Time-domain Approaches for Input Disturbance Observer

Kyung-Soo Kim*

*Department of Mechanical Engineering, Korea Polytechnic University, Kyonggi-do 429-793, Korea
(Tel: +82-31-496-8255; Fax: +82-31-496-8219; Email:kimkyungsoo@kpu.ac.kr)

Abstract: In the paper, algorithms for disturbance observers are newly presented in the time-domain. Attention is paid to observing a ramp disturbance by introducing an integral term to the output equation of a constant disturbance observer. In order to reduce the sensitivity to the measurement noise, the disturbance observer is combined with the state observer. It will be shown that the estimation dynamics can be arbitrarily chosen by assigning the eigenvalues of a characteristic equation. Also, we provide the analysis of observer behaviors subject to non-ramp-style disturbances. Finally, we propose the generalized disturbance observer that accurately estimates disturbances of higher order in time series expansion.

Keywords: Time-domain disturbance observer, ramp disturbance, state observer

1. Introduction

For past several decades, disturbance observation and rejection has been one of the most important topics in control theory. Depending on the characteristics of disturbances, different types of observers have been devised in literature. For instance, the periodic disturbance with unknown (or, known sometimes) frequencies can be identified by the adaptive algorithms [2]. In mechanical systems such as the beam-pointing system, robot arm moving systems or traction systems, the friction force is the major hindrance for precision control([7], [3], [4]).

In the paper, disturbance observers that will be derived in time domain are of major concern. It is noted that the advantages of time-domain approaches for disturbance observer are as follows.

- Visible transient behavior in time domain
- Easy implementation with digital signal processors

The system of concern in the paper is restricted to single-input-single-output (SISO) systems that have an auxiliary input disturbance. Even with the simplest description, the SISO system may represent many of practical applications such as the robot-arm control system, DC-motor control system and the actuator control in optical data storage devices ([6], [5], [1] among many).

The basic idea of the paper starts from the friction observer proposed in [7], which may be viewed as a constant disturbance observer. It is noted that the observer consists of a dynamic state equation and a static output equation. An interesting point is that the observer can be easily extended for estimating the ramp disturbance by introducing one integral term in the output equation. In order to reduce the noise effect, a state estimator is combined together with the disturbance observer. Through the frequency domain analysis, we also show that the observer can be applied to the non-ramp style disturbances. In the final section, a generalization of the approach is made for observing the n-th order disturbance, with predetermined dynamics, in time series expansion. In addition, a method is introduced to reduce the noise propagation in the disturbance estimate.

This paper was partially supported by the automotive DVD-Player development project funded by the company - FineMS Co. Ltd.

2. Preliminaries

2.1. Friction observer

For readability of the manuscript, we revisit the work by Friedland and Park [7] for estimating the magnitude of friction in a mechanical system

$$\dot{v} = a_0 x + a_1 v + bu - f \cdot sign(v) \tag{1}$$

where v, x, and u are the velocity, the position and the control input, respectively, and, f is the constant Coulomb friction. Then, using the estimator

$$\left\{ \begin{array}{rcl} \hat{f} & = & z - \gamma v sign(v) \\ \dot{z} & = & \gamma \left\{ a_0 x + a_1 v + bu - \hat{f} sign(v) \right\} sign(v) \\ & & + \gamma v \frac{d}{dt} sign(v) \end{array} \right. \tag{2}$$

for a $\gamma > 0$, it was shown that the constant friction can be estimated with the asymptotically exponential stability. Here, note that the nominal model is simply incorporated in the dynamics for a state variable z. As a matter fact, we will use the underlying structure in the friction observer for devising algorithms for observing unknown input disturbances.

2.2. Constant disturbance observer

Consider the system

$$\dot{x} = f(x, u; t) + d(t), \quad x(0) = x_o \tag{3}$$

where $f(x, u; t)$ and x_o are known and the state variable x is supposed to be measured. It is assumed that the disturbance is continuous and differentiable.

First, observe that the friction in (1) is constant when the motion is kept in a direction. For example, in case of $sign(v) = -1$, the observer (2) would be simplified as

$$\left\{ \begin{array}{rcl} \hat{f} & = & z + \gamma v \\ \dot{z} & = & -\gamma \left\{ a_0 x + a_1 v + bu + \hat{f} \right\} \end{array} \right. \tag{4}$$

Based on the observation, we introduce a disturbance observer in the following.

DOB0:

$$\hat{d} = z + \gamma_0 x \tag{5}$$

$$\dot{z} = -\gamma_0 \left\{ f(x, u; t) + \hat{d} \right\}, \quad z(0) = -\gamma_0 x_o \tag{6}$$

where $\gamma_0 > 0$ is a tuning parameter for estimation error dynamics.
One may show that, for $\epsilon = d - \hat{d}$,

$$\dot{\epsilon} = -\gamma_0 \epsilon + \dot{d}. \tag{7}$$

Then, it follows that

$$|\epsilon(t)| \leq e^{-\gamma_0 t} \cdot |\epsilon(0)| + \frac{1}{\gamma_0} \rho(t) \tag{8}$$

for an envelope function $\rho(t)$ such that $\rho(t) \geq \dot{d}(t),\ \forall t \geq 0$. It is noted that the estimation characteristics is asymptotically and exponentially stable for the initial error while there remains steady state error depending on the envelope of the time derivative of the disturbance. When the disturbance is constant, the disturbance observer would exactly estimate the disturbance in the steady state.
As a remark, the initial condition for the state variable z is chosen to produce the initial estimation output as zero, which eventually enhances the transient convergence at the beginning phase. This will be shown by an example later on.

3. Disturbance observer for a ramp disturbance

3.1. An observer algorithm

In this section, we suggest a simple method to estimate the disturbance having the characteristics

$$d(t) = d_0 + d_1 t \tag{9}$$

where d_i's $(i = 0, 1)$ are unknown but constant. To this end, we present a disturbance observer as follows.

DOB1: Given scalars $\gamma_0 > 0$ and $\gamma_1 > 0$,

$$\hat{d}(t) = z + \gamma_0 x + \gamma_1 \int_0^t \{z + \gamma_0 x(\tau)\}\, d\tau \tag{10}$$

where the state variable z is defined by (6).

Note that (10) includes an integral term differently from (5) while the state equation (6) is commonly used. To investigate the estimation error dynamics, one may have

$$\begin{aligned} \dot{\epsilon} &= \dot{d} - \dot{\hat{d}} \\ &= \dot{d} - \dot{z} - \gamma_0 \dot{x} - \gamma_1 (z + \gamma_0 x) \\ &= \dot{d} - \gamma_0 \epsilon - \gamma_1 (z + \gamma_0 x) \end{aligned} \tag{11}$$

Hence, through some manipulations, it follows that

$$\ddot{\epsilon} + \gamma_0 \dot{\epsilon} + \gamma_0 \gamma_1 \epsilon = \ddot{d}(t) \tag{12}$$

Note that the estimation error would converge to zero with the positive scalars γ_i's when the disturbance is of ramp as in (5). Also, the convergent dynamics can be assigned using the eigenvalue assignment using the characteristic equation

$$s^2 + \gamma_0 s + \gamma_0 \gamma_1 = 0 \tag{13}$$

Furthermore, the steady state error may be shown to be

$$|\epsilon_{ss}| \leq \frac{1}{\gamma_0 \gamma_1} \sup_{t \geq 0} |\ddot{d}(t)| \tag{14}$$

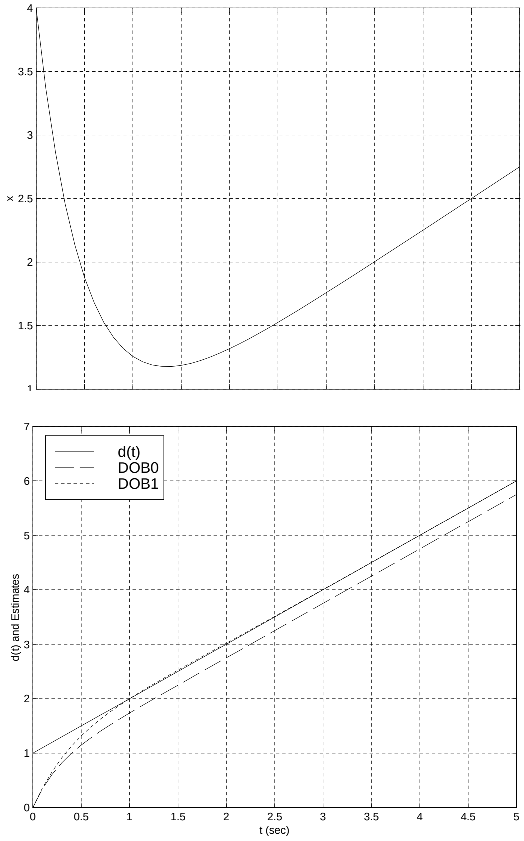

Fig. 1. Simulation results for Example 1. $x(t)$ (left). $d(t)$ and $\hat{d}(t)$ (right).

Example 1: Given the system

$$\dot{x} = -2x + d(t), \quad x(0) = 4 \tag{15}$$

where $d(t) = 1 + 0.5t$, we have the observer as follows.

$$\begin{cases} \hat{d} = z + \gamma_0 x + \gamma_1 \int_0^t (z + \gamma_0 x) d\tau \\ \dot{z} = -\gamma_0 \{-2x + \hat{d}\} \end{cases} \tag{16}$$

for $\gamma_0 = 4$ and $\gamma_1 = 1$ that place the eigenvalues at $\{-2, -2\}$. The simulation results are shown in Fig. 1. The estimation performance of the observer above is denoted by DOB1, which shows the asymptotic and exponential stability for the ramp disturbance estimation. Here, it is noted that the DOB0 denotes the estimate when $\gamma_1 = 0$ instead of $\gamma_1 = 1$, which results in an observer for a contant disturbance treated in Section 2. Observe that there remains the steady state error by DOB0. Now, to explain the effect of the initial state $z(0)$, let us set $z(0) = 0$ instead of choosing $z(0) = -\gamma_0 x(0)$. Then, in Fig. 2, one may observe the unsatisfactory transient response at the beginning of estimation. This phenomenon may severely distort the transient response whenever the disturbance observer is turned on from being off.

3.2. Combination with state estimation

Now, let us denote that the measured state as

$$x_m = x + m(t) \tag{17}$$

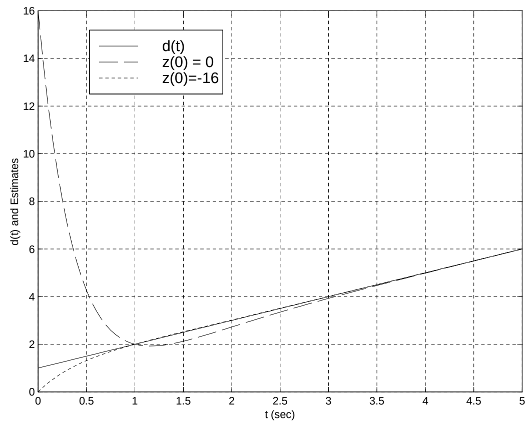

Fig. 2. Effect of the initial state variable $z(0)$.

where m is the measurement noise. With the noisy measurement, the disturbance estimate given by (from eqn. (10)):

$$\hat{d}(t) = z + \gamma_0 x_m + \gamma_1 \int_0^t \{z + \gamma_0 x_m(\tau)\}\, d\tau \tag{18}$$

would be corrupted by the noise because x_m directly puts through. To prevent this, we propose a disturbance observer (**DOB2**):

$$\hat{d} = z + \gamma_0 \hat{x} + \gamma_1 \int_0^t \{z + \gamma_0 x_m(\tau)\}\, d\tau \tag{19}$$

where the state estimate $\hat{x}$ and z are defined as

$$\dot{\hat{x}} = f(x_m, u; t) + \hat{d} + K_f(x_m - \hat{x}), \tag{20}$$

$$\dot{z} = -\gamma_0 \left\{ f(x_m, u; t) + \hat{d} \right\} \tag{21}$$

for $\hat{x}(0) = x_m(0),\ z(0) = -\gamma_0 x_m(0)$.

To investigate the stability of the disturbance and state observer, let us introduce the variables

$$\epsilon_0 = x - \hat{x}, \qquad \epsilon_1 = d - \hat{d} \tag{22}$$

Then, it follows that

$$\dot{\epsilon}_0 = -K_f \epsilon_0 + \epsilon_1, \tag{23}$$

and,

$$\ddot{\epsilon}_1 = \ddot{d} + n(m, \dot{m}) - \gamma_0 K_f \dot{\epsilon}_0 - \gamma_0 \gamma_1 \epsilon_1. \tag{24}$$

where $n(m, \dot{m}) = \gamma_0 K_f^2 m - \gamma_0 (K_f + \gamma_1) h(m) - \gamma_0 (K_f + 1)\dot{m}$ for $h \triangleq f(x, u; t) - f(x_m, u; t)$. It is noted that effects of the noise are lumped in the term $n(n, \dot{m})$, which becomes less effective to the state error after the double intergation. From these, the state and disturbance estimates are coupled with the following dynamics:

$$\frac{d}{dt}\begin{pmatrix} \epsilon_0 \\ \epsilon_1 \\ \dot{\epsilon}_1 \end{pmatrix} = \begin{bmatrix} -K_f & 1 & 0 \\ 0 & 0 & 1 \\ \gamma_0 K_f^2 & -\gamma_0(K_f + \gamma_1) & 0 \end{bmatrix} \begin{pmatrix} \epsilon_0 \\ \epsilon_1 \\ \dot{\epsilon}_1 \end{pmatrix} + h(t) \tag{25}$$

where $h(t)^T = [0,\ 0,\ \ddot{d} + n(m, \dot{m})]$. Then, through manipulations, the characteristic equations for the above system is given by

$$s^3 + K_f s^2 + \gamma_0 (K_f + \gamma_1) s + \gamma_0 \gamma_1 K_f = 0 \tag{26}$$

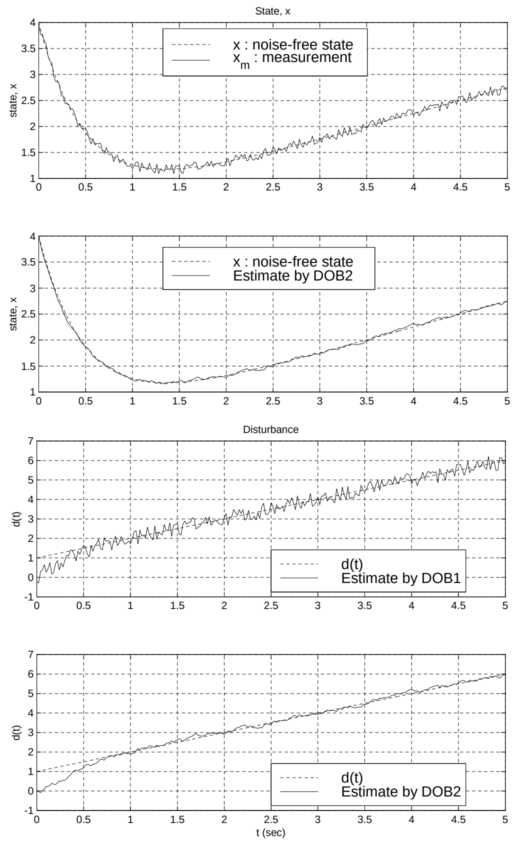

Fig. 3. Simulation results for Example 2. x_m & $\hat{x}$ (left). $\hat{d}_{DOB1}$ & $\hat{d}_{DOB2}$ (right).

In fact, it may be shown that the eigenvalues of (26) can be arbitrarily assigned based on the following rule

$$\begin{cases} K_f &= -(\lambda_1 + \lambda_2 + \lambda_3) \\ \gamma_0 &= -\frac{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_1+\lambda_3)}{(\lambda_1+\lambda_2+\lambda+3)^2} \\ \gamma_1 &= -\frac{\lambda_1\lambda_2\lambda_3(\lambda_1+\lambda_2+\lambda_3)}{(\lambda_1+\lambda_2)(\lambda_2+\lambda_3)(\lambda_1+\lambda_3)} \end{cases} \tag{27}$$

which places the eigenvalues at $\{\lambda_1, \lambda_2, \lambda_3\}$ for $\lambda_i \in C$.

Remark 1: As an extreme case, suppose that the state estimation is sufficiently faster than the disturbance estimate so that, from (23),

$$\epsilon_0 \approx \frac{1}{K_f}\epsilon_1. \tag{28}$$

Then, from (24), we have

$$\ddot{\epsilon}_1 + \gamma_0 \dot{\epsilon}_1 + \gamma_0 \gamma_1 \epsilon_1 = \ddot{d} \tag{29}$$

which matches the dynamics of DOB1. This implies that the dynamics of the disturbance and state observer approaches to that of DOB1, as long as the state estimate is sufficiently fast.

Example 2: For the same system in Example 1, we assume a uniformly random noise such that $|m(t)| \leq 0.1$ in the measured state. For the values such that $K_f = 9$, $\gamma_0 = 2.667$ and $\gamma_1 = 1.125$, which place the eigenvalues at $\lambda_i = -3$ $(i = 1, 2, 3)$, we simulate the performances of DOB1 and DOB2 in the presence of the measurement noise. The design parameters for DOB1 are kept equal to those in Example 1. As shown in Fig. 3, DOB2 provides the disturbance estimate much less corrupted by the noise than DOB1 does.

Table 1. Transfer function of estimation error to disturbance.

Observer Type	DOB0	DOB1	DOB2
$S(s)\left[\triangleq \frac{E(s)}{D(s)}\right]$	$\frac{s}{s+\gamma}$	$\left(\frac{s}{s+\gamma}\right)^2$	$\left(\frac{s}{s+\gamma}\right)^2 \cdot \frac{s+3\gamma}{s+\gamma}$

3.3. Performance analysis in frequency domain

So far, we have addressed the observer performance in the time-domain subject to a ramp disturbance. In this section, however, we address the performance in the frequency domain so as to envisage the observer characteristics in the presence of non-ramp style disturbances such as periodic disturbances.

For simplifying the analysis and, also, for convenience in practice, suppose that we choose the parameters as, for DOB1,

$$\gamma_0 = 2\gamma, \quad \gamma_1 = \frac{\gamma}{2} \tag{30}$$

for a $\gamma > 0$, which would result in the double eigenvalues at $s = -\gamma$. And, for DOB0, let us choose as

$$\gamma_0 = \gamma \tag{31}$$

Then, the transfer functions of the estimation error to the disturbance can be summarized as in Table 1. Assuming that the cutoff frequency is set to 100 (Hz), the frequency responses are plotted in Fig.4 in order to show the effectiveness of the DOB's. For the figure, we used $\gamma = 2\pi \times 100$, $\gamma = 2\pi \times 64.22$, and $\gamma = 2\pi \times 165$ for DOB0, DOB1 and DOB2, respectively, to have the same cutoff frequency.

The figure implies that even disturbances of non-ramp style can be estimated by the proposed observers as long as they are sufficiently *slow* with respect to the bandwidth of observers. Moreover, DOB1 (as well as DOB2) that are capable of estimating the ramp disturbance perfectly can further reduce the estimation error for the low frequency disturbance, compared with the constant observer, DOB0. This feature is expected to be useful for estimating the disturbance caused by the eccentricity in optical data storage devices. It is noted that the experimental feasibility has been shown with the constant disturbance observer in [8] and [6].

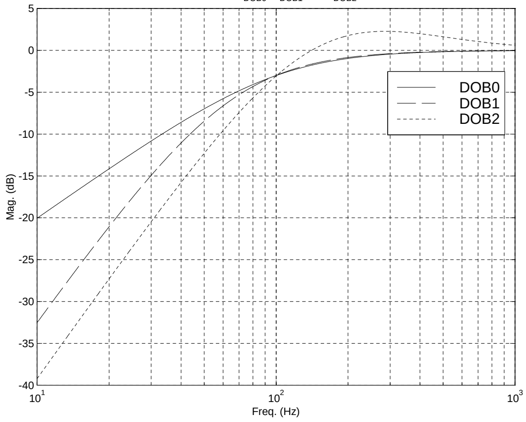

Fig. 4. FRFs for S-functions of DOB0, DOB1 and DOB2.

4. Concluding remarks

In the paper, we have proposed the input disturbance observers that can estimate higher order of disturbances than a constant. The dynamic features of the proposed approaches have been shown and proven by the examples and the analysis.

The proposed approaches are expected to be useful for solving practical issues such as the track-following problem in optical storage devices. The experimental validation of the proposed approaches remains as one of important topics for further study.

References

[1] J. R. Ryoo, K. B. Jin, J.-H. Moon, and M. J. Chung, "Track-following control using a disturbance observer with asymptotic disturbance rejection in high-speed optical disk drives," *IEEE Trans. Consumer Elect.*, Vol. 49, No. 4, pp. 1178-1185, 2003.

[2] R. Marino and P. Tomei,"Global adaptive compensation of noise with unknown frequency," *Proc. of the 39th IEEE Conf. on Decision and Control*, pp.4926-4927, 2000.

[3] D. Iannuzzi and R. Rizzo,"Disturbance observer for dynamic estimation of friction in railway traction systems," *Proc. of the IEEE Conf. on Decision and Control*, pp.2979-2982, 2003.

[4] W.-H. Chen, D. J. Ballance, P. J. Gawthrop, J. J. Gribble, J. O'Reilly,"A nonlinear disturbance observer for two link robotic manipulators," *Proc. of the 38th Conf. on Decision and Control*, pp.3410-3415, 1999.

[5] B. K. Kim and W. K. Chung,"Advanced disturbance observer design for mechanical positioning systems," *IEEE Trans. on Ind. Elect.*, vol. 50, no. 6, 2003.

[6] K.-S. Kim, "Eccentricity compensation in optical storage systems: Analysis and Experiments," *Japanese Journal of Applied Physics*, Vol. 41, Part 1, No. 10, pp. 6302-6303, 2002.

[7] B. Friedland and Y. Park, "On adaptive friction compensation," *IEEE Trans. Automat. Contrl.*, Vol. 37, No. 10, pp. 1609-1612, 1992.

[8] K.-S. Kim, "Using a disturbance observer for eccentricity compensation in optical storage systems," *Trans. on Contr. Autom, and Sys. Eng.*, Vol. 4, No. 4, pp. 319-323, 2002.