Control of Inverted Pendulum Using Continuous Time Deadbeat Control

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Abstract: Due to the asymptotic property, deadbeat control can hardly be applied to the continuous time system control. But some delay element method can deal such a problem. Besides delay element method, well-known digital deadbeat control can be used by the aid of some smoothing elements. In this paper, 2nd order smoothing element is used for the smoothing of the digital deadbeat controller. And this element is argumented to the plant, and so control problem is to control the argumented system digitally. We simulated this control system using Matlab language and finally apply this algorithm to the rotary inverted pendulum system.

1. INTRODUCTION

A deadbeat controller is well-known in digital control system but can 't be applied to continuous time system because of it's asymptotic property. So special method, for example delay elements method, must be used in continuous time system which is studied well by Japan researchers[1]-[3].

Additionally digital deadbeat control can be used. Firstly easily designed digital type controller is applied to the lower order smoothing element which is method suggested by Murata[4]-[8]. So the smoothed control output has the same effect of continuous time deadbeat controller. This smoothing element is the output of the controller and also is an input of the plant. If this smoothing element is regarded as one of plant elements, then plant including smoothing element become an argumented system. So digital deadbeat compensator must be firstly designed so that this argumented system attains digital deadbeat property. By S/W calculations two compensators are determined from this digital deadbeat controller. In references, researchers called this controller as CDBC(Continuous time DeadBeat Controller).

In this paper, we study on the CDBC implemented digitally which is suggested by Murata[4]-[8]. Especially, ZOH(Zero-order Hold) is applied to the digital deadbeat control and then resulted output is applied to the 2nd order smoothing element. As target plant rotary type 4th order inverted pendulum is used. Two unstable poles are relocated to LHP by using pole placement algorithms. Simulation is done using Matlab. DC servo motor is used to drive the pendulum.

2. CDBC THEORY

2.1 Continuous Time Parameters

Among various methods to obtain continuous time deadbeat control input, standard type second order delay elements are used for smoothing [4]-[7].

$$G_I(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \qquad (1)$$

Here ζ is a damping factor and ω_n is a natural frequency.

2.2 Problem Statements

To make continuous time deadbeat controller in the control system block of Fig. 1, $D_F(z)$ is introueced in local state

feedback loop and $D_I(z)$ is also introduced in the forward path as a form of serial feedforward form. This makes type 1 digital deadbeat control system[4]-[8].

Digital control input $\overline{u}_i$ is obtained in an optimal sense. And with symmetric weighing matrix, 2nd order performance index to be minimized is as follows[4]-[8].

$$P = \frac{1}{2}(e^T Q e + u_k^T R u_k) \qquad (2)$$

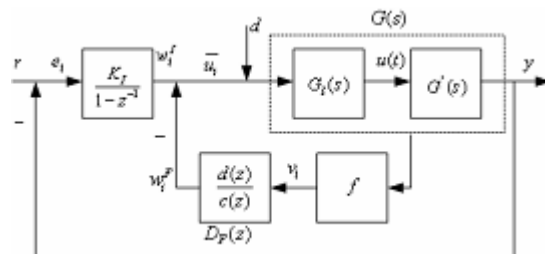Here u_k is control input which minimizes the index.

Fig. 1. Continuous time deadbeat control system

According to the rules, $\overline{u}_i$ is firstly calculated to meet the argumented system's desired response and then for this control input, feedback compensator in the local feedback and integral compensator in the forward path are cacluated accordingly.

3. DEDIGN OF CDBC

3.1 Derivation of CDBC

Assume digital deadbeat controller's control stage just before fininte time settling is N, and plant order, n and smoothing element's order, 2 and additional degree of freedom as k

Geneally for small N, large control input is obtained and user must

consider the size of N .

So $N = n + 2 + k(k \geq 0)$, and by the control input $u_i (i = 0 \sim N-1)$ after N stage $y_i = 1.0 (i \geq N)$, $u_{N+a} = u_N (\alpha \geq 1)$. In this case, input-output relations are

$$y_r = J_k u_k \tag{3}$$

$$u_k = [\overline{u}_0 \overline{u}_1 \cdots \overline{u}_N]$$

$$J_k = \begin{bmatrix} g_N & g_{N-1} & \cdots & g_{n+3} & g_{n+2} & \cdots & g_1 & 0 \\ g_{N+1} & g_N & \cdots & g_{n+4} & g_{n+3} & \cdots & g_2 & h_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_{N+n+2} & g_{N+n+1} & \cdots & g_{2n+5} & g_{2n+4} & \cdots & g_{n+3} & h_{n+2} \end{bmatrix}$$

$$\text{k} \qquad\qquad (\text{n}+3) \tag{4}$$

Here g_i is impulse response sequence and is obtained from uinit step response as $g_i = h_i - h_{i-1}$. In Eq. (4) impulse response sequences of inverted pendulum's argumented system are included. And output $y_i (i + N \sim N + n + 2)$ must be

$$y_r = [111.....1]^T \tag{5}$$

In minimum-time continuous deadbeat control case $(k = 0)$, Eq. (4) and Eq. (5) are reduced to

$$u_0 = J_0^{-1} y_r = [\overline{u}_0, \overline{u}_1, ... \overline{u}_{n+2}] \tag{6}$$

If $k \geq 1$, from Eq. (4) $rank J_k \neq n+3$ and u_k is not determined uniquely. So from Eq. (5) by using constraints of $y_r - J_k u_k = 0$ and Lagrange undetermined coefficients method, control input is calculated as follows[8].

$$u_k = M^{-1} \left[G^T Q i_r - J_k^T \left[J_k M^{-1} J_k^T \right]^{-1} \left[J_k M^{-1} G^T Q i_r - i \right] \right] \tag{7}$$

$$= [\overline{u}_0 \overline{u}_1 ... \overline{u}_N]^T$$

$$M = [R + G^T Q G] i_r = [1...1]^T \qquad ,$$

3.2 Design of CDBC

In Fig. 1 unit step function is used as reference input. Forward and feedback compensators are set as follows.

$$D_F(z) = \frac{d(z)}{c(z)}$$

$$d(z) = d_0 + d_1 z^{-1} + ... + d_{N-1} z^{-(N-1)} \tag{8}$$

$$c(z) = 1 + c_1 z^{-1} + ... + c_{N-1} z^{-(N-1)}$$

$$f(z) = f_1 z^{-1} + ... + d_{n+2} z^{-(n+2)}$$

If $i = 0$, initial value of integral forward compensator is 0 and $\omega_0^I = \overline{u}_0$. So gain of integrator $K_I \equiv \overline{u}_0$. If unit step input is applied as a reference input, closed-loop response is as follows

$$\overline{y}(z) = \overline{u}_0(z)\overline{b}(z) / (1 - z^{-1}) \tag{9}$$

Here one of denominator terms is regarded as follows

$$(1 - z^{-1})\{1 + \overline{a}(z) + d(z)f(z)\} + \overline{u}_0 \overline{b}(z) = 1 \tag{10}$$

So closed-loop transfer response is simplified as Eq. (9)[8]. In Eq. (10) polynomial $\overline{a}, \overline{b}$ have zero constant terms and have a degree of N . To obtain these polynomials in case of $n = 2, N = 5$, coefficient of powers from (-1) to (-9) are orders as follows.

$$(\overline{a}_1 - 1) + d_0 f_1 + \overline{u_0 b_1} = 0$$
$$(\overline{a}_2 - \overline{a}_1) + (d_0 f_2 + d_1 f_1 - d_0 f_1) + \overline{u_0 b_2} = 0$$
$$(\overline{a}_3 - \overline{a}_2) + (d_0 f_3 + d_1 f_2 + d_3 f_1 - d_0 f_3 - d_1 f_2 - d_2 f_1) + \overline{u_0 b_4} = 0$$
$$(\overline{a}_4 - \overline{a}_3) + (d_0 f_4 + d_1 f_3 + d_2 f_2 + d_3 f_1 - d_0 f_3 - d_1 f_2 - d_2 f_1) + \overline{u_0 b_4} = 0$$
$$(\overline{a}_5 - \overline{a}_4) + (d_1 f_4 + d_2 f_3 + d_3 f_2 + d_4 f_1 - d_0 f_4 - d_1 f_3 - d_2 f_2 - d_3 f_1) + \overline{u_0 b_5} = 0$$
$$-\overline{a}_5 + (d_2 f_4 + d_3 f_3 + d_4 f_2 - d_1 f_4 - d_2 f_3 - d_3 f_2 - d_4 f_1) = 0$$
$$d_3 f_4 + d_4 f_3 - d_2 f_4 - d_3 f_3 - d_4 f_2 = 0$$
$$d_4 f_4 - d_3 f_4 - d_4 f_3 = 0$$
$$-d_4 f_4 = 0$$

$$\tag{11}$$

From Eq. (11), $f_2, f_3, f_4 = 0$ because $d_i \neq 0$. So pulse sequence transfer function $D_F(z)$ is as follows

$$D_F(z) =$$

$$\frac{(w_1^{F-1} + ... + w_{n-1} z^{(-n-1)} + w_N^{F-N} z^{-N} / (1 - z^{-1}))}{v_1 z^{-1} + ... + v_{N-1} z^{-(N-1)} + v_N z^{-N}) / (1 - z^{-1})}$$

$$= z^{-1} \frac{1}{v_1} + \{w_1^F + (w_2^F - w_1^F) z^{-1} + ... + (w_N^F - w_{N-1} F) z^{-(N-1)}\}$$

$$\tag{12}$$

Coefficients in Eq. (12) are determined as follows by comparing the corresponding coefficients in ω_i^I , $\overline{y, u_0}$ [8]

$$w_i^F \overline{u}_0 \left((i+1) - \sum_{j=0}^{i} \overline{y}_j \right) - \overline{u}_i, \overline{y}_i = \overline{u}_0 \sum_{j=1}^{i} \overline{b}_j, f_1 = \frac{1}{\overline{u}_0} \tag{13}$$

4. MODELLING OF ROTARY INVERTED PENDULUM

The configuration of Rotary Inverted Pendulum is shown in Fig. 2. Measurements of angles θ, ϕ are done by using encoders with resolution 1024(pulse),2000(pulse) respectively.
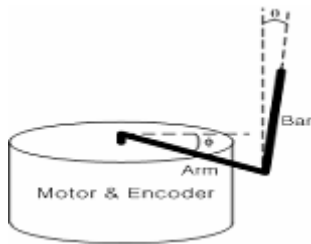
Fig. 2 Rotary type pendulum

Pendulum's model is described as follows by simplifying energy relations[11].

$$PE_{total} = m_b g \frac{l}{2} \cos\theta \qquad (14)$$

$$KE_{total} = \frac{1}{2}[J_t \dot{\phi}^2 + m_b lr \dot{\phi}\dot{\theta} + (J_b + J_{ex})\dot{\theta}^2] \quad (15)$$

Here $J_t = J_{Arm} + m_b r^2 + J_m + J_{ez}$. Above simplifed equations are combined to derive the next two equations by using Lagrangian Equations [9].

$$T = j_t \ddot{\phi} - \frac{1}{2} m_b lr (\ddot{\theta}\cos\theta - \dot{\theta}\sin\theta) \qquad (16)$$

$$0 = \frac{1}{2} m_b lr \ddot{\phi}\cos\theta - m_b g \frac{1}{2}\sin\theta + (J_b + J_{ex})\ddot{\theta} \quad (17)$$

Approximate values such as $\theta = 0, \sin\theta = \theta, \dot{\theta} = 0$ are used with Eq. (16) and Eq. (17) and then after pole relocation to derive the following transfer function

$$G(s) = \frac{-8.053_s^2}{s^4 + 24s^3 + 196s^2 + 720s + 1600} \qquad (18)$$

5. SIMULATION

5.1 Simulation(CDBC)

An inverted pendulum is naturally unstable system and is relocated using pole placement alogorithm in which new poles are $(-2 \pm 3.464 j, -10, -10)$. Overall augumented system prepositioned by smoothing element is as follows

$$G(s) =$$

$$\frac{w_n^2(-8.053s^2)}{(s^2 + 2\zeta w_n s + w_n^2)(s^4 + 24s^3 + 196s^2 + 720s + 1600)}$$

$$(19)$$

Fig. 3 shows digital deadbeat control input which is applied to

augmented system with initial value [0 0 0 0.1 0 0]' in case of shortest settling stage 6 and digital control input is

$$u_0 = [\overline{u_0 u_1 u_2 u_3 u_4 u_5 u_6}]^T$$
$$= [403.5 - 403.8 - 82.8, 234.9 - 94.0, 8.9 - 1.9] \qquad (20)$$

Fig. 4 shows resulting system output. Sampling time is set to 0.125sec and because control input is applied to insert after 1st sample, deadbeat time becomes (0.125[sec/stage]) * (6 [stages]+1[stage])=0.875[sec].
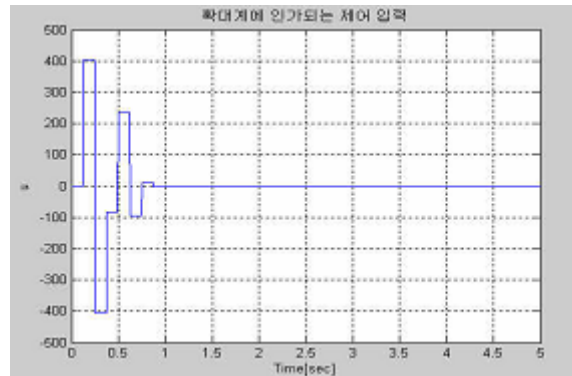

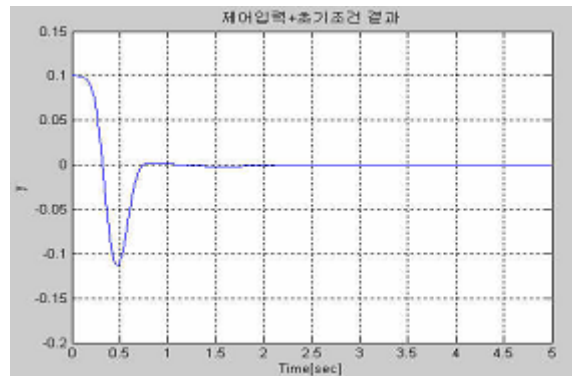Fig. 3 Digital control input applied to argumended system

Fig. 4 Plant output

5.2. Experiments(LQR control)

Experiments using LQR control theory is done as a prestage of CDBC. Experiments are implemented with 8bit processor 8051. Weighting matrix Q, R and resulting control gain are as follows.

$$Q = diag(0.025, 8, 0, 0, 0), R = 0.004$$
$$K = [-2.50 - 64.51 - 2.63 - 7.97]$$

Fig. 5 shows pendulum inverted. Fig. 6 is a circuits diagramon which two LS7166 are placed to measure the angles.
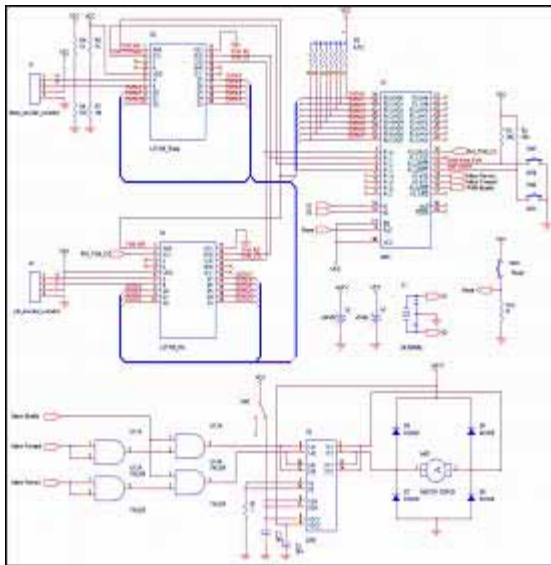
Fig. 5 Experimental apparatus

Fig. 6 Circuits diagram

6. CONCLUSION

Digital deadbeat controller is applied to argumented system. Controllers consist of feedforward integral compensator and local feedback compensator. Especially inverted pendulum has two unstable poles and so these unstable poles are relocated to LHP　by pole placement algorithm.

By using conventional digital control design methods, CDBC is easily designed. By comparing simulation it's well demonstrated, but experiments with SwingUP plus LQR is completed and replacing LQR with CDBC has been doing.

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