A Robust Adaptive Direct Controller for Non-Linear First Order Systems

Thi-Hong-Thanh Nguyen*, Xuan-Thinh CU*, Thi-Minh-Huong NGUYEN*, Thi-Hoan HA[**], Dac-Hai NGUYEN[**], Van-Truong TRAN[**]

*C&A Technology Application Support Co., Ltd., Hanoi, Vietnam
(Tel : +84-4-9261912; E-mail: thiminhhuong_nguyen@yahoo.com)
**Center for Automation Technology, Hanoi, Vietnam
(Tel : 84-4-7330806; E-mail: thihien_nguyen@yahoo.com)

Abstract: This paper presents new results on designing a robust adaptive direct controller for a class of non-linear first order systems. The designing method based on the use of dead zone in the parameters' update law. It is shown that the size of the dead zone does not depend on the upper bounds of the disturbances. That means that even if the bounds are large, the tracking error will always converge to a set of the dead zone size. However, in the ideal case, when the exogenous signal functions and the function represents un-modeled dynamics of the systems equal to zero, the proposed controller does not mean the convergence to zero of the tracking error.

Computer simulation results show the effectiveness of the controller in dealing with the stated problems.

Keywords: Adaptive Control, Dead zone, Robustness, Nonlinear System

1. INTRODUCTION

Concerning nonlinear systems, several results dealing with robust adaptive control of robot manipulators are now available in the literature (see [1], [2], [3], [4]). For the general case, however, few results have been obtained ([5], [6], [7], [8]), and robustness of those schemes has not been studied.

In this paper, the problem of controlling a first order nonlinear system subject to bounded input and output exogenous disturbances and un-modeled dynamics which are smooth enough is studied. The proposed scheme has been inspired from previous works (see [9], [5]), but the scheme in this paper allows us to considerably reduce the size of the dead zone, i.e. the size of the tracking error residual set. Furthermore, the a *priori* upper bounds knowledge on the system uncertainties and disturbances is significantly reduced, and is eliminated when the functions in the system equations are Lepschitz.

The paper is organized as follows: in the next section, the class of nonlinear plants to be studied and the structure of the controller are presented. In section 3, we propose three different robust adaptive control laws and we discuss their properties. In section 4, a numerical example simulated is presented. Finally, some conclusions are given in the last section.

2. PROBLEM STATEMENT AND ROBUST CONTROLLER STRUCTURE DESIGN

Consider the class of nonlinear systems:

$$x' = -D^T f(x) - bu + w_1(t) + g(x) \quad (1)$$
$$\bar{x} = x + w_2(t) \quad (2)$$

, where u, x and R denote the input, state and measured state, respectively; $D \in R^p$ and $b \in R$ are unknown parameters; $f(x) \in R^p$ is a known vector function and g(x) is an unknown scalar function which represents un-modeled dynamics.

Assume that the exogenous signal $w_1(t)$, $w_2(t)$ as well as f(x) and g(x) have standard smoothness and boundedness properties so that the existence of solutions for the ordinary differential equations involved in the remaining of the paper is guaranteed [10].

The aim of the control is to make plant (1) - (2) asymptotically match a linear first order reference model of the form:

$$x'_m = a_m x_m + r \quad (3)$$

, where $r \in R$ is the reference signal and $a_m < 0$. From (1) and (2) we have:

$$\bar{x}' = x' + w'_2(t) = -D^T f(\bar{x}) + D^T \Delta f(x, w_2) - bu + w'_2 + w_1 + g(x) \quad (4)$$

, where

$$\Delta f(x, w_2) = f(\bar{x}) - f(x) \quad (5)$$

is a measure of the sensitivity of the nominal model $(g(x)=w_1=w_2=0)$ with respect to the output noise $w_2(t)$.

Now, from (3) and (4) the error equation can be derived:

$$e' = \bar{x}' - x'_m = a_m e + b[-\overline{D}^T f - u + \frac{w_2}{b} + \frac{w_1}{b} + \frac{1}{b} D^T \Delta f(x, w_2)] \quad (6)$$

$$\overline{D}^T = \left[\frac{a_m}{b}, \frac{D^T}{b}, \frac{1}{b}\right] \quad (7)$$

, where

$$\bar{f}' = [\bar{x}, f^T(\bar{x}), r] \quad (8)$$

Assumption 1: b is unknown but its sign is known. We will consider b>0 in the fest of the paper without loss of generality.

Assumption 2: Suppose that for a suitable unknown parameter w_M and a known continuous-time function $h(\bar{x})$ we have

$$\frac{1}{b}|(w'_2 + w_1 + D^T \Delta f(x, w_2) + g(x))| \leq w_M h(\bar{x}) \quad (9)$$

, where w_M, $h(\bar{x}) > 0$.

Note that e and $\bar{f}$ are measurable and therefore available for control purpose. Now, the following control law is proposed:

$$u = -\hat{D}^T f(\bar{x}) + \frac{e|e|h(\bar{x})}{\varepsilon_0(e)}\hat{w}_M \qquad (10)$$

, where:

$\hat{D} \in R^{P+2}$ denotes an estimate for the unknown parameter vector in (7).

$\varepsilon_0(e)$ is a suitable function of the tracking error and $\hat{w}_M$ is an auxiliary parameter estimate. Both will be specified later.

To show the stability of the closed loop system (1), (10) we choose the following Lyapunov positive definite function:

$$V(t) = \frac{1}{2}e^2 + \frac{1}{2}b(\tilde{D}^T\tilde{D} + \tilde{w}_M^2) \qquad (11)$$

where:

$\tilde{D} = \hat{D} - \overline{D}$ and $\tilde{w}_M = \tilde{w}_M - w_M$ are parameter errors.

Taking derivative of V along the trajectory of the closed loop system combined with (6) and (10) then in conjunction with (9) we have:

$$V'(t) \le a_m e^2 + b\{-\frac{e^2|e|}{\varepsilon_0(e)}h(\bar{x})\hat{W}_M +$$

$$|e|w_M h(\bar{x})\} + b\tilde{D}^T(e\bar{f} + \overline{D}') + b\tilde{w}_M\tilde{w}'_M \qquad (12)$$

Since $\overline{D}$ and w_M are unknown but constant parameters we have $\tilde{D}' = \hat{D}'$ and $\tilde{w}' = \hat{w}'_M$ Now let us choose:

$$\hat{w}'_M = \begin{cases} |e|\frac{e^2}{\varepsilon_0(e)}h(\bar{x}) \ if\ |e| \ge \varepsilon_1 \\ 0 \qquad\qquad if\ |e| \le \varepsilon_1 \end{cases} \qquad (13)$$

$$\hat{D}' = \begin{cases} -e\bar{f}\ if\ |e| \ge \varepsilon_1 \\ 0 \qquad if\ |e| \le \varepsilon_1 \end{cases} \qquad (14)$$

, where ε_1 specifies the dead zone size. From (13), (14), (12) we get:

$$V' \le a_m e^2 + b|e|w_M[1 - \frac{e^2}{\varepsilon_0(e)}]h(\bar{x})\ \ for|e| \ge \varepsilon_1 \qquad (15)$$

If furthermore $\varepsilon_0(e)$ is chosen such that:

$$|e| \ge [\varepsilon_0(e)]^{1/2}\quad \forall\ |e| \ge \varepsilon_1 \qquad (16)$$

, we obtain:

$$V' \le a_m e^2 \le a_m \varepsilon^2\quad for|e| \ge \varepsilon_1 \qquad (17)$$

So V(t) decreases until $|e|$ reaches the bound of the dead zone, i. e $|e| = \varepsilon_1$ while assuming bounded initial conditions on e, D and $\hat{w}_M$.

To conclude boundedness of e, D and $\tilde{w}_M$ we follow the ideas in [5] and [9].

Let $\Omega_1 = (t/|e| \le \varepsilon_1)$ and $\Omega_2 = (t/|e| > \varepsilon_1)$. For $\tau \in \Omega_1$, $\hat{D}$ and $\tilde{w}_M$ are constant. Assume that for t=t₀∈Ω_1, $|e(t_0)| = \varepsilon_1$, i. e the trajectory lies on the boundary of Ω_1.

Assume also that $|e|$ leans Ω_1 at t₁, that there exists t₂>t₁ such that t₂ ∈ Ω_2. First note that the length of the interval (t₂,t₁) must be finite, see equation (17). Therefore, there exists a time t₃>t₂ such that $|e(t_3)| = \varepsilon_1$. Since e²(t₃)=e²(t₁), and V(t)is strictly decreasing for t ∈ Ω_2 it follows that:

$$\| \tilde{D}(t_3)\|^2 + \tilde{w}^2{}_M(t_3) < \| \tilde{D}(t_1)^2 + \tilde{w}_M^2(t_1) \qquad (18)$$

Remark:

Note that inequality (9) in assumption (2) is satisfied if the following three inequalities hold:

$$\frac{1}{b}|w_2' + w_1| \le w_{M1} \qquad (19a)$$

$$\frac{1}{b}|w_2' + w_1| \le w_{M1} \qquad (19b)$$

$$\frac{1}{b}|g(x)| \le w_{M3}h_3(\bar{x}) \qquad (19c)$$

, where w_{Mi} (i = 1, 2, 3) are unknown positive constant and $h_i(\bar{x})$ (i = 2,3) are known positive functions. The interest of the above inequalities is that the exogenous disturbances, the sensitivity term Δf and the un-modeled dynamics can be treated separately. Moreover, if f and g are globally Lepschitz, we can set $h_2(\frac{1}{b}|g(x)| \le w_{M3}h_3(\bar{x}))$=1 and $h_3(\bar{x})=\bar{x}$. In this case, one sees that no a priori upper bounds knowledge is required neither in the above equation nor in (9).

3. CHOICE OF THE DEAD ZONE SIZE

In the previous section we have shown that the system (1), (2) is globally stable with the control law (10), (13) and (14) provided that the function ε_0 (e) is chosen such that (16) holds. In this section, we present three different choices for $\varepsilon_0(e)$:

ε_0(e) = e² and ε_1 =0. In this case, we obtain:

$$\hat{w}'_M = |e|h(\bar{x}) \qquad (20)$$

$$\hat{D}' = e\bar{f} \qquad (21)$$

$$\hat{D}^T f + \text{sgn}(e)\hat{w}'_M h(\bar{x}) \qquad (22)$$

, where sgn denotes the sign function.

From (20), (21) it follows that the parameters are frozen when the sliding surface e=0 is reached. This condition can not met in practice because actual switchers only provide an approximation of the sign function. If a numerical implementation is used, round off errors in the tracking errors

in the tracking error may destroy the information of its sign in the neighborhood of e=0.

$$\varepsilon_0(e) = \{ e^2 \quad if \; |e| \ge \varepsilon_1 \quad , \text{ where } \; \varepsilon_1 > 0 \text{ and small enough}$$

The stability (16) is verified and in this case we have:

$$\hat{w}'_M = \begin{cases} |e| h(\bar{x}) & if \; |e| \ge \varepsilon_1 \\ 0 & if \; |e| < \varepsilon_1 \end{cases} \tag{23}$$

$$\hat{D}' = \begin{cases} -e\bar{f} & if \; |e| \ge \varepsilon_1 \\ 0 & if \; |e| < \varepsilon_1 \end{cases} \tag{24}$$

$$u = -\hat{D}\bar{f} + sat(e/\varepsilon_1)\hat{w}h(\bar{x}) \tag{25}$$

, where *sat* denotes the saturate function [11] (see Figure 1).
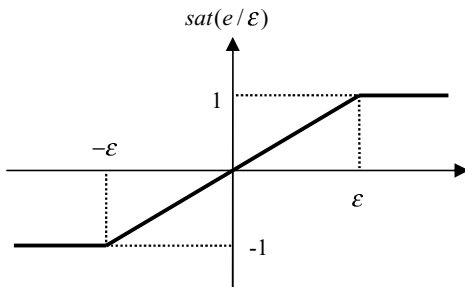

Fig. 1 The *Sat* function

This function has been used by several authors for robustness purposes (see [12]). As long as e is outside the dead zone, the controllers in 3.1, 3.2 are ideal. If e is inside the dead zone, the parameters in (23), (24) are frozen and the input in (25) is linear continuous function of the tracking error. Thus the problems encountered in 3.1 are avoided, provided that ε_1 designs a "sufficiently large "neighborhood of the origin e = 0.

$$\varepsilon_0(e) = \varepsilon_1^2 + \alpha(e^2 - \varepsilon_1^2) \quad \text{where} \; \alpha \in [0, e] \text{ and } \; \varepsilon_1 > 0 .$$

Then the stability condition in (16) is verified and (13), (14) yield:

$$\hat{w}'_M = \begin{cases} \dfrac{|e| e}{\varepsilon_1^2 + \alpha(e^2 - \varepsilon_1^2)} & if \; |e| \ge \varepsilon_1 \\ 0 & if \; |e| < \varepsilon_1 \end{cases} \tag{26}$$

$$\hat{D}' = \begin{cases} -e\bar{f} & if \; |e| \ge \varepsilon_1 \\ 0 & if \; |e| < \varepsilon_1 \end{cases} \tag{27}$$

$$u = \hat{D}^T \bar{f} + \frac{|e| e}{\varepsilon_1^2 + \alpha(e^2 - \varepsilon_1^2)} \hat{w}_M h(\bar{x}) \tag{28}$$

For a fixed value of ε_1 if, one sees that it approaches 1, then (e) approaches e^2. Hence u in (28) to the discontinuous input defined in (22). But now, I e is inside the dead zone, the parameters are frozen and the control input is a nonlinear continuous time function. Thus the problem in-countered in section 3.1 are again avoided.

4. SIMULATION EXAMPLE

The simulation results presented in this section were obtained by using MATLAB with nonlinear control tool box (NCD) added. Let us consider the following nonlinear system:

$$x' = \sum_{n=0}^{100} \frac{1}{(n+1)^2} \sin(nx) + u + w_1 \tag{29}$$

$$\bar{x} = x + w_2$$

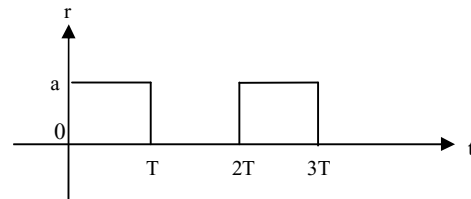The reference model is $x'_M = -a_M x_M + r$, where r is depicted shown in Figure 2.

Fig. 2 Reference signal r

For a = 1, T = 8 sec, $\varepsilon_1 = 10^{-3}$, h($\bar{x}$) =1, g(x)=0 we have got the simulation results in Figure 3.
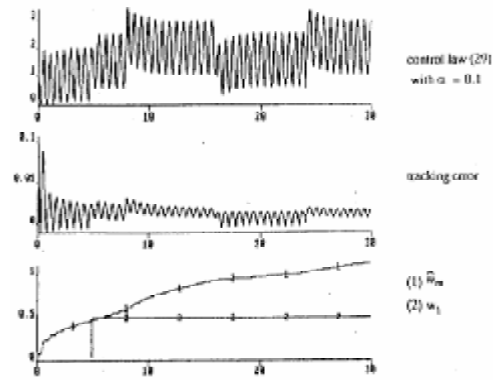

Fig. 3 Simulation result

5. CONCLUSIONS

In this work, we have presented a robust adaptive direct controller applied to a class of nonlinear first order systems. It is based on the use of a dead zone in the parameters update law. The size of the dead zone in (13) and (14) does not depend on the upper bounds of the disturbances. It follows that even If the bounds are large, the tracking error convergence to a set of size defined ε_1. In spite of this convergence, in the ideal case when $w_1 = w_2 = g(x) = 0$, the proposed scheme does not allow one to conclude convergence to zero of the tracking error. The extension of the results presented in this paper to systems of dimension greater than one is under currently research.

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