An INS Filter Design Considering Mixed Random Errors of Gyroscopes

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Abstract: We propose a filter design method to suppress the effect of gyroscope mixed random errors at INS system level. It is based on the result that mixed random errors can be represented by a single equivalent ARMA model. At first step, the time difference of equivalent ARMA process is performed, which consider the characteristic of indirect feedback Kalman filter used in INS filter. Next, a state space conversion of time differenced ARMA model is achieved. If the order of AR is greater than that of MA, the controllable or observable canonical form is used. Otherwise, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. At final step, a complete form state equation is presented. The simulation results shows that the proposed method gives less transient error and better convergence compared to the conventional filter which assume the mixed random errors as white noise.

Keywords: gyroscope, mixed random error, equivalent ARMA model, indirect feedback Kalman filter

1. INTRODUCTION

There is a kind of random errors in which a number of different forms of noises are mixed as commonly seen in gyroscopes. Gyroscopes are sensors used in INS(Inertial Navigation System) to measure the angular rate of vehicle. Various methods of modeling mixed random errors have been studied in the past[1-4]. Recently, the author showed that a mixture of several noises such as white noise, random walk, quantization noise, Markov process, and other forms of general ARMA processes can be represented by a single equivalent ARMA model[5]. Previous results are focused on the modeling and suppressing the gyro random errors at sensor level and the filtering methods at INS system level are rarely studied.

In this paper, based on the result that mixed random errors can be represented by a single equivalent ARMA model, we propose a filter design method to suppress the effect of gyro random errors at INS system level. First, the time difference of equivalent ARMA process is performed so that it is included in the INS filters as state variable. The reason to perform the time difference is to consider the characteristic of the indirect feedback Kalman filter of which the linear state space model is determined by small perturbation at each time update. The time difference results in the increase of the MA order by one. Secondly, a state space conversion of time differenced ARMA model is presented. At this step, there are two directions which depend on the orders of AR and MA parts. If the order of AR is greater than the one of MA, the ARMA model is converted to state equation easily using the controllable or observable canonical form. Otherwise, since the canonical form representation result in colored process noise, different type of model equation is needed. For this, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. Third, a complete form state equation is presented. Via simulation results, we compare the proposed filter with the conventional filter which assumes the mixed random errors as white noise.

The paper is composed as follows. In section 2, we explain the conventional INS filer. In section 3, as main result, a new INS filer which considers mixed random errors is proposed. In section 4, some simulation results are presented. Finally we conclude in section 5.

2. PRELIMINAR

A conventional INS filter is explained. To compensate navigation error, indirect feedback Kalman filter is widely used. Via feedback of the state estimate to the linearized filter, the indirect feedback Kalman filter maintain the linearity of the error propagation equation which is acquired by perturbing the nonlinear INS equation[6]. If we consider only the random constant error among the gyroscope and accelerometer errors, the filter can be represented as follows and the order is 12.

$$\begin{bmatrix} \dot{x}_f \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} {}_1 & {}_2 \\ 0_{6\times6} & 0_{6\times6} \end{bmatrix} \begin{bmatrix} x_f \\ x_a \end{bmatrix} + G w_f \quad (1)$$

$$x_f \equiv \begin{bmatrix} \delta v_N & \delta v_E & \delta v_D & \varepsilon_N & \varepsilon_E & \varepsilon_D \end{bmatrix}^T$$

$$x_a \equiv \begin{bmatrix} \nabla_x & \nabla_y & \nabla_z & \delta\varepsilon_x & \delta\varepsilon_y & \delta\varepsilon_z \end{bmatrix}^T$$

$$w_f = \begin{bmatrix} w_{fx} & w_{fy} & w_{fz} & w_{gx} & w_{gy} & w_{gz} \end{bmatrix}^T$$

The ∇_i and $\delta\varepsilon_i$ represent the accelerometer and gyroscope errors respectively. The process noise w_f is assumed as white noise in conventional INS filter. The matrices in Eq. (1) are as follows

$${}_1 \equiv \begin{bmatrix} 0 & 2\Omega_D + \rho_D & -\rho_E & 0 & -f_D & f_E \\ -2\Omega_D - \rho_D & 0 & 2\Omega_N + \rho_N & f_D & 0 & -f_N \\ \rho_E & -2\Omega_N - \rho_N & 0 & -f_E & f_N & 0 \\ & & & 0 & \Omega_D + \rho_D & -\rho_E \\ & 0_{3\times3} & & -\Omega_D - \rho_D & 0 & \Omega_N + \rho_N \\ & & & \rho_E & -\Omega_N - \rho_N & 0 \end{bmatrix}$$

$${}_2 \equiv \begin{bmatrix} C_b^n & 0_{3\times3} \\ 0_{3\times3} & -C_b^n \end{bmatrix}, \quad G \equiv \begin{bmatrix} C_b^n & 0_{3\times3} \\ 0_{3\times3} & -C_b^n \\ 0_{6\times3} & 0_{6\times3} \end{bmatrix}$$

where f_i is acceleration of vehicle, Ω_i is earth rotation angular velocity and ρ_i is navigation frame rotation angular velocity. To implement the filtering in the digital computer, the discrete version of Eq. (1) is needed.

$$\begin{bmatrix} x_{f,t+1} \\ x_{a,t+1} \end{bmatrix} = \begin{bmatrix} I_{6\times6} + \ _1\Delta t & _2\Delta t \\ 0_{6\times6} & I_{6\times6} \end{bmatrix} \begin{bmatrix} x_{f,t} \\ x_{a,t} \end{bmatrix} + G w_{f,t} \quad (2)$$

Δt is time interval at which the update is executed. If we use velocity aided navigation, the measurement equation is as follows.

$$h_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & 0 & 0_{3\times6} \\ 0 & 0 & 1 & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} x_{f,t} \\ x_{a,t} \end{bmatrix} + v_t$$

v_t is velocity measurement noise and is assumed white noise.

3. INS FILTER CONSIDERING G ROSCOPE MIXED RANDOM ERRORS

We reconstruct the INS filter considering the gyroscope mixed random errors. The gyroscope noise in Eq. (2) is no more white noise and must contain the effect of mixed random errors. The mixed random errors are the mixtures of white noise, random work, quantization noise, and Markov process.

These noises belong to the ARMA processes, we can represent the mixture of these noises as an equivalent ARMA model[5]. In this paper, we use the equivalent ARMA model in INS filter to consider the gyroscope mixed random errors. Since we use indirect feedback Kalman filter, the gyroscope errors are estimated and then extracted from the gyroscope outputs at next step before computing the navigation information. Therefore, if the equivalent ARMA process is z_t , the gyroscope noise part of Eq. (2) $w_{gi,\,t}$ can be represented as follows.

$$w_{gi,\,t} = z_t - z_{t-1}$$

z_{t-1} is a estimate of z_{t-1} . It is reasonable to assume that z_{t-1} is the sum of z_{t-1} and a white Gaussian noise. Then

$$w_{gi,\,t} = z_t - z_{t-1} + u_t \quad (3)$$

where u_t is a white Gaussian noise. That is, the time difference of the equivalent ARAM process is included in INS filter.

Assume that z_t is ARMA(p,q) process as follows.

$$(1 + a_1 B + \cdots + a_p B^p) z_t = (1 + c_1 B + \cdots + c_q B^q) w_t$$

Let s define $\tilde{z}_t = z_t - z_{t-1}$. Then, we get

$$(1 + a_1 B + \cdots + a_p B^p) \tilde{z}_t = (1 - B)(1 + c_1 B + \cdots + c_q B^q) w_t$$

We notice that the order of the MA part of $\tilde{z}_t$ is increased by one due to the time difference compared to z_t . If $p > q+1$, $\tilde{z}_t$ can be realized into state space form using observability or controlability canonical representation[6]. In the case $p \le q+1$, we introduce a difference realization method based on the fact that $\tilde{z}_t$ can be represented as the sum of on step ahead predictor of $\tilde{z}_t$ at $t-1$ and white noise[7]. The resulting state space equation is

$$_t = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & \cdots & e_k & \cdots & e_1 \end{bmatrix} {}_{t-1} + \begin{bmatrix} 1 \\ \psi_1 \\ \vdots \\ \psi_{q+1} \end{bmatrix} w_t \quad (4)$$

where $_t = [\tilde{z}_t \quad \tilde{z}_{1/t} \quad \cdots \quad \tilde{z}_{q+1/t}]^T$ and $\tilde{z}_{i/t}$ is a i-step ahead predictor of $\tilde{z}_t$ at time t . ψ_i is acquired by dividing the AR part by MA part of $\tilde{z}_t$ as follows.

$$1 + \psi_1 B + \cdots = \frac{(1-B)(1 + f_1 B + f_2 B^2 + \cdots + f_l B^l)}{1 + e_1 B + e_2 B^2 + \cdots + e_k B^k}$$

Now, we are ready to reconstruct the INS filter considering the gyroscope mixed random errors. The resulting filter model is as follows.

$$\begin{bmatrix} x_{f,t+1} \\ x_{a,t+1} \\ \bar{}_{t+1} \end{bmatrix} = \begin{bmatrix} I_{6\times6} + \ _1\Delta t & _2\Delta t & 0_{6\times3(q+2)} \\ 0_{6\times6} & I_{6\times6} & \bar{C}_b^n \\ 0_{3(q+2)\times6} & 0_{3(l+2)\times6} & \bar{E} \end{bmatrix} \begin{bmatrix} x_{f,t} \\ x_{a,t} \\ \bar{}_t \end{bmatrix} \quad (5)$$

The definition of additional vector and matrices are as follows.

$\bar{}_t$: an extended vector of $_t$ considering gyroscope 3 axes.

$\bar{C}_b^n$: coordinate transform matrix which transform gyroscope 3 axes $\tilde{z}_t$ s to navigation frame.

$\bar{E}$: an extended matrix of system matrix in Eq. (4) considering gyroscope 3 axes.

$$\bar{G} \equiv \begin{bmatrix} C_b^n & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & -C_b^n \\ 0_{6\times3} & 0_{6\times3} & 0_{6\times3} \\ 0 & \Psi & 0 \end{bmatrix}$$

(Ψ :an extended matrix of process noise matrix in in Eq. (4) considering gyroscope 3 axes)

$$_t \equiv [\bar{w}_{v,t} \quad \bar{w}_t \quad \bar{u}_t]^T$$

($\bar{w}_{v,\,t}$: white noise of accelerometer 3 axes

$\bar{w}_t$: an extended vector of w_t in Eq. (4) considering gyroscope 3 axes

$\bar{u}_t$: an extended vector of u_t in Eq. (3) considering gyroscope 3 axes)

The gyroscope mixed random errors estimated through $\bar{}_t$ and one element $\tilde{z}_t$ of $\bar{}_t$ is connected to the original INS filter equation. The order of proposed filter in Eq. (5) is increased by 3(q+2) compared to the conventional filter. If the equivalent ARMA process of one gyroscope axis is ARMA(1,1), then the total order of INS filter will be 21. Although the order of proposed filter is increased, it gives good performance compared to conventional filter as shown in the simulation result.

4. SIMULATION

Through simulation, we compare the performance of the proposed filter compared to the conventional filter which assume the gyroscope mixed random errors simply as white noises. We use the order of ARMA(1,1) process as the equivalent ARMA model. The vehicle trajectory used is descending S curve as shown in Fig. 1. The total flight time is 600 sec. As simulation conditions, we use 1 ft/sec initial velocity error, 0.1 deg initial attitude error, 1000 μg accelerometer bias, 1 deg/h gyroscope and initial covariance corresponding initial errors. The white noise covariance of accelerometer is $(30\ \mu g)^2$. The parameters of ARMA(1,1) are -0.98 for AR , 0.6 for MA and $(1\ \deg/h)^2$ for the white noise covariance. The white noise covariance of measurement is $(0.01\ ft/\sec)^2$.

The simulation results show that the proposed filter give less transient navigation error and faster convergence to zero compared to the conventional filter. In Fig. 2 and Fig. 3, the attitude error is presented. As seen in the figures, the proposed filter give less transient yaw angle error and faster convergence to zero.

5. CONCLUSION

In this paper, based on the result that mixed random errors can be represented by a single equivalent ARMA model, we propose a filter design method to suppress the effect of gyro random errors at INS system level. At first step, considering the characteristic of the indirect feedback Kalman filter, the time differenced form of equivalent ARMA process is included in the INS filter. At next step, the time differenced ARMA process is represented as state space form. If the order of AR is greater than the one of MA, the ARMA model is converted to state equation easily using the controllable or observable canonical form. Otherwise, we introduce the state equation of which the state variable is composed of the ARMA model output and several step ahead predicts of that. Third, a complete form state equation is presented by including the system and measurement equations of the state space representation of time differenced ARMA process. Via simulation results, we showed that the proposed method gives less transient error and better convergence. From the results we can confirm that the proposed method is effective to reduce the effect of mixed random errors when they are not eliminated at sensor level.

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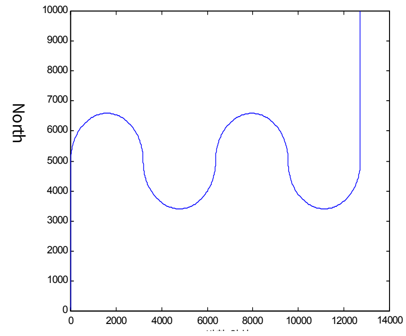

Fig. 1 The vehicle trajectory used in simulation

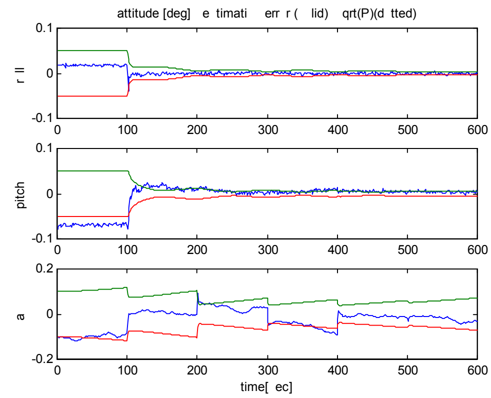

Fig. 2 The attitude error using the proposed filter

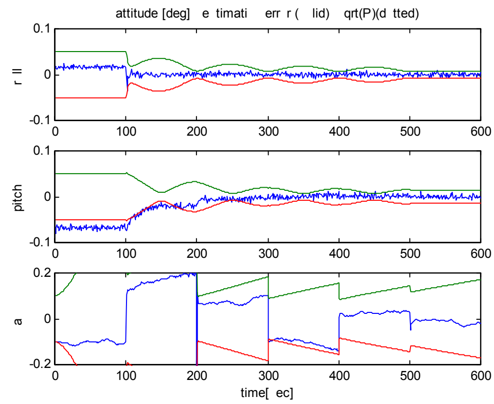

Fig. 3 The attitude error using the conventional filter