An Implementation of Discrete Mathematical Model for ECG waveform

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Abstract: This paper proposes a new design of the ECG simulator with high resolution by using small amount of memories based on discrete least square estimation equations instead of reading the stored data inside the look-up table. The experimental results have shown that the ECG simulator using discrete least square estimation equations can display the bipolar limb leads ECG signals with low PRD (percent root-mean-square difference) while taking the less amount of memories than the previous method which used the look-up table to store ECG data for ECG simulation.

Keywords: Discrete least square, Electrocardiogram (ECG).

1. INTRODUCTION

Electrocardiogram (ECG) detection is the medical diagnosis often applied for diagnosing the conditions of the patients. The measured ECG signals come from the heart muscle contraction and relaxation while the characteristics and shapes of the ECG signals will show the condition of heart muscle contraction and relaxation. The normal condition of heart will show the ECG signals standard shape and characteristics while the abnormal condition of heart will show the signal deviated form the standard pattern.

The ECG signal measurement requires ECG simulator with high accuracy which requires regular calibration before using this simulator. The old ECG simulator has stored data from the patient and normal ECG has been saved in the memory units as full table and then the ECG simulator machine will read the data inside the memory through a look-up table before displaying the simulated results for ECG machine calibration. The resolution of the simulated signal depends upon the number of bits in the stored data and the bit rate of data transfer, thus the high resolution ECG simulator requires a large amount of memories to store the data [1-3].

2. THEORY

2.1 Previous Design

The old principle for the construction of simulated ECG signal is to use the look up table to read the digitized ECG data stored in memory units as shown in Fig. 1, through the calculation in CPU before sending to DAC to convert the data into analog signal which is displayed on the screen. However, the resolution of digitized data depends upon the amount of stored data in the memory. Therefore, the higher resolution requires more memory space to store the data in look-up table.

2.2 Proposed Design

The new method uses mathematical equation of discrete least square estimation to implement the simulated ECG signals through the following procedures. The first step is to measure the ECG signal [4] from normal patient as shown in Fig. 2 and digitize the ECG signal by using AD-612 Data Acquisition Card (Humusoft S.R.O.) [5] to store the data in memory by using MATLAB and SIMULINK programs [6].

Digitization process will define that each ECG signal data pulse has 1024 values as shown in Fig. 3, and it has 11 sections of ECG data pulse as shown in Table 1.

After the division, the construction of polynomial term from each sections of ECG Data will be shown in Eq. (1) [7].

$$y_{k(m)} = a_0 + a_1 x_{k(m)} + a_2 x_{k(m)}^2 + \dots + a_n x_{k(m)}^n \tag{1}$$

The matrix for the calculation of coefficients $a_0, a_1, a_2, ..., a_n$ can be shown in Eq. (2).

$$\begin{bmatrix} \sum_{i=1}^{m} x_{ki}^0 & \sum_{i=1}^{m} x_{ki}^1 & \sum_{i=1}^{m} x_{ki}^2 & \cdots & \sum_{i=1}^{m} x_{ki}^n \\ \sum_{i=1}^{m} x_{ki}^1 & \sum_{i=1}^{m} x_{ki}^2 & \sum_{i=1}^{m} x_{ki}^3 & \cdots & \sum_{i=1}^{m} x_{ki}^{n+1} \\ & & \vdots & & \\ \sum_{i=1}^{m} x_{ki}^n & \sum_{i=1}^{m} x_{ki}^{n+1} & \sum_{i=1}^{m} x_{ki}^{n+2} & \cdots & \sum_{i=1}^{m} x_{ki}^{2n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{m} y_{ki} x_{ki}^0 \\ \sum_{i=1}^{m} y_{ki} x_{ki}^1 \\ \vdots \\ \sum_{i=1}^{m} y_{ki} x_{ki}^n \end{bmatrix} \tag{2}$$

3. DESIGN AND IMPLEMENTATION

The ECG signal is divided into 11 sections as shown in Fig. 3, will be applied to implement the polynomial terms as shown in Eq. (1) with $n = 4$ and the equations for each section of ECG data can be shown in Eqs.(3) – (13).

The actual implementation of ECG simulated signal will be executed on PC with the connection to MCS-51 microcontroller [8] as shown on the diagram in Fig. 4.

Table 1 Division of ECG Data.

y_{k1}	y_{k2}	y_{k3}	...	...	$y_{k(m)}$
x_{k1}	x_{k2}	x_{k3}	...	...	$x_{k(m)}$

$y_{k(m)}$: ECG signal pulse in each sampling.

$x_{k(m)}$: Time for each sampled ECG date.

m : Positive number.

k : ECG data in each section.

4. CALCULATION OF SIGNAL ERROR

The measurement of the error in simulated ECG signal generated by mathematical equations will be in percent root mean square difference (PRD) [9]. The calculation of PRD comes from the comparison between simulated ECG signals and the real ECG signals acquired by using AD-612 data acquisition card with the equation for calculating PRD can be shown in Eq. (14).

$$PRD = \left\{ \frac{\sum_{i=1}^{m} \left(y_{(org)ki} - y_{(rec)ki} \right)^2}{\sum_{i=1}^{m} \left(y_{(org)ki} \right)^2} \right\}^{\frac{1}{2}} \times 100\% \qquad (14)$$

When $y_{(org)ki}$: original ECG data.

$y_{(rec)ki}$: simulated ECG data calculated from polynomial

The 1st Section:

$$y_{1(m)} = 51.73401448 + 0.03887848 x_{1(m)} - 0.00030923 x_{1(m)}^2 + 1.3450652\text{e-}006 x_{1(m)}^3 - 1.95799658\text{e-}009 x_{1(m)}^4 \qquad (3)$$

The 2nd Section:

$$y_{2(m)} = 2.9121843\text{e-}005 - 6.8337501 x_{2(m)} + 0.0496220 x_{2(m)}^2 - 0.0001153 x_{2(m)}^3 + 8.7629799\text{e-}008 x_{2(m)}^4 \qquad (4)$$

The 3rd Section:

$$y_{3(m)} = 1.4140387\text{e-}009 - 29.3065286 x_{3(m)} + 0.1774381 x_{3(m)}^2 - 0.0003562 x_{3(m)}^3 + 2.3799828\text{e-}007 x_{3(m)}^4 \qquad (5)$$

The 4th Section:

$$y_{4(m)} = -5.3505822\text{e-}010 - 1413.0712891 x_{4(m)} + 8.0298958 x_{4(m)}^2 - 0.0152058 x_{4(m)}^3 + 9.5960359\text{e-}006 x_{4(m)}^4 \qquad (6)$$

The 5th Section:

$$y_{5(m)} = -3.5422509\text{e-}010 + 2686.0149841 x_{5(m)} - 14.6001690 x_{5(m)}^2 + 0.0264319 x_{5(m)}^3 - 1.5936042\text{e-}005 x_{5(m)}^4 \qquad (7)$$

The 6th Section:

$$y_{6(m)} = -4.2968056\text{e-}009 - 6786.7694931 x_{6(m)} + 34.9145802 x_{6(m)}^2 - 0.0598386 x_{6(m)}^3 + 3.416646\text{e-}005 x_{6(m)}^4 \qquad (8)$$

The 7th Section:

$$y_{7(m)} = 2.3887337\text{e-}010 + 5391.35252380371 x_{7(m)} - 26.7057103 x_{7(m)}^2 + 0.0440907 x_{7(m)}^3 - 2.4261708\text{e-}005 x_{7(m)}^4 \qquad (9)$$

The 8th Section:

$$y_{8(m)} = 1.12740083935137\text{e-}009 - 222.5883789 x_{8(m)} + 1.0463156 x_{8(m)}^2 - 0.0016382 x_{8(m)}^3 + 8.5454128\text{e-}007 x_{8(m)}^4 \qquad (10)$$

The 9th Section:

$$y_{9(m)} = 1.3616778\text{e-}006 - 0.509923697216436 x_{9(m)} + 0.0026026 x_{9(m)}^2 - 3.7805940\text{e-}006 x_{9(m)}^3 + 1.7894277\text{e-}009 x_{9(m)}^4 \qquad (11)$$

The 10th Section:

$$y_{10(m)} = 1.4763155\text{e-}006 - 41.1950299 x_{10(m)} + 0.1392266 x_{10(m)}^2 - 0.00015591 x_{10(m)}^3 + 5.7959711\text{e-}008 x_{10(m)}^4 \qquad (12)$$

The 11th Section:

$$y_{11(m)} = 4.43476367\text{e-}010 + 43.4618751 x_{11(m)} - 0.1322752 x_{11(m)}^2 + 0.0001343 x_{11(m)}^3 - 4.5454443\text{e-}008 x_{11(m)}^4 \qquad (13)$$

5. EXPERIMENTAL RESULTS

The researchers used the AD-612 data acquisition card converting ECG signals from lead I, lead II and lead III which consist of 1024 samples for each data. After that, the researchers applied (1)-(13) to implement the simulated ECG signals to display by MATLAB Program as shown in Fig. 5 while the calculated PRD will be as shown in Table 2. The comparison between the simulated ECG signals derived from data in the look-up table and the simulated ECG signal generated from equation will be shown in Table 3.

6. CONCLUSION

The experimental results have shown that simulated ECG signals derived from mathematical equations by applying discrete least square has low PRD with high accuracy. Furthermore, the simulated ECG signals use much less memory than the old look-up table method of ECG simulation. The results from this experiment become useful for the construction of a high resolution ECG simulator which takes much less memory and has smaller size of hardware.

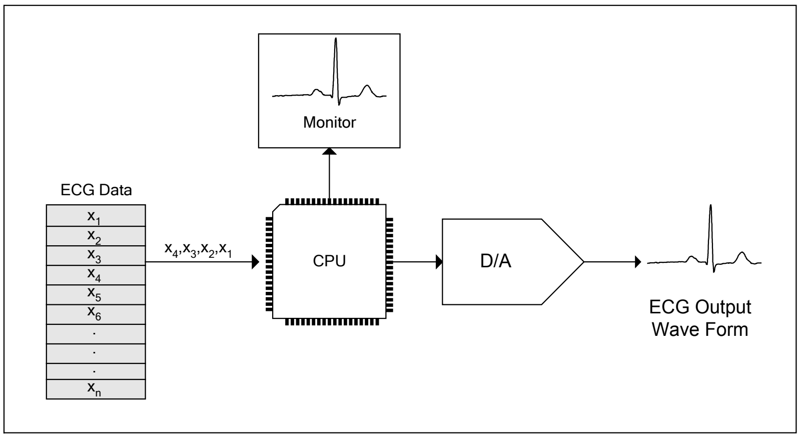

Fig.1 Diagram for the implementation of the simulated ECG signal by using look-up table

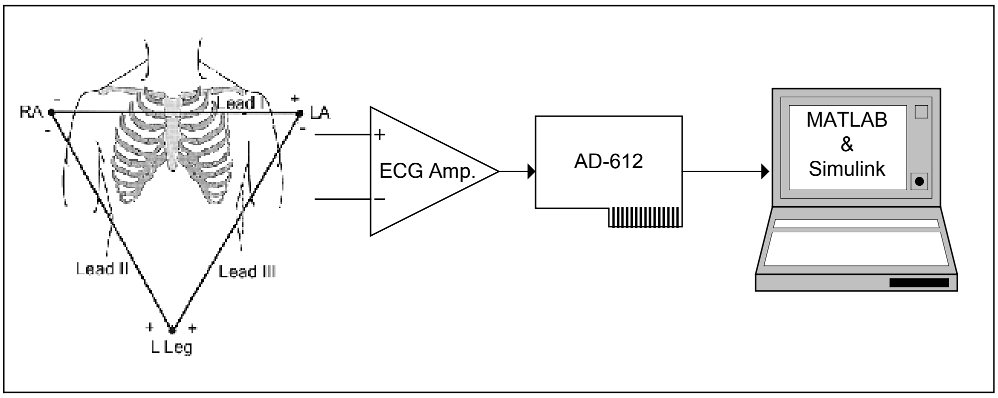

Fig.2 Measurement and digitization of ECG signals by using AD-612 data acquisition card

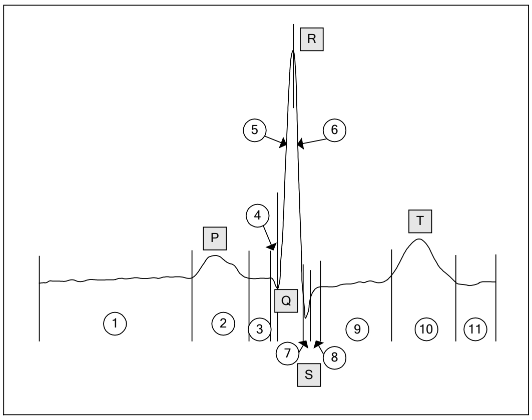

Fig.3 Division of ECG data into 11 sections

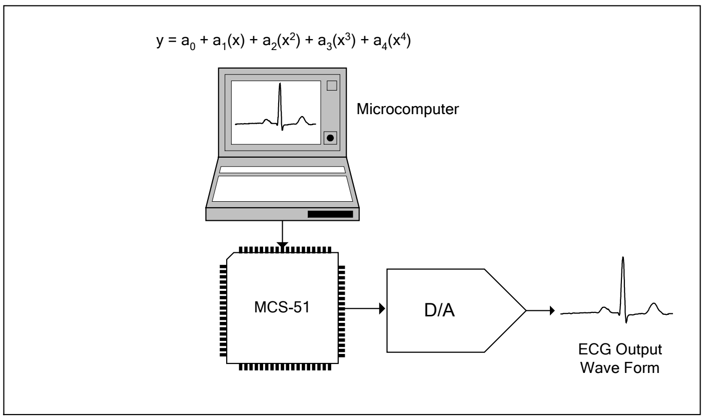

Fig. 4 Diagram for the reconstruction ECG signal by mathematical model

Table 2 Percent root-mean-square differences in each ECG signal

Lead	PRD (%)
I	17.55%
II	17.04%
III	16.94%

Table 3 Simulated ECG signal comparison

Resolution	Number of Data	
	Previous Design	ProposedDesign
1024	1024	55
2048	2048	55
4096	4096	55

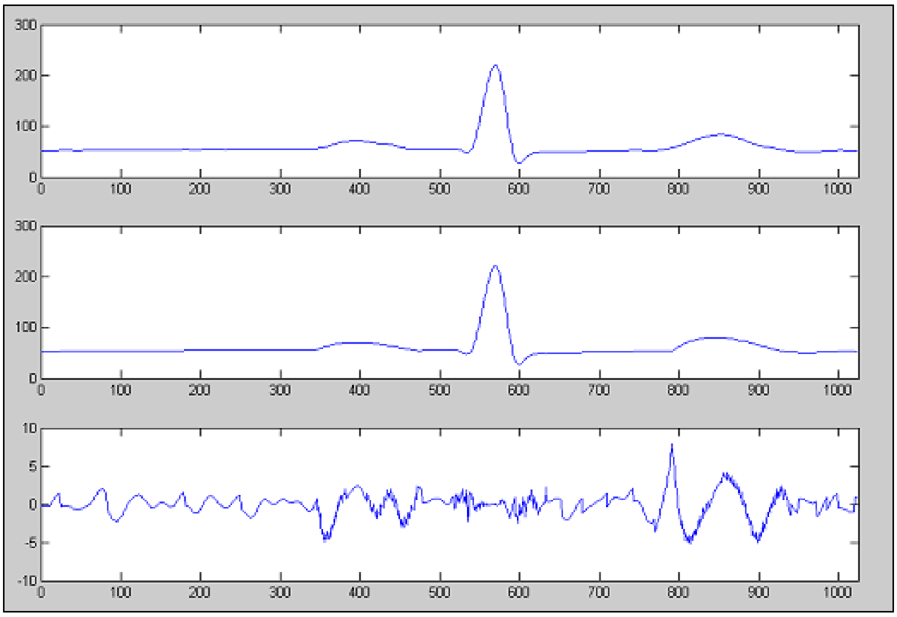

Fig. 5 Upper trace : Original signal lead I ECG signal
Middle trace : Reconstruction lead I ECG signal
Lower trace : Point-to-point of error (%)

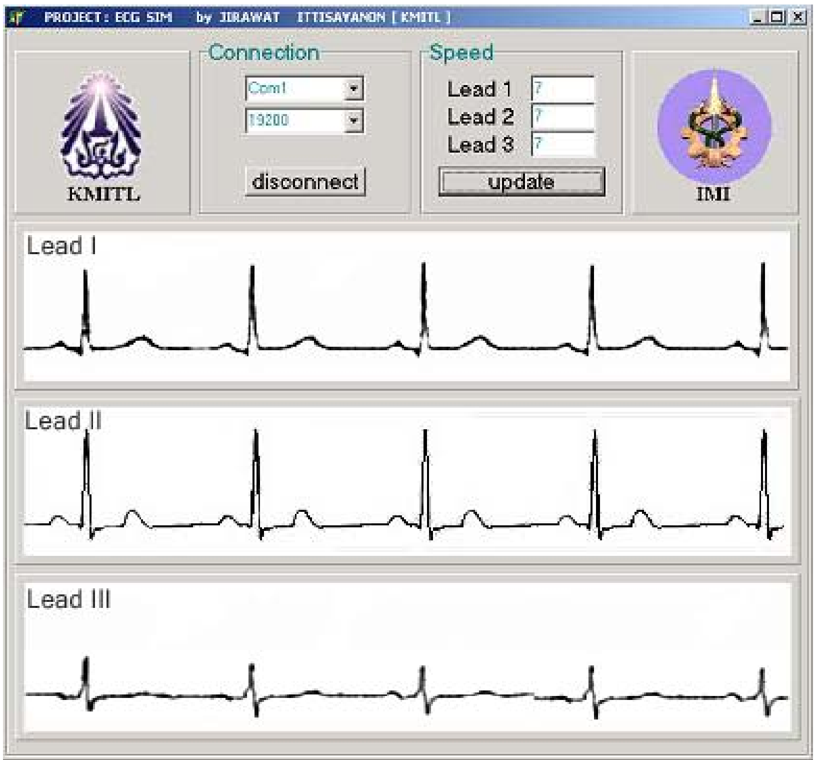

Fig. 6 Results of ECG lead I, II, III ECG reconstruction signal

REFERENCES

[1] W. J. Tompkins, “Twelve-lead simulation for testing interpretive ECG machines,” *Proc. of the Third Annual IEEE Symposium on Computer-Based Medical Systems*, pp.178-181, 1990.

[2] I. Sadighi and M. Kejariwal, “A Generalized ECG simulator: an educational tool,” *Images of the Twenty-First Century. Proc. of the Annual International Conference of the IEEE Engineering on Engineering in Medicine and Biology Society*, Vol. 6, pp.1963-1964, 1989.

[3] L. Shuqian and W. J. Tompkins, “Microcomputer-based simulator of the 12-lead electrocardiogram,” *Proc. of the Annual International Conference of the IEEE on Engineering in Medicine and Biology Society*, Vol. 1, pp.25-26, 1988.

[4] R. F. Coughlin, R.S. Villanucci, “*Introductory of operational amplifiers and linear ICs theory and experimentation,*” Prentice-Hall, 1990.

[5] User manual, “*Data acquisition card for PCI AD-612,*” Humusoft, 2003.

[6] User manual, “Real time toolbox for use with MATLAB R14,” Humusoft, 2003.

[7] W. Hager, “*Applied numerical linear algebra*,” Prentice Hall 1988.

[8] I. S. Mackenzie, “*The 8051 microcontroller,*” Macmillan 1992.

[9] W. J. Tompkins, “*Biomedical digital signal processing C- language examples and laboratory experiments for the IBM PC*,” Prentice Hall, 1993.