Design of a controller for input time-delay nonlinear system

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Abstract: In most physical processes, the transfer function includes a time-delay, and in the general distributed control system using a computer network, an inherent time-delay exists due to the spatial separation between controllers and actuators. Under the circumstance where an input time-delay exits, the system response overshoots and tends to diverge. For this reasons described above, a controller design method is proposed for a discrete nonlinear system including input time-delay, which adopts the time-discretization using Taylor series. Controllers are synthesized using an input/output linearization method. Finally, several cases of the computer simulations were conducted, and the results validate the proposed methods.

Keywords: nonlinear systems, time-delay systems, digital control, input/output linearization

1. INTRODUCTION

Time-delay means that a delay occurred at the gap from the beginning of an order to the response for the order due to a time interval, or spatial distance between the components of a control system [1]. In other words, time-delay is a certain section where a system doesn't respond to the signal of control elements. Time-delay using a state feedback in a closed system can be classified as two types: a state time-delay from sensors to a controller, and time-delay between the controller and actuators.

Studies on the time-delay have traditionally been conducted in the fields of chemical process control, however, the importance of the study for the time-delay has been recently recognized by introducing a remote control system using networks [2][3][4]. It is impossible to design an controller without considering the time-delay that inevitably occurred due to the spatial distance in signal transmission routes and network congestion.

Time-delay reduces gain and phase margins in a continuous system, causes a lowering of system performance, and makes the system unstable [1][8]. Although it is a simple linear time-invariable system that has time-delay for an input or state, the system becomes an unlimited dimensionless state. This time-delay makes it impossible to apply a classical controller design method. Thus, a controller design method to compensate for the effects of time-delay is required.

In order to remove this time-delay, a method, which designs a controller after reanalyzing the system including the time-delay element, and obtains a dynamics model using a Pade approximation method, was proposed [6].

A method that stabilizes a remote robot system using an asymptotic stability of reflecting torque for the design of a force-reflecting robot system was proposed. This method calculates the conditions for the moment of inertia and marginal range for reflecting torque according to time passed in a master-slave system, which has time-delay. In addition, this method makes it easy to design a force-reflecting robot system by analyzing the stability region of a linear system, and focuses on the improvement of the relative stability of a time-delay system [7].

A typical control method for a system, which has time-delay, is a predictive control [10]. A control method using the Smith predictor has been proposed in the fields of process control. This method configures each model with the objective to control the system and time-delay. Thus, it designs a system to remove the effects of time-delay in the characteristic equation of the whole closed loop transfer function through a structural method. Thus, the Smith predictor makes it possible to design a controller by considering a system, which has time-delay, to a system, which doesn't include time-delay. this method has the merit that a controller can be designed using a structural method, regardless of the effects of time-delay, however, it can only be applied to a linear system. In addition, this method has the demerit that an exact model equation for the system and time-delay is required [1][2][8][9].

An estimator is also proposed as an alternative method of predictive control. This estimator calculates state changes in the delayed time using an analysis of the time region of a state equation, and obtains an undelayed and exact plant state for the time that is required to calculate control signals. However, it is impossible to compensate for the time-delay for the input of the controller [4][5].

Studies on time-delay have been largely conducted for a linear system in an continuous time region. However, an actual physical system basically has nonlinearity, and most control systems in the present time are designed using a digital computer system. Thus, an analysis, which is performed in the discrete-time region for a nonlinear system that has time-delay, and the design of a controller according to the analysis is an important step. Accordingly, this paper deals with designing a controller through a system discretization using a Taylor-series, and linearization for inputs and outputs for a nonlinear system that has input time-delay.

2. DISCRETIZATION OF A NONLINEAR SYSTEM

A discrete-time model for a nonlinear continuous-time control system that has time-delay can be obtained using a Taylor-series under the assumption of zero-order hold. This discretization method provides a relatively more exact discrete model compared to a continuous-time nonlinear system, and makes it possible to apply the existing nonlinear control method to a discrete system, which includes time-delay.

A continuous-time nonlinear control system, which has a single input, can be presented as Eq. (1) using a state-space expression.

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D) \tag{1}$$

where $x \in X \subset R^n$ represents the system state, $u \in R$ is an input variable, D is time-delay, and $f(x)$ and $g(x)$ are nonlinear functions for x, respectively. In addition, the zero-order hold was assumed for a fixed sampling period, and constant input in a single sampling region.

$$u(t) = u(kT) \equiv u(k) = \text{constant}, \quad kT \le t < kT + T$$
$$D = qT + \gamma \tag{2}$$

where T is sampling phase, q is an integer multiple of $q \in \{1, 2, 3, \dots\}$ for the sampling period, γ is a small time-delay of $0 < \gamma \le T$. The delayed input variable was applied to the system that has values for the different sampling regions, as presented in Eq. (3).

$$u(t-D) = \begin{cases} u(k-q-1) & \text{if } kT \le t < kT + \gamma \\ u(k-q) & \text{if } kT + \gamma \le t < kT + T \end{cases} \tag{3}$$

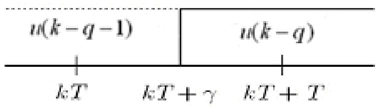

Fig. 1 Delayed input signal

A discrete system for the nonlinear system that has input time-delay can be configured as Eq. (4).

$$x(k+1) = x(k) + \sum_{l=1}^{M} A^l(x(k), u(k-q-1))\frac{\gamma^l}{l!}$$
$$+ \sum_{l=1}^{M} A^l\Big((x(k) + \sum_{i=1}^{M} A^i(x(k), u(k-q-1))\frac{\gamma^i}{i!}), u(k-q)\Big)\frac{(T-\gamma)^l}{l!}, \tag{4}$$

where $x(k)$ is the value of a state vector of x at $t = t_k = kT$, M is truncation order of the Taylor-series. $A^{[l]}(x,u)$ s cyclically defined by Eq. (5).

$$A^{[1]}(x,u) = f(x) + ug(x)$$
$$A^{[l+1]}(x,u) = \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x) + ug(x)). \tag{5}$$

The discrete expression for Eq. (1), which is the original continuous-time systems, is presented by Eq. (6).

$$x(k+1) = \Phi_T^M(x(k), u(k-q-1), u(k-q)) \tag{6}$$

where the function Φ_T^M depends on the sampling period of T and truncation order of M. As mentioned above, the discretization of a nonlinear system using a Taylor-series presented better results than that of the existing Euler method. The comparison can be performed using discretization errors.

3. Design of a controller

3.1 Linearization of input/output for a continuous-time linear system

Linearization of inputs/outputs linearizes a nonlinear system using an algebraical means through an exact state transition and feedback, instead of a liner approximation method for a nonlinear system. Then, it applies a linear control method. This method transfers the original system into a simple equivalent model, and has been used to control many industrial fields, such as aircrafts, robots, medical purposes, and other various fields.

Let's consider issues for the design of a controller for a nonlinear system as follows.

$$\dot{x} = \Phi[x, u] \tag{7}$$
$$y = h[x] \tag{8}$$

where the output y and input u of the system can be obtained using differentiation as follows. This is expressed as a relative order of r. If the output y of the system has non limited order, the handled input will not affect the output. The output y of the system in general issues of a control system should be applied to a limited and relative order for the input u.

$$\frac{d^l}{dt^l} y = h^{(l)}[x], \quad l = 0, \dots, r-1 \tag{9}$$

$$\frac{d^r}{dt^r} y = h^{(r-1)}\left[\Phi(x,u)\right] = f_2(x)u + f_1(x) \tag{10}$$

Eq. (10) can be obtained through the differentiations with relative orders in order to verify the relationship between inputs and outputs. f_1 and f_2 are the functions for a system state. In Eq. (10), if the input u is configured by Eq. (11), the nonlinearity of Eq. (10) will be removed. then, a simple linear differential equation for the output y and new internal input ν can be obtained, as presented in Eq. (12).

$$u = \frac{1}{f_2}(\nu - f_1) \tag{11}$$

$$y^{(r)} = \nu \tag{12}$$

Because a linear control method can be applied to Eq. (12), the tracking control problem can be solved using the method as follows. When the internal input ν can be configured as Eq. (14) using Eq. (12) through the differentiation of Eq. (13), which presents the output y and target value y_d, whit relative orders. In this case, the tracking error for the entire closed circuit system is presented as Eq. (15).

$$e = y - y_d \tag{13}$$
$$\nu = y_d^{(r)} - k_r e^{(r-1)} - k_{r-1} e^{(r-2)} - \cdots - k_1 e \tag{14}$$
$$e^{(r)} + k_r e^{(r-1)} + k_{r-1} e^{(r-2)} + \cdots + k_1 e = 0 \tag{15}$$

If the coefficient k in each item of the equations is configured to satisfy a stable dynamics equation, in which Eq. (15) converges to zero, the input u that makes it possible to obtain a tracking property will be performed by applying a reverse process from Eq. (11) to Eq. (15).

Fig. 2 presents the configuration of design for input/output linearization controller of a nonlinear system.

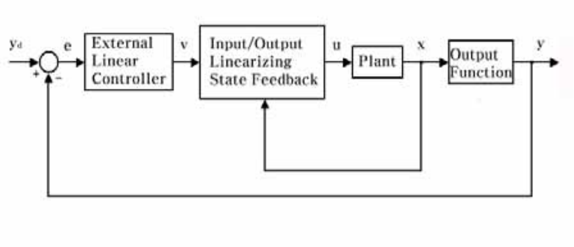

Fig. 2 Configuration of the design for input/output linearization

3.2 Input/output linearization of a discrete-time nonlinear system

Design of a controller using input/output linearization for a nonlinear system in the discrete-time region follows the same method in a continuous-time nonlinear system. Using the discrete-time nonlinear system presented in Eq. (16), the relative order and relative relationship between input and output can be verified as expressed in Eq. (17).

$$x(k+1)=\Phi[x(k),u(k)],\quad y(k)=h[x(k)] \qquad (16)$$

$$y(k+r)=f_2(x(k))u(k)+f_1(x(k)) \qquad (17)$$

where f_1 and f_2 are the functions related to the state of a discrete-time system. If the control input u is configured by Eq. (18), the nonlinearity of Eq. (17) will be removed. In addition, the system can be expressed as a simple linear differential equation for the output y and internal input ν, as presented in Eq. (19).

$$u(k)=\frac{1}{f_2(x(k))}\left\{\nu(k)-f_1(x(k))\right\} \qquad (18)$$

$$y(k+r)=\nu(k) \qquad (19)$$

As described above, the tracking issue for a linearized system is to be solved using a linear control method as follows. The internal input ν can be obtained as expressed in Eq. (21) using a transferring calculation with relative orders for Eq. (20). then, the coefficient k is adjusted to satisfy a convergence property, which is required to a dynamics equation for the tracking error of the entire closed circuit system presented in Eq. (22).

$$e(k)=y(k)-y_d(k) \qquad (20)$$

$$\nu(k)=y_d(k+r)-k_re(k+r-1)-k_{r-1}e(k+r-2)-\cdots-k_1e(k) \qquad (21)$$

$$e(k+r)+k_re(k+r-1)+k_{r-1}e(k+r-2)+\cdots+k_1e(k)=0 \qquad (22)$$

In the tracking control issue of a discrete-time nonlinear system, the input u can be obtained using reverse substitution of the mentioned processes. In addition, the system output converges to the target value of y_d by applying this value.

3.3 Design of a controller for a discrete-time nonlinear system with time-delay

The supplementary variables for the past input variables defined as Eq. (23).

$$\begin{aligned} z_1(k)&=u(k-q-1)\\ z_2(k)&=u(k-q)\\ &\vdots\\ z_q(k)&=u(k-2)\\ z_{q+1}(k)&=u(k-1), \end{aligned} \qquad (23)$$

Then, the dynamics equations are configured by Eq. (24).

$$\begin{aligned} z_1(k+1)&=z_2(k)\\ z_2(k+1)&=z_3(k)\\ &\vdots\\ z_q(k+1)&=z_{q+1}(k)\\ z_{q+1}(k+1)&=u(k) \end{aligned} \qquad (24)$$

Thus, Eq. (1) can be noted as a discrete-time nonlinear system as presented in Eq. (25), in which the equation presents an expanded state space.

$$\begin{bmatrix} x(k+1)\\ z_1(k+1)\\ \vdots\\ z_{q+1}(k+1)\end{bmatrix}=\begin{bmatrix}\Phi_T^M(x(k),z_1(k),z_2(k))\\ z_2(k)\\ \vdots\\ u(k)\end{bmatrix} \qquad (25)$$

Let's define, $\bar{x}=\left[x,z_1,\cdots,z_{q+1}\right]^T$ and $\bar{\Phi}_T^M(\bar{x},u)=\begin{bmatrix}\Phi_T^M(x,z_1,z_2)\\ z_2(k)\\ \vdots\\ u\end{bmatrix}$, then Eq. (25) can be expressed as a simple formula, as presented in Eq. (26).

$$\bar{x}(k+1)=\bar{\Phi}_T^M(\bar{x}(k),u(k)) \qquad (26)$$

The relative order of the system for the output equation of Eq. (27) can be derived using a transferring equation, as presented in Eq. (28). In addition, if the control input is configured as Eq. (29), the linearity of the system will be numerically removed.

$$y(k)=h(\bar{x}(k)) \qquad (27)$$

$$y(k+r)=h^{r-1}\left(\bar{\Phi}_T^M(\bar{x}(k),u(k))\right)=F(k)+G(k)u(k)=\nu(k) \qquad (28)$$

$$u(k)=\frac{1}{G(k)}\left\{\nu(k)-F(k)\right\} \qquad (29)$$

$F(k)=F\left\{\bar{x}(k)\right\}$ and $G(k)=G\left\{\bar{x}(k)\right\}$ in Eq. (29) are the function for the present state value and past input variables of $z(k)=[u(k-q-1),u(k-q),\cdots,u(k-1)]$, and $\nu(k)$ is the internal input.

In regards to the tracking issue, if the dynamics equation and internal input are defined by Eq. (30) and Eq. (31), the output property of the system is only followed by an error dynamics equation.

$$e(k+r)+k_re(k+r-1)+k_{r-1}e(k+r-2)+\cdots+k_1e(k)=0 \qquad (30)$$

$$\nu(k)=(1+k_1+k_2+\cdots k_r)y_d-k_ry(k+r-1)-k_{r-1}y(k+r-2)-\cdots-k_1y(k) \qquad (31)$$

Fig. 3 presents the configuration of the proposed control system.

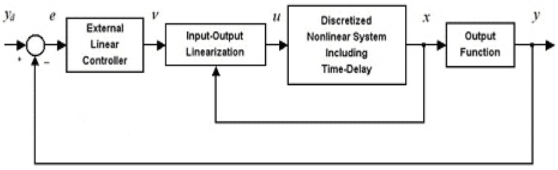

Fig. 3 Design of a controller for a nonlinear system with input time-delay

4. SIMULATION

In order to verify the proposed method, this study performed two simulations for nonlinear system, which has an input time-delay. The systems used in this simulation were a simple CSTR system and a Van der Pol equation. Van der Pol system is a typical nonlinear system. This system can be analyzed using a mass-spring-damper system, which has a position-dependent damping coefficient, and a RLC electric circuit. If this system has an initial value besides an equilibrium point, a periodical vibration will be maintained in a limited region. This periodical vibration is called a limit cycle. Fig. 4 presents a phase portrait of the system.

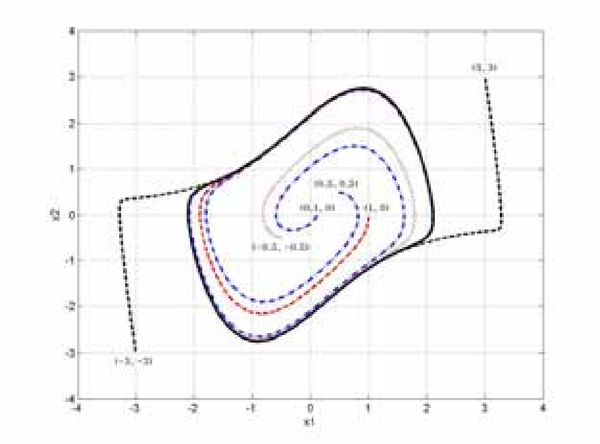

Fig. 4 Phase portraits of the Van der Pol system

The system can be expressed using a dynamics equation as presented in Eq. (32). The state space expression is Eq. (33).

$$\ddot{x} = \dot{x}(1-x^2) - x + u \qquad (32)$$
$$y = x,$$

$$\dot{X}_1 = f_1(X) + g_1(X)u = X_2$$
$$\dot{X}_2 = f_2(X) + g_2(X)u = X_2(1-X_1^2) - X_1 + u. \qquad (33)$$

where the state vector is $X = [X_1\ X_2]^T = [x\ \dot{x}]^T$. In the case of the existing input time-delay, such as $D = qT + \gamma$, the discrete expression is expressed as Eq. (34) using the Taylor's discretization method.

$$X_1(k+1) = X_1(k) + \sum_{l=1}^{M} A_1^l(X(k), u(k-q-1))\frac{\gamma^l}{l!} + \sum_{l=1}^{M} A_1^l((X(k) + \sum_{j=1}^{M} A_1^j(X(k), u(k-q-1))\frac{\gamma^j}{l!}), u(k-q))\frac{(T-\gamma)^l}{l!}$$

$$X_2(k+1) = X_2(k) + \sum_{l=1}^{M} A_2^l(X(k), u(k-q-1))\frac{\gamma^l}{l!} + \sum_{l=1}^{M} A_2^l((X(k) + \sum_{j=1}^{M} A_2^j(X(k), u(k-q-1))\frac{\gamma^j}{l!}), u(k-q))\frac{(T-\gamma)^l}{l!} \qquad (34)$$

where the time of the partial differentiation of $A^l(x,u)$ is to be cyclically defined as follows.

$$A_1^{\,l}(x,u) = f_1(X) + g_1(X)u$$
$$A_1^{\,l+1}(X,u) = \frac{\partial A_1^{\,l}(X,u)}{\partial X_1} f_1 + \frac{\partial A_1^{\,l}(X,u)}{\partial X_2} f_2$$

$$A_2^{\,l}(X,u) = f_2(X) + g_2(X)u$$
$$A_2^{\,l+1}(X,u) = \frac{\partial A_2^{\,l}(X,u)}{\partial X_1} f_1 + \frac{\partial A_2^{\,l}(X,u)}{\partial X_2} f_2 \qquad (35)$$

Fig. 5 presents the output errors of a continuous system and discrete system for the behaviors of a limited phase, in which the system has an initial value beside an equilibrium point. In addition, there is no input time-delay. This revealed that the discrete system using a Taylor-series presented superior characteristics to the existing Euler equation when applied to a discrete system.

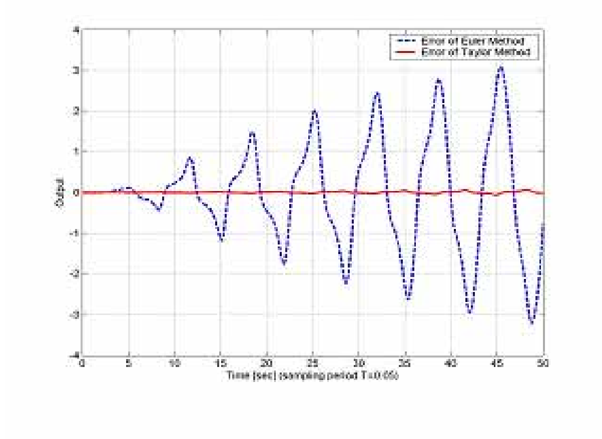

Fig. 5 Output error of the system with an initial value $x_{(0,0)} = [0.1 \quad 0]^T$

A controller was designed using the input/output linearization method previously mentioned above for the input time-delay, such as *0T, 1T, 2T* and *3T*. The discretization was performed using the truncation order of *M=2*. The relative orders for each case were obtained as Eq. (36). In addition, the output error dynamics equation was defined as Eq. (37), and the controller was designed as Eq. (38).

$$y(k+1) = F\{X_1(k), X_2(k)\} + (T^2/2!)u(k) = v(k)$$
$$y(k+2) = F\{X_1(k), X_2(k), u(k-1)\} + (T^2/2!)u(k) = v(k)$$
$$y(k+3) = F\{X_1(k), X_2(k), u(k-2), u(k-1)\} + (T^2/2!)u(k) = v(k)$$
$$y(k+4) = F\{X_1(k), X_2(k), u(k-3), u(k-2), u(k-1)\} + (T^2/2!)u(k) = v(k) \qquad (36)$$

$$e(k+1) + k_1 e(k) = 0, \quad k_1 = -0.7$$
$$e(k+2) + k_2 e(k+1) + k_1 e(k) = 0, \quad k_2 = -1.4, k_1 = 0.53$$
$$e(k+3) + k_3 e(k+2) + k_2 e(k+1) + k_1 e(k) = 0$$
$$k_3 = -0.7, k_2 = -0.45, k_1 = 0.371$$
$$e(k+4) + k_4 e(k+3) + k_3 e(k+2) + k_2 e(k+1) + k_1 e(k) = 0$$
$$k_4 = -2.8, k_3 = 3.02, k_2 = -1.484, k_1 = 0.2809 \qquad (37)$$

$$u(k) = \frac{1}{(T^2/2!)}\{v(k) - F(k)\} \qquad (38)$$

Fig. 6 presents the output of the system for each case where the output shows a simple time-transition removed by the time-delay effect.

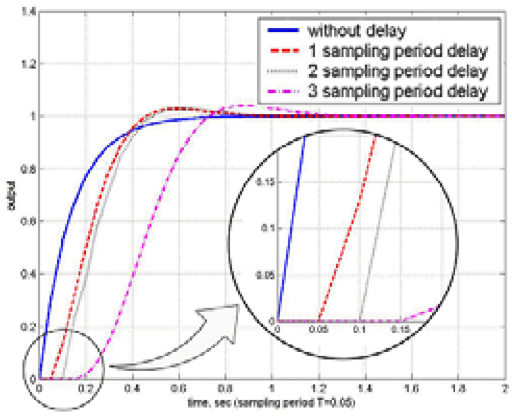

Fig. 6 Output of the nonlinear control system with input time-delay

Second computer simulation was conducted for CSTR system. CSTR system is a common nonlinear system in chemical process. Dynamic equation of the system is presented in Eq. (39).

$$x' = -x^2 - 3x + (1-x)u \qquad (39)$$
$$y = x$$

In the case of the existing time-delay, such as $D = qT + \gamma$, the discrete expression is expressed as Eq. (4) and Eq. (5) using the Taylor's discretization method. A controller was designed using the input/output linearization method previously mentioned above for the input time-delay, such as *0T*, *1T* and , *2T*. The discretization was performed using the truncation order of *M=2*.

Fig. 7 presents the output of the system for each case where the output shows a simple time-transition removed by the time-delay effect as the first simulation result.

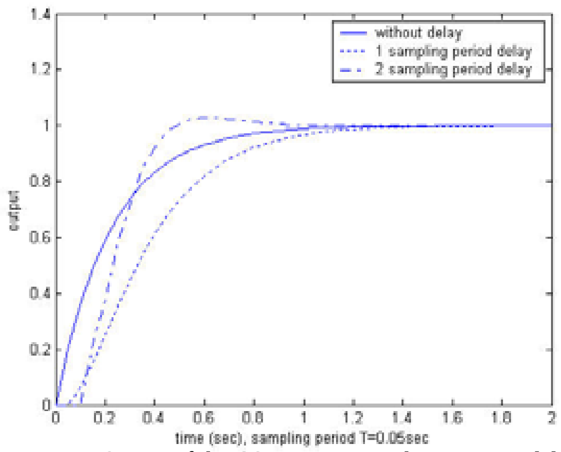

Fig. 7 Output of the CSTR system with input time-delay

5. CONCLUSIONS

In order to compensate for the input time-delay of a nonlinear system, this study proposed a design method, which supported an exact discretization for a nonlinear system that has time-delay, and designed a controller for a discrete system that included time-delay using a Taylor-series. The proposed control system that has the characteristics of an independent output for time-delay was verified using a simulation. This method has the demerit that the system is to be reanalyzed according to the time-delay. However, it has the merit that the system can be applied to a large time-delay and variable time-delay.

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