A Kalman Filter Localization Method for Mobile Robots

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Abstract: In this paper, we investigate an improved mobile robot localization method using Kalman filter. The highlight of the paper lies in the formulation of combined Kalman filter and its application to mobile robot experiment. The combined Kalman filter is a kind of extended Kalman filter which has an extra degree of freedom in Kalman filtering recursion. It consists of the standard Kalman filter, i.e., the predictor-corrector and the perturbation estimator which reconstructs unknown dynamics in the state transition equation of mobile robot. The combined Kalman filter (CKF) enables to achieve robust localization performance of mobile robot in spite of heavy perturbation such as wheel slip and doorsill crossover which results in large odometric errors. Intrinsically, it has the property of integrating the innovation in Kalman filtering, i.e., the difference between measurement and predicted measurement and thus it is so much advantageous in compensating uncertainties which has not been reflected in the state transition model of mobile robot. After formulation of the CKF recursion equation, we show how the design parameters can be determined and how much beneficial it is through simulation and experiment for a two-wheeled mobile robot under indoor GPS measurement system composed of four ultrasonic satellites. In addition, we discuss what should be considered and what prerequisites are needed to successfully apply the proposed CKF in mobile robot localization.

Keywords: Mobile robot, Localization, Kalman filter, Perturbation estimator, Indoor GPS.

1. INTRODUCTION

As a first step solution for the robot to navigate environments, localization problem answers the question “where I am.” Although there were great research works on the mobile robot for the commercialization and industrialization of non-industrial robots such as home service robot, guard robot, and entertainment robot, one of the most fundamental problem, the localization to find robot’s position is still a hot issue. To get a reliable localization solution with cheap cost, an innovation in sensor technology or practical alternatives such as RFID environmental sensor seems to have priority. However, software algorithms to best utilize given sensors information should be evolved also to get best estimates of robot location.

Due to the simplification when representing the probability distribution of the robot’s belief about where it is, the Kalman filter localization method is very efficient comparing with the general Markov localization method [1]. Moreover, the Kalman filter provides the framework for sensor fusion which takes into account all the information from heterogeneous sensors and produces an optimal output given the statistics of system noise and measurement noise. Although the Kalman filter has been successfully applied for mobile robot localizations [1-3], a drawback of the Kalman filter is that it guarantees minimum variance output of state estimation error under the assumption that the system model is perfect and the system and measurement noise processes are white and Gaussian [4,5]. However, as we know, in many cases it is not reasonable to assume white noise, and the perfect system knowledge is impossible as well. Hence, as the deviation of the assumptions from the real gets larger, the resulting outputs will be quite different from the optimal case. Then, what is the practical approach to get reliable estimates by Kalman filtering even when the assumptions are not valid any more?

The motivation of this paper is to discuss a practical Kalman filtering method which is able to achieve robust localization of mobile robots. The highlight is to adopt a perturbation estimator [6,7] in the Kalman filter framework, which reconstructs total amount of perturbation which distorts nominal system. In the state estimation context, as the system model is closer to the actual system behavior, the more accurate estimates can be expected. By adding the perturbation estimates to the prediction equation (the copy of system model) in Kalman filter, the state transition of prediction equation will get closer to the real system and resultantly, the combined Kalman filter-perturbation estimator can produce enhanced localization performance.

After describing system model in the following Section 2, a set of recursive equations of the Combined Kalman Filter (CKF) which includes perturbation estimation process is formulated in Section 3 in terms of the given state transition model and measurement model of mobile robot. In Section 4, characteristics of differential-type mobile robot are considered before the CKF is applied to the localization problem. The validity of the CKF algorithm is demonstrated through simulations and experiment for ultrasonic GPS mobile robot in Section 5. Finally, this paper is concluded in Section 6.

2. SYSTEM MODEL OF MOBILE ROBOT

First, consider the system model for state transition of wheeled mobile robot:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \tag{1}$$

which describes the nonlinear kinematic relationship how the robot pose is updated given the prior position $\mathbf{x}_{k-1}$, the relative displacement $\mathbf{u}_{k-1}$ during sampling interval, and the unknown system noise $\mathbf{w}_{k-1}$. Generally, the state vector of mobile robot is composed of planar location and orientation as $\mathbf{x}_k = \begin{bmatrix} x_k & y_k & \phi_k \end{bmatrix}^T$ with respect to a global coordinates system and the relative displacement vector can be determined using odometry data (encoder output). Considering the system noise $\mathbf{w}_{k-1}$ (perturbation) corrupting the nominal system, it includes both deterministic and random errors. Here, we assume that the perturbation estimation error $\tilde{\mathbf{w}}_k = \mathbf{w}_k - \hat{\mathbf{w}}_k$ to be discussed next is white Gaussian with zero-mean and covariance $\mathbf{Q}_k$, i.e., $\tilde{\mathbf{w}}_k \sim N(0, \mathbf{Q}_k)$ rather than assuming perfect system model and white Gaussian system noise $\mathbf{w}_k$

as in the conventional Kalman filter (KF). This assumption is necessary for the CKF derivation and actually more reasonable in practical situations.

Second, the measurement model for external sensors which detect the global or relative location of the robot or environment landmarks or features can be written as

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \tag{2}$$

with the assumption of white Gaussian measurement noise, $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$, the same as in the conventional KF.

To construct an extended Kalman filter (EKF) for the nonlinear system (1) and (2), they should be linearized along the nominal trajectory as follows.

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_{\mathbf{x},k}\mathbf{x}_{k-1} + \mathbf{A}_{\mathbf{u},k}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \\ \mathbf{z}_k &= \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{3}$$

where the following Jacobians are evaluated at every nominal state in real-time.

$$\mathbf{A}_{\mathbf{x},k} = \left(\frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{x}}\right)_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \mathbf{u}=\bar{\mathbf{u}}}}, \mathbf{A}_{\mathbf{u},k} = \left(\frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}}\right)_{\substack{\mathbf{x}=\bar{\mathbf{x}} \\ \mathbf{u}=\bar{\mathbf{u}}}} \\ \mathbf{H}_k = \left(\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right)_{\mathbf{x}=\bar{\mathbf{x}}} \tag{4}$$

3. COMBINED KALMAN FILTER-PERTURBATION ESTIMATOR

For the location update model (1) and measurement model (2) of mobile robot, the overall procedure of the CKF formulation and how the perturbation estimator works in CKF is addressed.

3.1 Predictor-Corrector

As is well known, the conventional KF has two steps in estimation, predictor and corrector. But the CKF in this paper will take the form of predictor-corrector- perturbation estimator. First, in terms of the state transition model (1), the *predictor* for prior estimate update ($\hat{\mathbf{x}}_k^-$) in CKF is given by

$$\hat{\mathbf{x}}_k^- = \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+, \hat{\mathbf{u}}_{k-1}) + \hat{\mathbf{w}}_{k-1}, \tag{5}$$

which requires the perturbation estimate $\hat{\mathbf{w}}_{k-1}$ additionally unlike in the conventional KF with the posterior estimate $\hat{\mathbf{x}}_{k-1}^+$ one step before and the relative displacement estimate.

Second, the *corrector* for posterior estimate update ($\hat{\mathbf{x}}_k^+$) corresponds to the measurement update process which feedbacks the residual between current measurement and prior estimate one step before as

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - h(\hat{\mathbf{x}}_k^-)\right) \tag{6}$$

where the feedback gain $\mathbf{K}_k$ called Kalman gain is an optimal weighting which minimizes the variance of the posterior estimation error. Let $\tilde{\mathbf{x}}_k^+ = \mathbf{x}_k - \hat{\mathbf{x}}_k^+$ the posterior estimation error and $\tilde{\mathbf{z}}_k = \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-$ the predicted measurement error. Then, the orthogonality principle which states that the state estimation error is orthogonal to the linear vector space of measurements [3,4] implies that $E[\tilde{\mathbf{x}}_k^+\tilde{\mathbf{z}}_k^{\mathbf{T}}] = 0$. Using the linear system model (3) and the corrector (6), we obtain the error equations:

$$\tilde{\mathbf{x}}_k^+ = \tilde{\mathbf{x}}_k^- - \mathbf{K}_k(\mathbf{H}_k\tilde{\mathbf{x}}_k^- + \mathbf{v}_k), \quad \tilde{\mathbf{z}}_k = \mathbf{H}_k\tilde{\mathbf{x}}_k^- + \mathbf{v}_k, \tag{7}$$

and thus

$$E[\{\tilde{\mathbf{x}}_k^- - \mathbf{K}_k(\mathbf{H}_k\tilde{\mathbf{x}}_k^- + \mathbf{v}_k)\}\{\mathbf{H}_k\tilde{\mathbf{x}}_k^- + \mathbf{v}_k\}^T] = 0 \tag{8}$$

Considering the uncorrelatedness assumption between state and measurement noise, this relationship leads to the Kalman gain:

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \tag{9}$$

with $\mathbf{P}_k^- \triangleq E[\tilde{\mathbf{x}}_k^-\tilde{\mathbf{x}}_k^{-T}]$ the definition of the prior error covariance and the assumption of $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$.

To update the Kalman gain at sampling times, we need the propagation equations for the prior error covariance ($\mathbf{P}_k^-$) and the posterior error covariance $\mathbf{P}_k^+ \triangleq E[\tilde{\mathbf{x}}_k^+\tilde{\mathbf{x}}_k^{+T}]$. From the linear model (3) and predictor (5), we have

$$\tilde{\mathbf{x}}_k^- \triangleq \mathbf{x}_k - \hat{\mathbf{x}}_k^- = \mathbf{A}_{\mathbf{x},k}\tilde{\mathbf{x}}_{k-1}^+ + \mathbf{A}_{\mathbf{u},k}\mathbf{u}_{k-1} + \tilde{\mathbf{w}}_{k-1} \tag{10}$$

where the last term means perturbation estimation error $\tilde{\mathbf{w}}_{k-1} \triangleq \mathbf{w}_{k-1} - \hat{\mathbf{w}}_{k-1}$. Here, it is necessary to assume that $\tilde{\mathbf{x}}_k^-$, $\mathbf{u}_k$, $\tilde{\mathbf{w}}_k$, and $\mathbf{v}_k$ are mutually uncorrelated. Then, by using (10), the propagation equation of $\mathbf{P}_k^-$ can be derived as

$$\mathbf{P}_k^- = E[\tilde{\mathbf{x}}_k^-\tilde{\mathbf{x}}_k^{-T}] = \mathbf{A}_{\mathbf{x},k}\mathbf{P}_{k-1}^+\mathbf{A}_{\mathbf{x},k}^T + \mathbf{A}_{\mathbf{u},k}\mathbf{U}_{k-1}\mathbf{A}_{\mathbf{u},k}^T + \mathbf{Q}_{k-1} \tag{11}$$

with the definitions of $\mathbf{Q}_k \triangleq E[\tilde{\mathbf{w}}_k\tilde{\mathbf{w}}_k^T]$ and $\mathbf{U}_k \triangleq E[\mathbf{q}_k\mathbf{q}_k^T]$, where it is assumed that the relative displacement $\mathbf{u}_k$ has random noise uncertainty $\mathbf{q}_k$ with $\mathbf{q}_k \sim N(0, \mathbf{U}_k)$. Also by using (7), we have the propagation equation of $\mathbf{P}_k^+$:

$$\mathbf{P}_k^+ \triangleq E[\tilde{\mathbf{x}}_k^+\tilde{\mathbf{x}}_k^{+T}] = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^- \tag{12}$$

3.2 Perturbation estimator

As a matter of fact, the formulation is the former section is just the same as the conventional KF except that the predictor in (5) includes the perturbation estimate. Now, it is discussed how the perturbation estimation process is formulated and what benefits it give in Kalman filtering.

In the state transition equation in (1), the system noise $\mathbf{w}_{k-1}$ denotes all kind of unmodeled effects which perturbs the nominal system, $\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})$. Although the conventional KF assumes white Gaussian noise, actually the system noise contains unknown deterministic error sources coming from the system and environment as well as random error sources which are colored noises rather than white. The perturbation estimator in this section is to reconstruct the deterministic noise quantities indirectly.

A simple and effective way to detect the perturbation $\mathbf{w}_{k-1}$ is to use the inverse model of nominal system. From (1), the equivalent perturbation in time-domain can be described as

$$\mathbf{w}_{eq,k-1} = \mathbf{x}_k - \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \tag{13}$$

Then, we can estimate the perturbation by the equation

$$\hat{\mathbf{w}}_k = \mathbf{w}_{eq,k-1} = \mathbf{x}_k - \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \tag{14}$$

using one-step delayed signals because $\mathbf{x}_{k+1}$ is not available at the *k*-th step. This kind of schemes making use of inverse model technique to estimate unmodeled dynamics can be found in [6-9] according to their applications. When the sampling is sufficiently fast comparing with the perturbation change, Eq. (14) works well.

By the way, too high frequency components in (14) exceeding the bandwidth of Kalman filter dynamics may cause adverse effect in the state estimation accuracy of (5). Hence, it is reasonable to adjust the perturbation estimate input by applying low-pass-filtering to (14) like

$$\hat{\mathbf{w}}_k = \mathbf{F}\cdot\mathbf{w}_{eq,k-1} = \mathbf{F}\cdot\left(\mathbf{x}_k - \mathbf{f}(\mathbf{x}_{k-1},\mathbf{u}_{k-1})\right) \qquad (15)$$

where the diagonal terms of the matrix $\mathbf{F}$ are low pass filters (LPFs) with unity DC gain. For example, in case of the mobile robot system model, we can let

$$\begin{aligned}\mathbf{F} &= diag\left(F_1(z), F_2(z), F_3(z)\right) \\ &= diag\left(\frac{b_1}{1-a_1z^{-1}}, \frac{b_2}{1-a_2z^{-1}}, \frac{b_3}{1-a_3z^{-1}}\right)\end{aligned} \qquad (16)$$

with 1st order filters, where $F_i(1)=1 \to a_i + b_i = 1\ (i = 1 \sim 3)$. In the above, if $a_i \to 0, b_i \to 1$, we have $\mathbf{F}=\mathbf{I}$ and the filter does not work at all and if $a_i \to 1, b_i \to 0$, it is the same as the situation that the perturbation estimator is not applied.

When applying the perturbation estimator (15) in Kalman filtering, it is desirable to use more accurate posterior estimates after measurement update by the corrector (6). Then, we finally have the perturbation estimate update equation for CKF as

$$\hat{\mathbf{w}}_k = \mathbf{F}\cdot\mathbf{w}_{eq,k-1}(\hat{\mathbf{x}}_k^+) = \mathbf{F}\cdot\left(\hat{\mathbf{x}}_k^+ - \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+,\mathbf{u}_{k-1})\right) \qquad (17)$$

3.3 Characteristics of perturbation estimator

By subtracting (5) from (1), we have the following error equation.

$$\tilde{\mathbf{x}}_k^- = \mathbf{f}(\mathbf{x}_{k-1}) - \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+) + \tilde{\mathbf{w}}_{k-1} \qquad (18)$$

As shown, the disturbing term of the error equation has been changed from the real perturbation $\mathbf{w}_{k-1}$ in conventional KF to the residual perturbation $\tilde{\mathbf{w}}_{k-1}$ in CKF, which implies that the accuracy of the prior estimate can be improved as far as the norm of the error maintains $\|\tilde{\mathbf{w}}_{k-1}\| < \|\mathbf{w}_{k-1}\|$ by the successful action of the perturbation estimator (17).

Second, by substituting (5) into (6), we obtain

$$\hat{\mathbf{x}}_k^+ = \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+,\hat{\mathbf{u}}_{k-1}) + \mathbf{K}_k\left(\mathbf{z}_k - h(\hat{\mathbf{x}}_k^-)\right) + \hat{\mathbf{w}}_{k-1} \qquad (19)$$

and using (19), Eq. (17) can be written as

$$\hat{\mathbf{w}}_k = \mathbf{F}\cdot\left(\hat{\mathbf{w}}_{k-1} + \mathbf{K}_k\left(\mathbf{z}_k - h(\hat{\mathbf{x}}_k^-)\right)\right), \qquad (20)$$

which illustrates that the perturbation estimator (17) intrinsically has the property of integrating the innovation $\tilde{\mathbf{z}}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_k^-)$, i.e., the residual of predicted measurement error. While, the low pass filter $\mathbf{F}$ in (20) can be treated as an integral gain.

The innovation feedback term in the corrector (6) make it possible that the estimates converge to the real states in an optimal way which produces minimum variances of the estimation error. However, the optimality is true only when the Kalman filter assumptions are valid and not the case when there exist non-white system noises. In fact, the innovation process in the corrector (6) corresponds to the proportional (P) control for the estimation error and the Kalman gain to the optimal P gain. On the other hand, the perturbation estimator (17) achieves integral control for the innovation as we have shown in (20).

In feedback control problems, it is common-sense that the proportional (P) control is not sufficient and an integral action is indispensable to regulate the control error to zero when there exist modeling error and external disturbance. Likewise, when we have not a reliable system model, other than the innovation feedback, another mechanism to compensate the effect of uncertainties will help to find true states in Kalman filtering and other estimation problems also.

In view of estimation problem, it is also common-sense that as the system model is closer to the actual system, the more accurate estimates can be expected. In this sense, the perturbation estimate added in (5) tries to make the prediction behavior close to the actual system behavior. As a simple example, if a biased deterministic noise is inserted in the system noise $\mathbf{w}_{k-1}$ in (1), the KF produces biased estimates since it has no integral function, but the CKF with perturbation estimator will show better results due to the integral property which has been implied in (18) and (20).

3.4 Combined Kalman filter-Perturbation estimator

By arranging the results in the former section, the CKF recursion equations to apply to the mobile robot localization are given in Table 1, where it should be noted that the last equation can be implemented after expanding the LPF $\mathbf{F}$ an example of which was taken in (16).

When evaluating the Jacobian matrices to update two covariances and Kalman gain, the most recent state estimates should be used as follows.

$$\mathbf{A}_{\mathbf{x},k} = \left(\frac{\partial\mathbf{f}(\mathbf{x})}{\partial\mathbf{x}}\right)_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^+} \quad \mathbf{A}_{\mathbf{u},k} = \left(\frac{\partial\mathbf{f}(\mathbf{x})}{\partial\mathbf{u}}\right)_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^+} \quad \mathbf{H}_k = \left(\frac{\partial\mathbf{h}(\mathbf{x})}{\partial\mathbf{x}}\right)_{\mathbf{x}=\hat{\mathbf{x}}_k^-} \qquad (21)$$

On the other hand, performance tuning parameters of CKF are $\mathbf{Q}_k$, $\mathbf{R}_k$, $\mathbf{U}_k$, and additionally $\mathbf{F}$ for perturbation estimator. Conclusively, the perturbation estimation process in CKF enhances performance robustness of state estimation by providing one more freedom in performance tuning.

TABLE I
RECURSIVE EQUATIONS OF CKF

Prior estimate update : $\hat{\mathbf{x}}_k^- = \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+,\hat{\mathbf{u}}_{k-1}) + \hat{\mathbf{w}}_{k-1}$
Prior error covariance update : $\mathbf{P}_k^- = E[\tilde{\mathbf{x}}_k^-\tilde{\mathbf{x}}_k^{-T}] = \mathbf{A}_{\mathbf{x},k}\mathbf{P}_{k-1}^+\mathbf{A}_{\mathbf{x},k}^T + \mathbf{A}_{\mathbf{u},k}\mathbf{U}_{k-1}\mathbf{A}_{\mathbf{u},k}^T + \mathbf{Q}_{k-1}$
Kalman gain update : $\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1}$
Posterior estimate update : $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - h(\hat{\mathbf{x}}_k^-)\right)$
Posterior error covariance update : $\mathbf{P}_k^+ \triangleq E[\tilde{\mathbf{x}}_k^+\tilde{\mathbf{x}}_k^{+T}] = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^-$
Perturbation estimate update: $\hat{\mathbf{w}}_k = \mathbf{F}\cdot\mathbf{w}_{eq,k-1}(\hat{\mathbf{x}}_k^+) = \mathbf{F}\cdot\left(\hat{\mathbf{x}}_k^+ - \mathbf{f}(\hat{\mathbf{x}}_{k-1}^+,\mathbf{u}_{k-1})\right)$

4. WHEELED MOBILE ROBOT

In case of differential-type wheeled mobile robots, the nonlinear state transition equation in (1) can be described as [1,2]

$$\begin{bmatrix} x_k \\ y_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \phi_{k-1} \end{bmatrix} + \begin{bmatrix} \Delta s_{k-1}\cdot\cos(\phi_{k-1} + \frac{1}{2}\Delta\phi_{k-1}) \\ \Delta s_{k-1}\cdot\sin(\phi_{k-1} + \frac{1}{2}\Delta\phi_{k-1}) \\ \Delta\phi_{k-1} \end{bmatrix} + \begin{bmatrix} w_{x,k-1} \\ w_{y,k-1} \\ w_{\phi,k-1} \end{bmatrix} \qquad (22)$$

where Δs_{k-1} and $\Delta\phi_{k-1}$ respectively denote linear and angular displacement by differential wheels and are the components of which the relative displacement vector in (1) consists as $\mathbf{u}_{k-1} = \left[\Delta s_{k-1} \quad \Delta\phi_{k-1}\right]^T$.

Then, the Jacobians in (21) for the linear model are given by

$$\mathbf{A}_{\mathbf{x},k} = \begin{bmatrix} 1 & 0 & -\Delta s_{k-1}\cdot\sin(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) \\ 0 & 1 & \Delta s_{k-1}\cdot\cos(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) \\ 0 & 0 & 1 \end{bmatrix}$$
$$\mathbf{A}_{\mathbf{u},k} = \begin{bmatrix} \cos(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) & -\Delta s_{k-1}\cdot\sin(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) \\ \sin(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) & \Delta s_{k-1}\cdot\cos(\hat{\phi}_{k-1}^{+} + \frac{1}{2}\Delta\phi_{k-1}) \\ 0 & 1 \end{bmatrix} \tag{23}$$

with latest posterior estimates.

Second, if external measurement sensors can produce full-state of the robot directly like the indoor GPS rather than detecting artificial landmarks or natural features in the environment, Eq. (2) simply can be written as

$$\mathbf{z}_k = \begin{bmatrix} x_{m,k} \\ y_{m,k} \\ \phi_{m,k} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \\ \phi_k \end{bmatrix} + \begin{bmatrix} v_{x,k} \\ v_{y,k} \\ v_{\phi,k} \end{bmatrix} \tag{24}$$

and the Jacobian describing the linear relationship between measurements and states becomes the identity matrix:

$$\mathbf{H}_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{25}$$

For a two-wheeled mobile robot, the linear and angular displacement in (22) and (23) can be determined by $\Delta s_{k-1} = (\Delta s_R + \Delta s_L)/2$ and $\Delta\phi_{k-1} = (\Delta s_R - \Delta s_L)/b$ given the distance b between the two wheels and the right and left wheel displacement $(\Delta s_R, \Delta s_L)$ which can be calculated, e.g., by using encoder pulses. In deriving (11), the uncertain effects in relative displacement determination were assumed white Gaussian noise $\mathbf{U}_k \triangleq E[\mathbf{q}_k\mathbf{q}_k^T]$. But, this can be set to zero if the errors can be considered as purely deterministic.

5. SIMULATION AND EXPERIMENT

5.1 Simulation conditions

Let the robot follow the circular trajectory with diameter of 1 *m* shown in Fig. 5 with zero initial pose $x_k = y_k = \phi_k = 0$ and at the rate of 50 degrees turn per second. Then, the normal displacements $(\Delta s_R, \Delta s_L)$ of right and left wheel during sampling time can be easily determined. To generate odometric errors which perturb nominal state transition equation in (22), arbitrary disturbances are added to the nominal wheel displacements $(\Delta s_R, \Delta s_L)$ as

$$\begin{aligned} \Delta s'_R(t) &= \Delta s_R(t) + d_1(t) \\ \Delta s'_L(t) &= \Delta s_L(t) + d_2(t) \end{aligned} \tag{26}$$

where the disturbances can be considered to represent all kinds of error sources which cannot be detected by odometry and which result in range error, turn error and drift error by distorting normal odometric displacements. They include both deterministic and stochastic factors coming from robot's effectors and also from the environment such as uneven floor and obstacle.

Figure 1 shows the odometric disturbances for right and left wheel after 2 *sec* where the magnitudes are equal to normal displacements $(\Delta s_R, \Delta s_L)$ between 2 and 3.5 *sec* and half of them after 3.5 *sec*. It can be assumed that robot meets a threshold at 2 *sec* and heavy slip occurs after 3.5 *sec*. In simulations, actual behaviour of robot is determined by inserting the disturbances in Fig. 1 into the nonlinear system (22) without the last unknown term. Then, the system noises $\mathbf{w}_{k-1} = (w_{x,k-1}\, w_{y,k-1}\, w_{\phi,k-1})$ are effectively generated. But the normal odometric displacements $(\Delta s_R, \Delta s_L)$ corresponding to the circle trajectory are used in computing all recursion equations in Table 1.

Secondly, assuming the standard deviation of measurement error as $\sigma_{v,x} = \sigma_{v,y} = 5mm$ and $\sigma_{v,\phi} = \frac{\pi}{180}$ rad, which is related to the sensor spec, the sensor noise covariance is given as $\mathbf{R}_k = diag(5^2, 5^2, \left(\frac{\pi}{180}\right)^2)$. To make the measurement noise in (24) close to the Gaussian noise assumption $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$, a random number generation function is used. While, the update rate (sampling time) is 10 *msec* both in odometry and measurement.

Performance tuning of CKF can be performed in such a way: First fix the value of $\mathbf{R}_k$ by assuming sensory performance as the value and let $\mathbf{U}_k = 0$ by considering deterministic errors of relative displacements. Then, adjust the value of $\mathbf{Q}_k$ until good results come about. In succession, make the noise level satisfactory by adjusting the cut-off frequency of $\mathbf{F}$. Finally, the tuning parameters of CKF which result in a good performance after many trials have been obtained as the following values.

$$\begin{aligned} \mathbf{R}_k &= diag(\sigma_{v,x}^2, \sigma_{v,y}^2, \sigma_{v,\phi}^2) = diag(5^2, 5^2, \left(\tfrac{\pi}{180}\right)^2) \\ \mathbf{Q}_k &= diag(\sigma_{\tilde{w},x}^2, \sigma_{\tilde{w},y}^2, \sigma_{\tilde{w},\phi}^2) = diag(3^2, 3^2, \left(\tfrac{\pi}{360}\right)^2) \\ \mathbf{U}_k &= diag(\sigma_{q,x}^2, \sigma_{q,y}^2) = diag(0,0) \\ \mathbf{F} &= diag\left(\tfrac{b_1}{1-a_1 z^{-1}}, \tfrac{b_2}{1-a_2 z^{-1}}, \tfrac{b_3}{1-a_3 z^{-1}}\right) = \tfrac{0.5}{1-0.5z^{-1}} diag(1,1,1) \end{aligned} \tag{27}$$

The KF parameters are the same as the above except that the last filters are excluded.

5.2 Simulation results

The numerical results in Figs. 2~4 compare the estimation error between conventional KF and proposed CKF in X-, Y-position, and orientation, respectively. As shown, when large disturbances in Fig. 1 are occurred, the perturbation estimator in CKF detects them and the estimates are reflected in the prediction equation (5). In the sequel, the prior and posterior state estimates have been improved as much comparing with KF case. The CKF will become more advantageous when the disturbance is the larger. But the effect will not be so observable when the system noise is very weak.

In the Figs. 2~4, it is natural that prior errors are larger than posterior errors after measurement updates. It is notable that KF produced large errors which were over measurement errors but CKF maintained smaller error values than the measurement error. This means that the KF output is not reliable if too large perturbation is occurred to the robot. As arranged in Table 2, CKF shows better standard deviation of estimates. Finally, Fig. 5 shows how the actual robot trajectory deviates from the odometry trajectory computed in the algorithm when the disturbances in Fig. 1 corrupts the state transition equation (22).

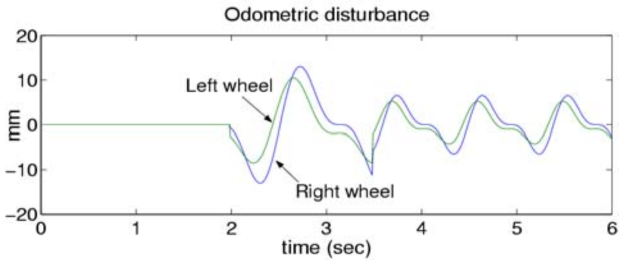

Fig. 1 Odometric disturbances.

$$\text{right } d_1(t) = |\Delta s_R(t)|(\sin(5t) + 0.5\sin(10t))$$
$$\text{left } d_2(t) = |\Delta s_L(t)|(\sin(5t+\alpha) + 0.5\sin(10t+\alpha))$$

In fact, Kalman theory explains that it produces estimates with smaller covariance than sensory covariance for perfect system model and white Gaussian noise. Hence, the CKF in this paper can be considered as a practical Kalman filtering approach which is more reliable when the system noise contains excessive deterministic errors.

TABLE II
COMPARISON OF STANDARD DEVIATION OF ERRORS

		KF	CKF
Prior error	X (mm)	5.00	2.86
	Y (mm)	7.36	2.89
	Phi (deg)	1.16	0.44
Posterior error	X (mm)	3.12	2.12
	Y (mm)	2.89	2.12
	Phi (deg)	0.44	0.34

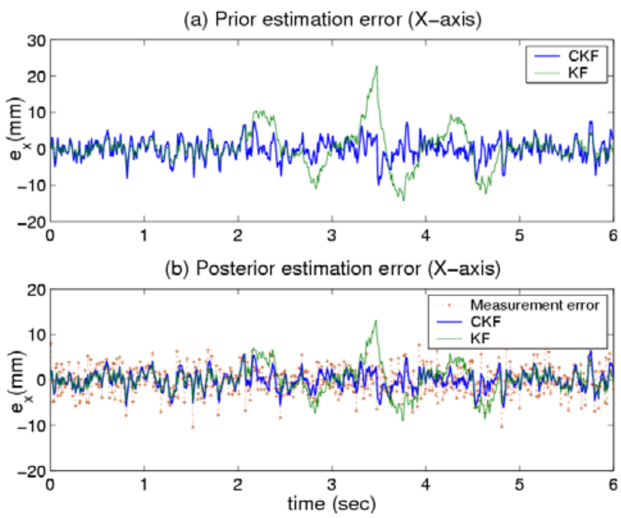

Fig. 2 X-position estimation erorr: (a) prior estimates, (b) posterior estimates.

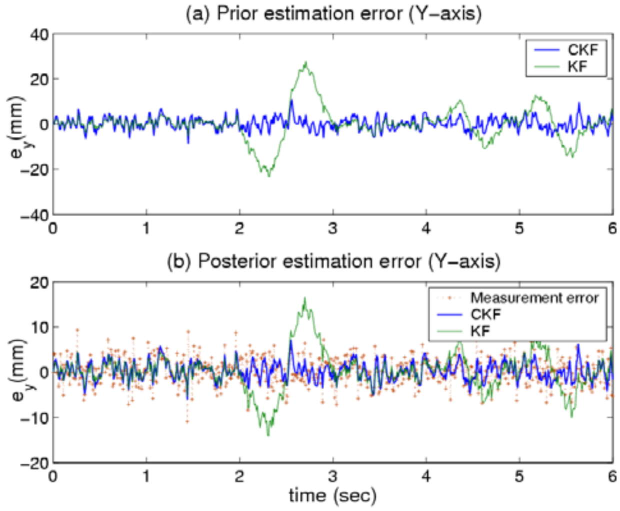

Fig. 3 Y-position estimation erorr: (a) prior estimates, (b) posterior estimates.

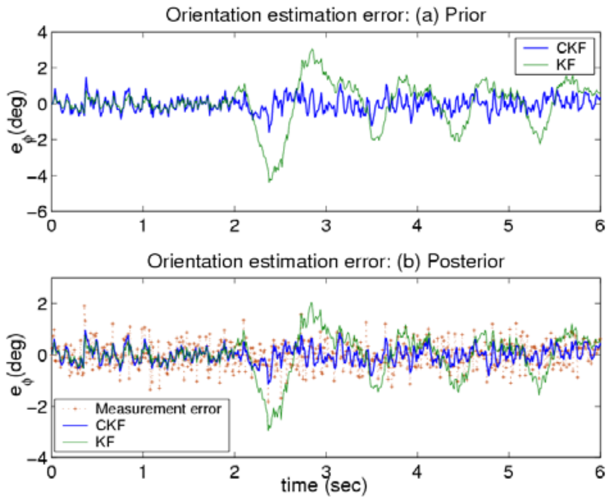

Fig. 4 Orientation estimation erorr: (a) prior estimates, (b) posterior estimates.

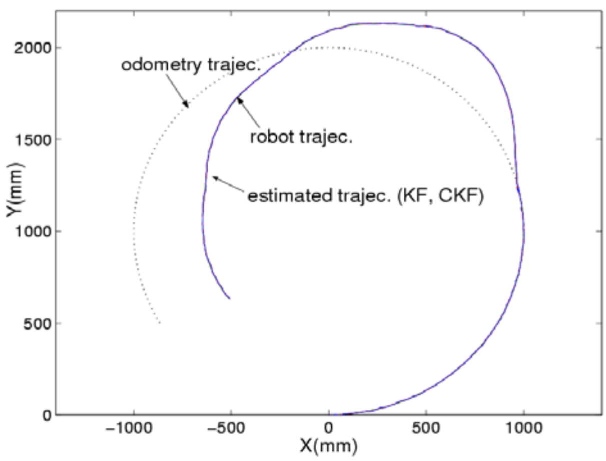

Fig. 5 Robot trajectory and odometry trajectory.

5.3 Experiment

The CKF was applied to the localization of the two-wheeled mobile robot in Fig. 6. The measurement system is an indoor GPS which consists of four ultrasonic satellites (U-SAT) installed in ceiling and two receivers equipped in robot. The robot navigates rectangular shaped trajectory on the quite even floor and the measurement update rate is 500 *msec*. While the tuning parameters used in the experiment were given by

$$\mathbf{R}_k = diag(10^2, 10^2, \left(\tfrac{\pi}{90}\right)^2) \quad \text{and} \quad \mathbf{Q}_k = diag(2^2, 2^2, \left(\tfrac{0.5\pi}{180}\right)^2)$$

as the variances for measurement error and for odometric movement error, respectively.

From the experimental result in Fig. 7, it was found that CKF estimates (circle) follows the measurements (cross) better than KF estimates (triangle). This indicates indirectly that the CKF estimates can be closer to the real trajectory. However, it is very hard to say that CKF is better from this result since the variance of measurements are too big and above all, we have not precision external sensor to produce exact robot location and to calibrate the measurements. In order to verify the CKF performance more clearly, a different experiment which can result in abrupt location change of robot is necessary such that the robot goes over a doorsill or bumps with some obstacles.

Fig. 6 Two-wheeled mobile robot with two U-SAT receivers.

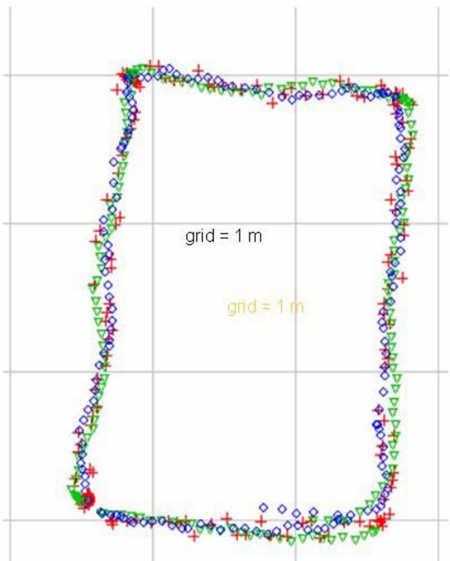

Fig. 7 Localization experiment:
cross(red): U-SAT sensor, triangle(green): KF, circle(blue): CKF

6. CONCLUSION

In this paper, we discussed the Kalman filtering localization in terms of the proposed combined Kalman filter (CKF). Through the investigation of the characteristics of CKF and simulation and experimental results, the CKF was proved to be very promising for mobile robot localization, specifically when there are large modeling errors and large environmental disturbances such as from uneven floor, doorsill, and other obstacles. One prerequisite so that the CKF with perturbation estimator can be successfully applied is that the measurement update should be as fast as the robot's deviation from nominal system model can be detected by the inverse model (14). In conclusion, the CKF make it possible to enhance the robustness of localization performance by providing one more freedom, the perturbation estimator, in Kalman filtering.

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