I-PDA controller design for Robotic Manipulator based on Coefficient Diagram Method with FFC

Young-su Lee[*], Dae-hyun Kim[*], Seung-chul Kim[*] , Young-do Lim[*]

* Department of Electronic and Computer Engineering, Donga University, Busan, Korea
(Tel : +82-2-200-6962; E-mail: lys4868@nate.com)

Abstract : In this paper, I-PDA controller based on Coefficient Diagram Method incorporating feedforward controller is applied by robotic manipulators. Robotic manipulator models contain uncertain elements, which are not known exactly. Therefore, the dynamics of robotic manipulators are generally classified as uncertain dynamic system. The controller considered for the robotic manipulators need to move payloads of different masses from one point to another with good balance of the stability and response, consequently we propose I-PDA controller based on Coefficient Diagram Method incorporating FFC. The effectiveness of the controller for different system type of robotic manipulators is demonstrated by the simulation results.

Keywords: I-PDA controller, FFC, FBC, CDM(Coefficient Diagram Method), Manipulator

1. INTRODUCTION

Given the dynamic equations of motion of a manipulator, the purpose of robot arm control is to maintain the dynamic response of the manipulator in accordance with some prespecified performance criterion. Although the control problem can be stated in such a simple manner, its solution is complicated by inertial forces, coupling reaction forces, and gravity loading on the links. In general, the control problem consists of (1) obtaining dynamic models of the manipulator, and (2) using these models to determine control laws or strategies to achieve the desired system response and performance [1].

The dynamics of *n*-link robotic manipulators are usually modeled by *n* coupled second-order nonlinear differential equations. Robotic manipulator models contain uncertain elements, which are not known exactly. Therefore, the dynamics of robotic manipulators are generally classified as uncertain dynamic system, and can be described by ordinary differential equation where the bound of uncertainty is known [2].

Thus, we present I-PDA controller based on Coefficient Diagram Method with Feedforward controller. The parameters of I-PDA controller are designed by the basis of the stability and speed of the object controlled.

The stability and speed are designed by Standard Stability Index γ_i and Equivalent Time Constant τ. It is effective that we control the most significant parts for Robotic manipulator as the stability and the response speed.

2. STRUCTURE OF I-PDA CONTROL SYSTEM WITH FFC

The I-PDA control system with FFC shown in Fig. 1 consists of a FFC, a feedback controller (FBC), an integral controller and a third-order plant. K_{pf}, K_{df} and K_{af}, and K_p, K_d and K_a are the proportional gain, derivative gain and acceleration gain of the FFC and of the FBC, respectively, K_I is the integral gain of the integral controller. $D_1(s)$ and $D_2(s)$ are the process step disturbance to be applied to the system. The transfer function from $R(s)$ to $C(s)$ is then given as

$$\frac{C(s)}{R(s)} = \frac{G_p(s)\left[\dfrac{K_I}{s} + K_{pf} + K_{df}s + K_{af}s^2\right]}{1 + G_p(s)\left[\dfrac{K_I}{s} + K_p + K_d s + K_a s^2\right]} \quad (1)$$
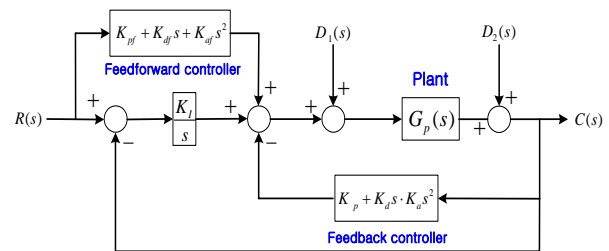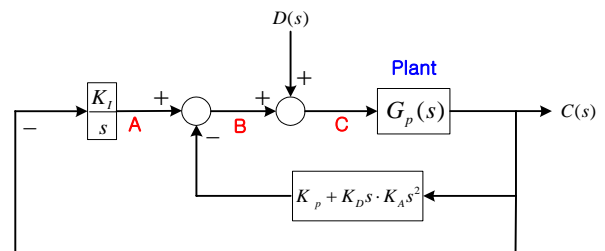

Fig. 1 Structure of I-PDA control system with FFC.

Fig. 2 Consideration of Disturbance elements in Fig.1.

Fig.2 is to simplify the consideration of disturbance elements in Fig.1.

$$A = \frac{K_I}{s}C(s) \quad (2)$$

$$B = A - C(s) \cdot \left(K_p + K_d s + K_a s^2\right) \quad (3)$$

$$C = D(s) \cdot \frac{K_I}{s}C(s) - \left(K_p + K_d s + K_a s^2\right) \cdot C(s) \quad (4)$$

The transfer function in Fig.2 from $R(s)$ to $C(s)$ is then given as

$$\frac{C(s)}{R(s)} = \frac{G_p(s)}{1 + G_p(s)\left[\frac{K_I}{s} + K_p + K_d s + K_a s^2\right]} \tag{5}$$

The proposed control system structure consisting of the CDM standard block diagram with the FFC and FBC is shown in Fig. 3. $A_p(s)$ and $B_p(s)$ are the polynomials of the plant $G_p(s)$, $A_c(s)$, $B_c(s)$ and $B_a(s)$ are the polynomials of the CDM controller, $B_{ff}(s)$ and $B_{fb}(s)$ are the polynomials of the Feedforward Controller and the Feedback Controller.

By rearranging the plant and the FBC, the $G_p*(s) = \dfrac{B_p*(s)}{A_p*(s)}$ can be obtained and called here as a modified plant. Then the transfer function from $R(s)$ to $C(s)$ of Fig. 3 is

$$\frac{C(s)}{R(s)} = \frac{B_p*(s)\left[B_a(s) + F_{ff}(s)A_c(s)\right]}{A_c(s)A_p*(s) + B_c(s)B_p*(s)} \tag{6}$$
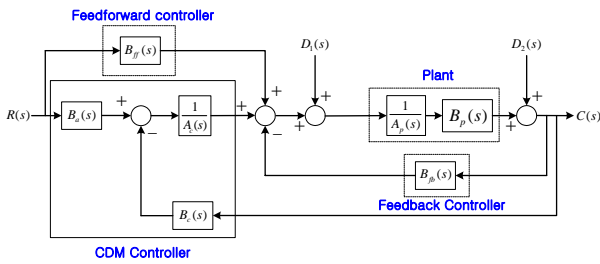

Fig. 3 SISO system with FFC and FBC.

3. COEFFICIENT DIAGRAM METHOD

The CDM is used to design the controller so that the step response of the controlled system satisfies both transient and steady state response specifications, and also satisfy the requirement of stability, faster response and robustness. Generally, the order of the controller designed by CDM is less than the order of the plant. However, when using the I-PDA controller for the third order plant, the order of the controller is equal to the order of the plant, but the integrator of the integral controller virtually makes the plant to be fourth order. Thus CDM condition is satisfied.

The polynomials form of the controller and the plant are generally be respectively written in the form (1)

$$\begin{aligned} A_c(s) &= l_\lambda s^\lambda + l_{\lambda-1}s^{\lambda-1} + \cdots + l_0 \\ B_c(s) &= k_\lambda s^\lambda + k_{\lambda-1}s^{\lambda-1} + \cdots + k_0 \end{aligned} \tag{7}$$

and

$$\begin{aligned} A_p(s) &= p_k s^k + p_{k-1}s^{k-1} + \cdots + p_0 \\ B_p(s) &= q_m s^m + q_{m-1}s^{m-1} + \cdots + q_0 \end{aligned} \tag{8}$$

Where $\lambda > k$ and $m < k$

The characteristic polynomials of the closed-loop controlled system shown in Fig. 3 can be given in the following form

$$\begin{aligned} P(s) &= a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \\ &= \sum_{i=0}^{n} a_i s^i \end{aligned} \tag{9}$$

Where $a_0, a_1, \cdots, a_n$ are the real coefficients.

The stability index γ_i, the equivalent time constant τ and the stability limit γ_i^* are defined as follows :

$$\gamma_i = \frac{a_i^2(s)}{a_{i+1}a_{i-1}} \tag{10}$$

$$\tau = \frac{a_1}{a_0} \tag{11}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \gamma_0, \gamma_n = \infty \tag{12}$$

Where $i = 1, \cdots, n-1$.

To meet the specifications, equivalent time constant τ and the standard values of the stability index γ_i are chosen as

$$t_s = 2.5\tau \sim 3\tau \tag{13}$$

$$\gamma_{n-1} = \cdots = \lambda_3 = \lambda_2 = 2, \quad \lambda_1 = 2.5 \tag{14}$$

In general, the settling time $t_s = 2.5\tau$, and the stability index $\lambda_1 = 2.5$, $\lambda_2 = \lambda_3 = 2$ are strongly recommended due to the stability and the step response requirement. However, it is not necessary to always define $t_s = 2.5\tau$ and $\gamma_4 \sim \gamma_{n-1} = 2/$

Then the condition for the stability index can be relaxed to

$$\gamma_i > 1.5\gamma_i^* \tag{15}$$

The standard values of the stability index γ_i in (14) can be used if the following condition in (15) is satisfied.

$$p_k / p_{k-1} > \tau / (\gamma_{n-1}\gamma_{n-2}\cdots\gamma_1) \tag{16}$$

Where p_k and p_{k-1} are the coefficients of the plant at k^{th} and $(k-1)^{th}$, respectively.

If (16) is not satisfied, the γ_{n-1} is first increased, then γ_{n-2} and so on, until (16) is satisfied. From (10)~(12), the coefficient a_i and the characteristic polynomial $P(s)$ are

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1}\cdots\gamma_2^{i-2}\gamma_1^{i-1}} = a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{\left(\gamma_{i-j}\right)^j} \tag{17}$$

$$P(s) = a_0 \left\{ \left[\sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right)(\tau s)^i \right] + \tau s + 1 \right\} \tag{18}$$

From (9) and (18), the characteristic polynomial $P(s)$ is

$$P(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \qquad (19)$$

The design procedures for the I-PDA controller by CDM are summarized as follows:

1) Determine the equivalent time constant τ from the desired settling time t_s

2) Determine the proper values of the stability index γ_i from the standard values of the stability index in (14)

4. CONTROLLER DESIGN FOR ROBOTIC MANIPULATOR & SIMULATION

So far the properties and design procedures of CDM controller are given in general. The important characteristic of CDM is its simplicity, robustness and disturbance rejection, which are inherently needed for the manipulators. To illustrate the preceding results, let us consider a one-link robotic manipulator that has the total length of $2l$, mass m and torque of u governed by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -D/I \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \end{bmatrix}u + \begin{bmatrix} 0 \\ 1/I \end{bmatrix}Mgl\sin x_1 \qquad (20)$$

Where $(x_1, x_2) = (\theta, d\theta/dt)$ and θ is the angular displacement of the arm from the vertical, D is the viscous damping coefficient and I is the inertia moment of $4ml^2/3$. Here it is assumed that x_1 is measurable and the system output is

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (21)$$

Letting $M = 0.75kg$, $l = 1m$, $D = 1 N\,ms$ and $g = 9.8 m/s^2$ yields the following second-order system :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} 0 \\ 1 \end{bmatrix}7.35\sin x_1 \qquad (22)$$

Where $7.35\sin x_1$ is assumed to be unknown, but its bound is known, $\rho = |7.35\sin x_1| = 7.35$.

Here the problem is to get state x_1 to track a specific time varying or set point $x_d = \gamma$ in the resence of the model uncertainty. For set point problem of $x_d = 1.5\,rad$ a controller based on CDM with FFC is designed. The polynomials represent the nominal system given in Fig. 3 for the manipulator are $N(s) = 1$, $D(s) = s^2 + s$ and the controller transfer polynomials are chosen to be in the form of

$$A(s) = l_1 s, \;\; B(s) = k_2 s^2 + k_1 s + k_0, \;\; F(s) = k_0 \qquad (23)$$

Where l_1, k_0, k_1, k_2 are controller design parameters. Then the closed-loop characteristic polynomial in terms of the design parameters is

$$P(s) = l_1 s^3 + (l_1 + k_2)s^2 + k_1 s + k_0 \qquad (24)$$

From standard form of CDM the stability indexes $\gamma_i = [2, 2.5]$, $\gamma_0 = \gamma_3 = \infty$ and stability limits $\gamma_i^* = [0.4\ 0.5]$ are chosen so $l_1 = [0.08\ 0]$, $k_1 = [0.32\ 1\ 1]$ obtained for settling time, $t_s \cong 2.5$. Finally the closed-loop characteristic polynomial is becomes as

$$P(s) = 0.08s^3 + 0.4s^2 + s + 1 \qquad (25)$$

The system time response is depicted in Fig. 4. As seen from this figure that the settling time satisfies for both with and without disturbance levels. Fig. 4 shows that the closed loop system achieve the desired performance in the presence of uncertainty level of $\rho = 0,\ 4\ and\ 7$. Also it has been shown that the controller is robust in presence of time variable disturbance. Fig. 4 illustrates large ρ is resulting the large overshoot.
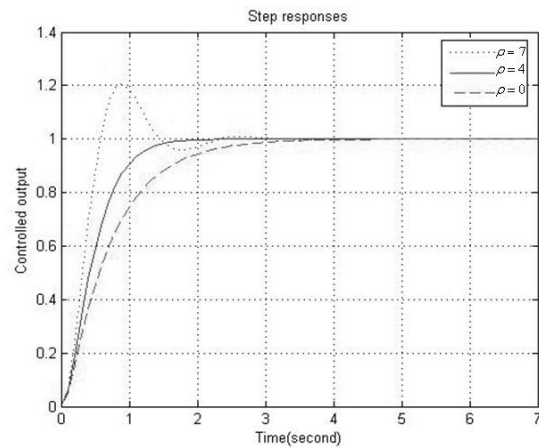

Fig. 4 Time response of the system for $\rho = 0,\ 4,\ and\ 7$.

Fig. 5 shows that it leads slow closed loop response. Here designer can use his/her engineering insight to improve the system response by adjusting the stability index of γ_2 .

Fig. 6 shows that I-PDA control system can fastly recover from the disturbance.
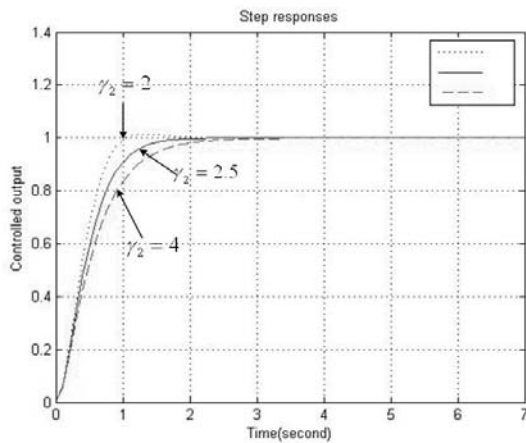
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Fig. 5 Time response of the closed-loop system for $\rho = 4$ and stability index $\gamma_2 = 2,\ 2.5,\ and\ 4$.
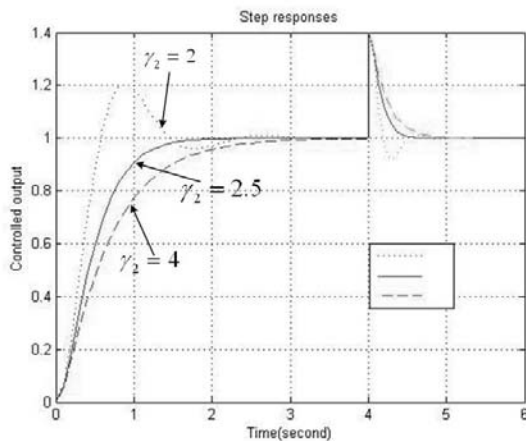
Fig. 6 Response with disturbance effects for $\rho = 4$ and stability index $\gamma_2 = 2,\ 2.5,\ and\ 4$.

5. CONCLUSION

The I-PDA controller based on for robotic manipulator with feed-forward controller that can improve the speed of step response for the third-order plant(n -link manipulator) has been proposed. Robotic manipulators need to maintain the stability and the speed of system response without overshoot. This paper shows that the stability and speed of response for robotic manipulators can improve them through simulation using MATLAB.