{k+1} - r_1 u_k. \tag{14}$$

But, this is only possible after the control signal is determined. Therefore, the approximation

$$f_{k-1}(y, u) = y_k - r_1 u_{k-1} \tag{15}$$

is used in a control input. The following examples confirm the validity of this method.

(Example 3) Consider the following system and the desired system.

$$\begin{aligned} y_{k+1} &= 0.5 y_k - 0.06 y_{k-1} \\ &\quad + 0.8 u_k - 0.3 u_{k-1} + 0.29 \sin u_k \\ y_{d,k+1} &= 1.4 y_k - 0.48 y_{d,k-1} + 0.05 u_{d,k} + 0.03 u_{d,k-1} \end{aligned}$$

Fig.8 and Fig.9 are a step and a ramp responses respectively, and both show a fairy large offset because there is an approximation and the identification error in eq. (15) and, eq. (15) should be expressed exactly as

$$f_{k-1}(y, u) = f_{k-1}(y, u) = y_k - r_1 u_{k-1}. \tag{16}$$

This means that there is no output feedback and the control signal (10) is used as an open loop control. This is more important and critical defect when an identification result by the neural network is utilized.

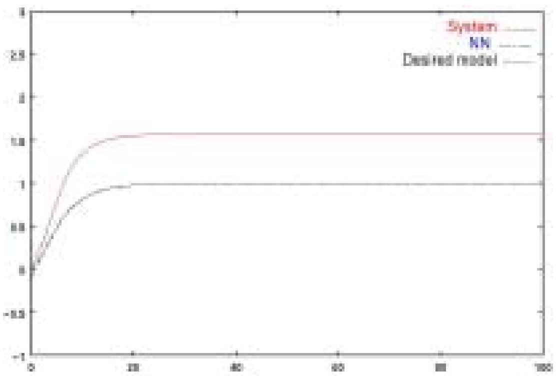

Fig.8 Step response

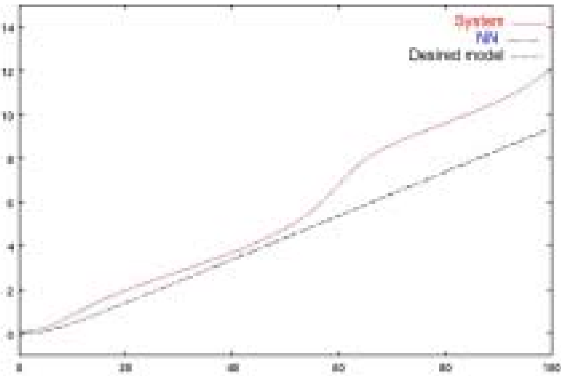

Fig.9 Ramp response

This defect can be recovered if an integral action is used. Fig.10 and Fig.11 are the results using an integral action corresponding to Fig.8 and Fig.9, respectively.

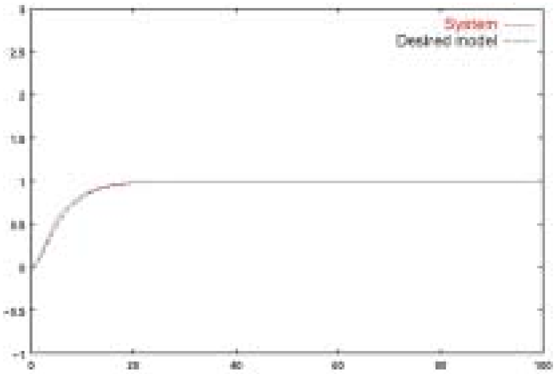

Fig.10 Step response with integral action

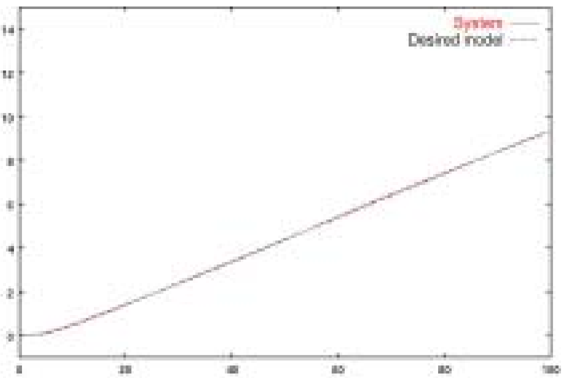

Fig.11 Ramp response with integral action

Similar results are obtained for a system with time lag in example 4.
(Example 4)

$$\begin{aligned} y_{k+1} &= 0.5 y_k - 0.06 y_{k-1} \\ &\quad + 0.0 u_k + 0.3 u_{k-1} + 0.29 \sin u_k \\ y_{d,k+1} &= 1.4 y_k - 0.48 y_{d,k-1} + 0.0 u_{d,k} + 0.08 u_{d,k-1} \end{aligned}$$

Figs.12 and 13 are results without integral action and Figs.14 and 15 are with integral action. Similar results are obtained for a system with time lag.

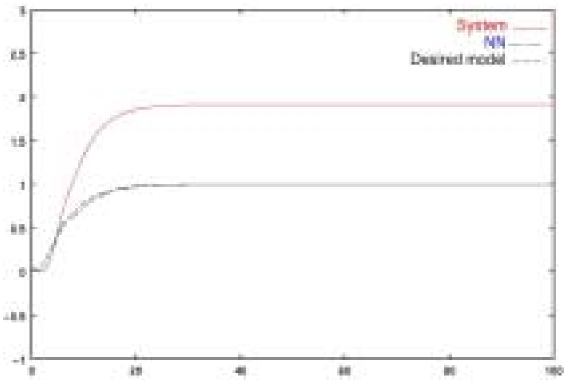

Fig.12 Step response

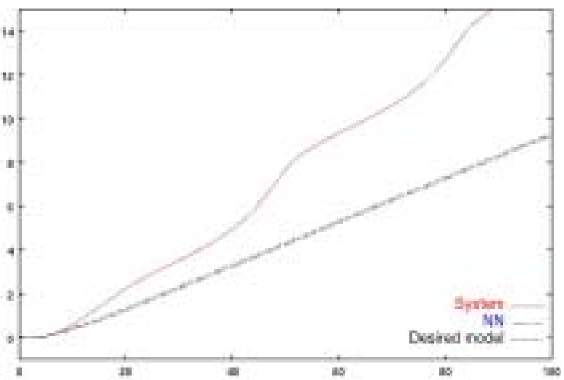

Fig.13 Ramp response

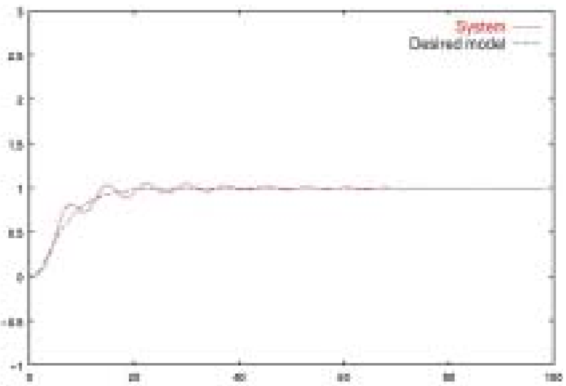

Fig.14 Step response with integral action

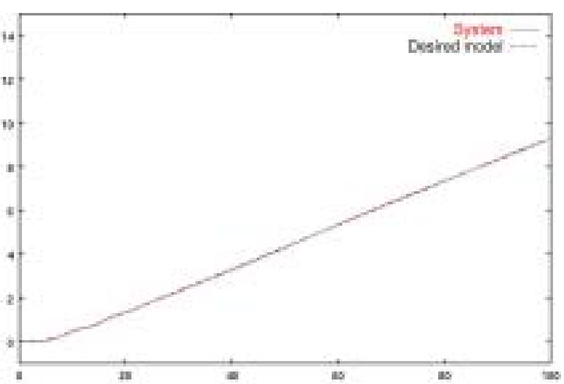

Fig.15 Ramp response with integral action

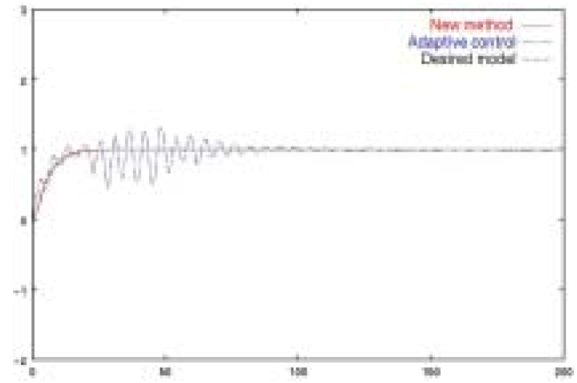

Fig.16 Step response vs adaptive control

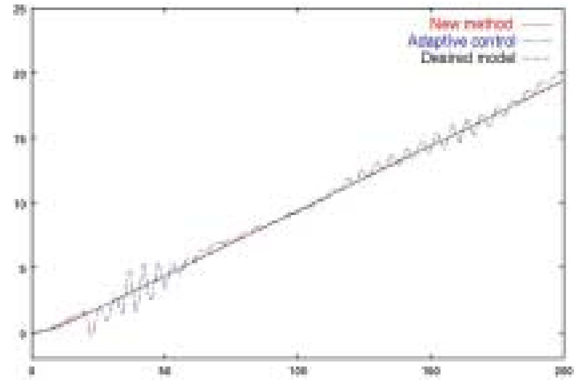

Fig.17 Ramp response vs adaptive control

Figs.16 and 17 are results compared with an adaptive control where the identification is recursively performed in each step. It is seen that the combination of Identification and control is much better than the adaptive control.

4. CONCLUSIONS

A new scheme for identification for nonlinear discrete time system was proposed using an interconnected neural network and the EBP-EWLS learning algorithm. It was shown that the coefficient of the linear input variable can be obtained numerically. This result was applied for control, which showed an integral action was very powerful to recover the defect of the open loop control and was superior to the adaptive control..

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