Global Synchronization of Two Different Chaotic Systems via Nonlinear Control

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Abstract: This paper presents chaos synchronization between two different chaotic systems using nonlinear control method. The proposed technique is applied to achieve chaos synchronization for the Lorenz and Rossler dynamical systems. Numerical simulations are also implemented to verify the results.

Keywords: Chaotic systems, Synchronization, Nonlinear control, Noisy Channel

1. INTRODUCTION

Because of its importance, chaos synchronization has been studied extensively in recent years. Pecora and Carroll introduced a method (PC method) to synchronize two identical chaotic systems with different initial conditions [1]. This kind of synchronization can be realized only under the condition that all the Lyapunov exponents of the response system under the action of the drive system (the conditional Lyapunov exponents) are negative. Consequently, the PC method does not work well in every case [2].

Synchronization has been widely exploited in a variety of fields including physical [3], chemical and ecological [4, 5] systems, secure communications [6–8], etc. Meanwhile, various synchronization schemes such as adaptive control [9], backstepping design [10], active control [11], and nonlinear control [12] have been successfully applied to chaos synchronization.

Secure communication has been an interesting field of application of chaotic synchronization during the last decade [13-15]. Due to their unpredictability and broadband spectrum, chaotic signals have been used to encode information and transmit the message by simple masking (addition) or using modulation [16]. In practice the channel is subjected to noise, therefore the investigation of the signal transmission in the case of noisy channel is very important.

Nonlinear control is an effective method for making two different chaotic systems be synchronized. However, this method usually assumes that the Lyapunov function of error dynamic of synchronization is formed as $V = (1/2)e^T e$ [2]. In this paper, modification based on Lyapunov stability theory to design a controller is proposed in order to overcome this limitation. The synchronization can be robustly achieved without the requirement to calculate the conditional Lyapunov exponents. The proposed method will be applied to make two different chaotic systems (Lorenz and Rossler) globally asymptotically synchronized.

The paper is organized as follows. In Section 2, Lorenz and Rossler chaotic systems are introduced. In Section 3, theory of nonlinear control is presented. The theory is adopted to synchronize the systems in Section 4. In Section 5 numerical simulations are presented in order to show effectiveness of the synchronization method. The effect of transmission channel noise on the synchronization is studied in Section 6, and finally the paper is concluded in Section 7.

2. SYSTEMS DESCRIPTION

The Lorenz system is known to be a simplified model of several physical systems [17]. Originally, it was derived from a model of the earth's atmospheric convection flow heated from below and cooled from above [17]. Furthermore, it has been reported that Lorenz equations may describe such different systems as laser devices, disk dynamos and several problems related to convection [18]. The Lorenz system is described by the following nonlinear differential equations.

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - xz - y \\ \dot{z} = xy - bz \end{cases} \tag{1}$$

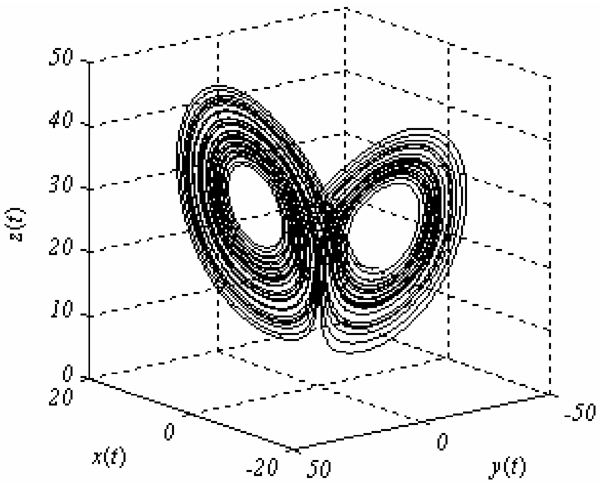


Fig. 1 Lorenz attractor.

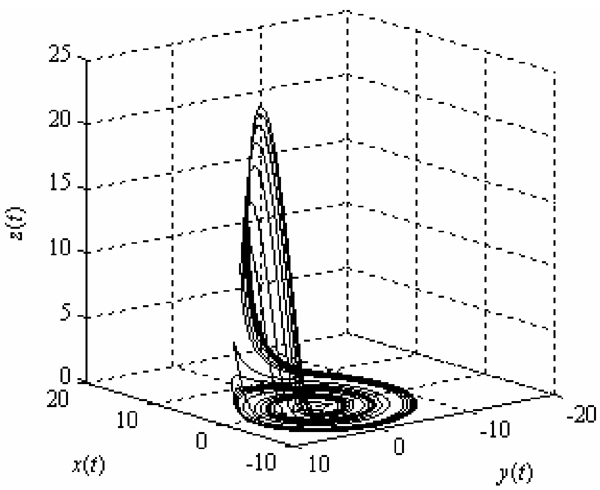


Fig. 2 Rossler attractor.

Which has a chaotic attractor as shown in Fig. 1 when $a = 10$, $b = 8/3$, and $c = 28$.

The so called Rossler system is credited to Otto Rossler and arose from work in chemical kinetics. The system is described with 3 coupled nonlinear differential equations [19].

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + \alpha y \\ \dot{z} = \beta + z(x - \gamma) \end{cases} \tag{2}$$

Which has a chaotic attractor as shown in Fig. 2 when $\alpha = 0.2$, $\beta = 0.2$, and $\gamma = 5.7$.

3. DESIGN OF THE CONTROLLER

Most of the synchronization methods belong to the drive-response type. By one system driving another we mean that two systems are coupled so that the behavior of the second is dependent on the behavior of the first, but the first is not influenced by the behavior of the second [20]. The first system is called the drive and the second one is the response.

Consider a chaotic system described by the following relation.

$$\dot{X}_1 = A_1 X_1 + f_1(X_1) \tag{3}$$

Where $X_1(t) \in R^n$ is the n-dimensional state vector of the system, $A_1 \in R^{n\times n}$ is the matrix of the system parameters, and $f_1 : R^n \to R^n$ is the nonlinear part of the system. Relation in (3) represents the drive system.

The controller $U \in R^n$ is added into the response system, so it is given by

$$\dot{X}_2 = A_2 X_2 + f_2(X_2) + U \tag{4}$$

Where $X_2(t) \in R^n$ represents the state vector of the system, $A_2 \in R^{n\times n}$ is the matrix of the response system parameters, and $f_2 : R^n \to R^n$ is the nonlinear part of the response system. If $A_1 = A_2$ and $f_1(X) = f_2(X)$, then X_1 and X_2 are the states of two identical chaotic systems. If $A_1 \neq A_2$ and/or $f_1(X) \neq f_2(X)$, then X_1 and X_2 represent the states of two different chaotic systems.

The synchronization problem is to design a controller U, which synchronizes the states of the drive and response systems. The dynamics of the synchronization errors can be expressed as:

$$\dot{e} = A_2 X_2 + f_2(X_2) - A_1 X_1 - f_1(X_1) + U \,. \tag{5}$$

Where $e = X_2 - X_1$. The objective of synchronization is to make $\lim_{t\to\infty} \| e(t) \| = 0$.

Now, the problem of synchronization between the drive and response systems can be translated into a problem of how to realize the asymptotical stabilization of the system (5). So the aim is to design a controller U for stabilizing the error dynamical system (5) at origin.

If let the Lyapunov function be $V(e) = e^T P e$, where P is a positive definite matrix, then $V(e)$ is a positive definite function. We assume that the parameters of the drive and response systems are known and the states of both systems are measurable. One may achieve the synchronization by selecting a nonlinear controller U to make $\dot{V}(e) = -e^T Q e$ be a negative definite function, i.e., Q is a positive definite matrix. Now, based on the Global Stability Theorem [21], since 1) $V(e)$ is positive definite 2) $\dot{V}(e)$ is negative definite 3) $V(e) \to \infty$ as $\| e \| \to \infty$, then states of the response and drive systems are globally asymptotically synchronized.

4. SYNCHRONIZING LORENZ AND ROSSLER SYSTEMS

In this section, the nonlinear control method is used to synchronize Lorenz and Rossler chaotic systems. To observe the synchronization behavior in these systems, we assume that Lorenz system drives the Rossler system. Therefore, we define the master and slave systems as follows.

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = c x_1 - x_1 z_1 - y_1 \\ \dot{z}_1 = x_1 y_1 - b z_1 \end{cases} \tag{6}$$

and

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 + u_1 \\ \dot{y}_2 = x_2 + \alpha y_2 + u_2 \\ \dot{z}_2 = \beta + z_2 (x_2 - \gamma) + u_3 \end{cases} . \tag{7}$$

We have introduced three control functions u_1, u_2 and u_3 in (7). Our aim is to determine the controller $U = [u_1, u_2, u_3]^T$ for the global synchronization of these two different chaotic systems. In order to estimate the mentioned functions, we subtract (6) from (7). We define the error system as the differences between the Lorenz (6) and the controlled Rossler (7) systems. Let us define the state errors between the *slave* system (7) that is to be controlled and the controlling system (6) as:

$$\begin{cases} e_1 = x_2 - x_1 \\ e_2 = y_2 - y_1 \\ e_3 = z_2 - z_1 \end{cases} . \tag{8}$$

Subtracting (6) from (7) and using the notation (8) yields.

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 + a x_1 - (a+1) y_1 - z_1 + u_1 \\ \dot{e}_2 = e_1 + \alpha e_2 + (1-c) x_1 + (1+\alpha) y_1 + x_1 z_1 + u_2 \\ \dot{e}_3 = e_1 e_3 - \gamma e_3 + x_1 e_3 - x_1 y_1 + (x_1 + e_1 + b - \gamma) z_1 + \beta + u_3 \end{cases} \tag{9}$$

Consider a Lyapunov function candidate as:

$$V(e) = e^T P e \tag{10}$$

where P is

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \tag{11}$$

It is clear that $V(e)$ is a positive definite function. Now, we choose a controller $U = [u_1, u_2, u_3]^T$ as follows.

$$\begin{cases} u_1(t) = -a\,x_1 + (1+a)y_1 + z_1 - e_1 \\ u_2(t) = -(1-c)x_1 - (1+\alpha)y_1 - x_1 z_1 - (\alpha+1)e_2 + e_1 \\ u_3(t) = -e_1 e_3 - x_1 e_3 + x_1(y_1 - z_1) - (e_1 + b - \gamma)z_1 - \beta + e_1 \end{cases} \tag{12}$$

With this choice, the time derivative of the Lyapunov function (10) is

$$\dot{V}(e) = e^T \begin{bmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4\gamma \end{bmatrix} e \tag{13}$$

i.e.,

$$\dot{V}(e) = -e^T Q e \quad \text{and} \quad Q = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 22.8 \end{bmatrix}.$$

Based on Lyapunov stability theory, this means that $\lim_{t\to\infty} \| e(t) \| = 0$. Thus, the drive and response systems are globally asymptotically synchronized.

5. SIMULATION RESULTS

In this section, numerical simulations are given using MATLAB. The fourth order Runge-Kutta integration method is used to solve two systems of differential equations (6) and (7). In addition, a time step size 0.001 is employed. We select the parameters of Lorenz system as $a = 10$, $b = 8/3$, $c = 28$ and the parameters of Rossler system as $\alpha = 0.2$, $\beta = 0.2$, $\gamma = 5.7$. Therefore, both Lorenz and Rossler systems exhibit chaotic behavior. The initial values of the *master* and *slave* systems are $x_{10} = -10$, $y_{10} = -14$, $z_{10} = 16$ and $x_{20} = 1$, $y_{20} = 0.5$, $z_{20} = 1$, respectively. These choices result in initial errors of $e_{10} = 11$, $e_{20} = 14.5$, and $e_{30} = -15$. The diagram of the Rossler system controlled to be Lorenz system accompanied with the control functions are shown in Fig. 3(a-c). The dynamics of the synchronization errors for the *master* and *slave* systems are given in Fig. 4.

6. SYNCHRONIZATION IN PRESENCE OF A NOISY CHANNEL SYNCHRONIZING

In our simulations, it is assumed that a Gaussian white noise $\xi(t)$ with zero mean and intensity σ is added to the transmitter output.

Although boundedness of the noise is important for theoretical investigations, it is interesting how the proposed schemes work in case of Gaussian noise which is not bounded [22]. The problem is examined by computer simulations.

Fig. 5(a-c) illustrates the influence of the noise with $\sigma = 10^{-2}$ in the transmission channel. Fig. 6 shows the dynamics of synchronization errors. One can notice that the message can be properly recovered in this case too.

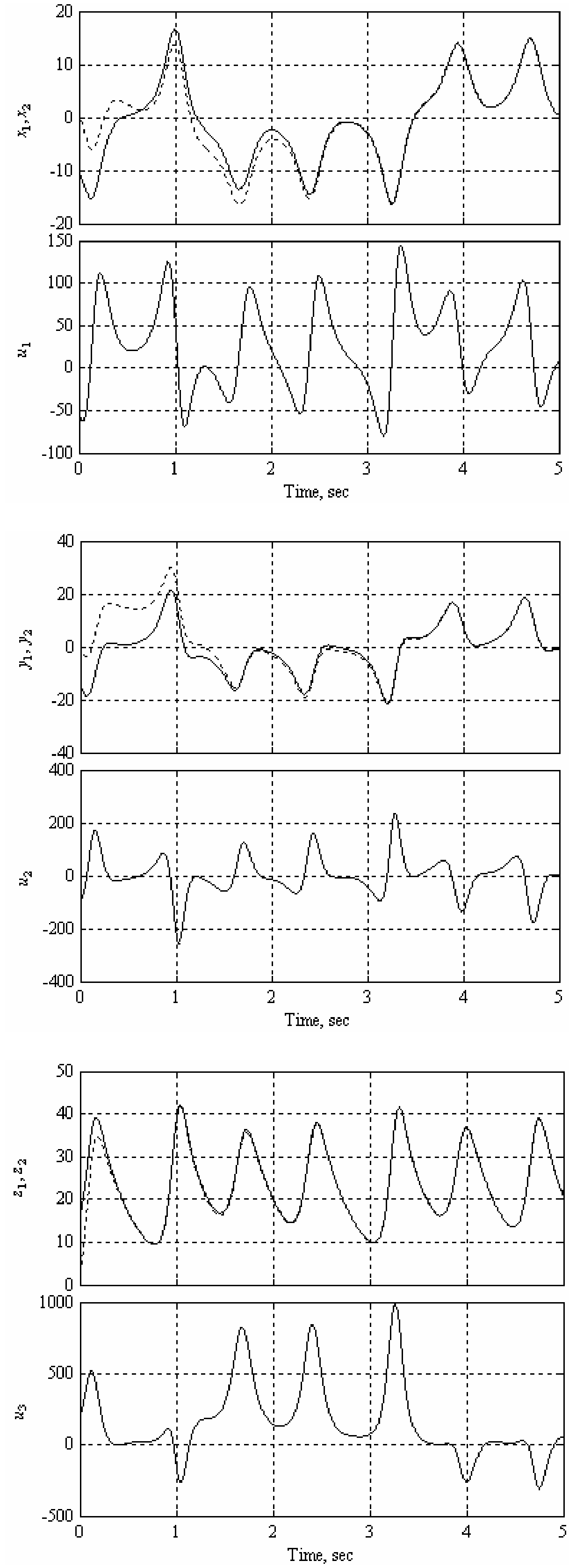


Fig. 3 Results of synchronizing the Rossler and Lorenz systems using nonlinear control method.

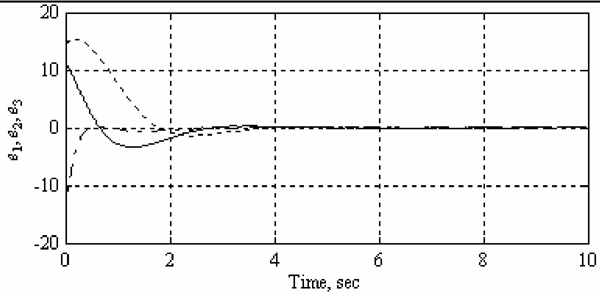


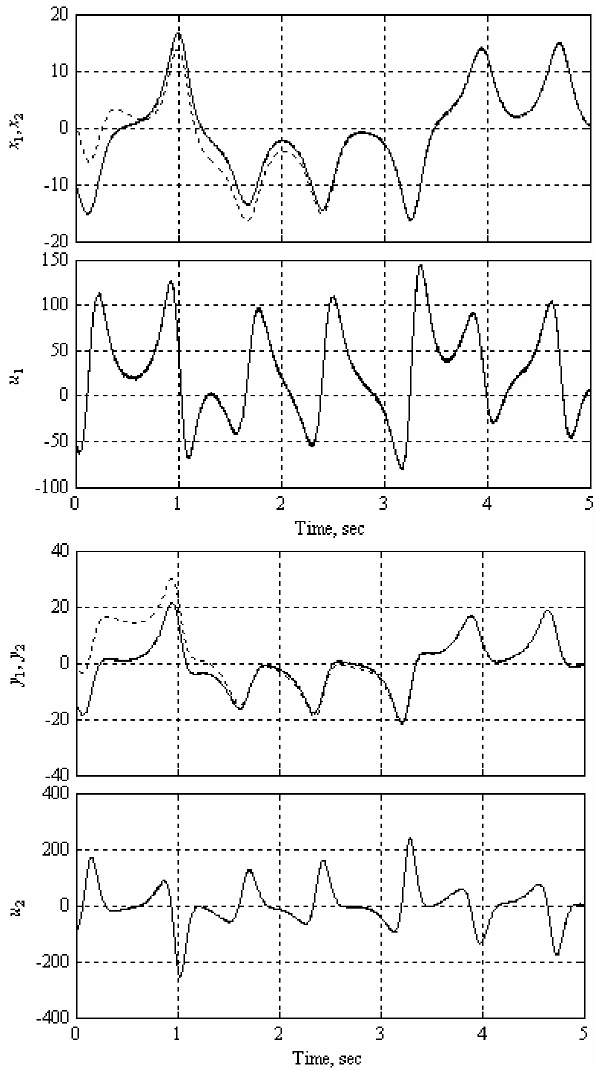
Fig. 4 Synchronization error (e_1, e_2, e_3) between the Lorenz and Rossler systems.

7. CONCLUSION

This paper demonstrates that chaos synchronization between two different chaotic systems is achieved using nonlinear control method. Rossler system is controlled to be Lorenz system. Since the Lyapunov exponents are not required for the calculation, this method is effective and convenient to synchronize two different chaotic systems. It also provides reasonable values of synchronization error in noisy case.

ACKNOWLEDGMENTS

The results presented in this paper come from a project that is supported in part by Iran Communication Research Center.



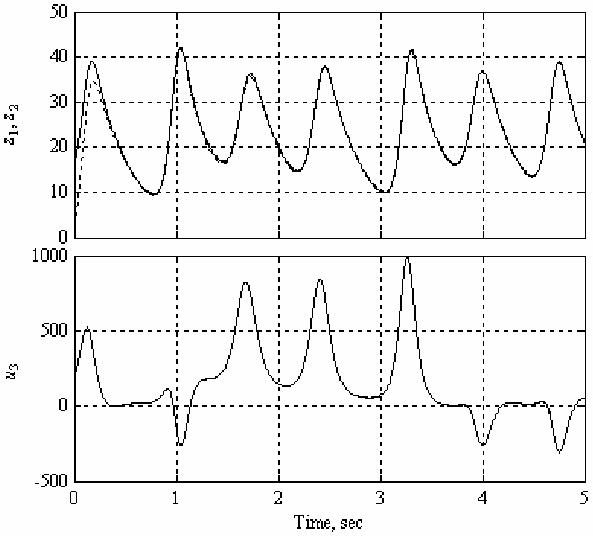


Fig. 5 Results of synchronizing the Lorenz and Rossler systems in presence of noise using nonlinear control method.

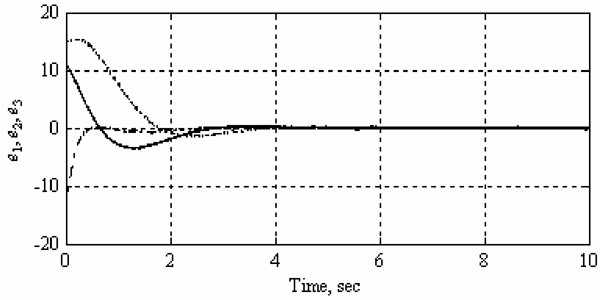


Fig. 6 Dynamics of synchronization error (e_1, e_2, e_3) for the Lorenz and Rossler systems, in presence of noise.

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