Performance of Selective Rake Receivers for Weak Signals in Impulsive Fading Channels

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Abstract

In this paper, based on a detection criterion proposed recently, we investigate the performance of selective rake receivers (SRRs) in fading environment exhibiting impulsive nature. Optimum and suboptimum SRRs for the impulsive fading channel are derived, and suboptimum SRRs with reduced complexity are obtained for practical purposes. Simulation results confirm that, when the noise is impulsive, the SRRs designed for impulsive noise offer performance improvement over those optimized for Gaussian environment. The suboptimum SRR is observed to exhibit almost the same performance as the optimum SRR.

I. Introduction

Recent studies [1] in the area of wireless systems indicate that the ultra wideband multiple access (UWB-MA) system is a viable technology for short-range multiple access communications. The fine time resolution of multipath induced by the channel is exploited by the use of a rake receiver to capture significant amount of energy found in the multipath components and to benefit from multipath diversity gain.

In many detection problems, most of the rake receivers have been studied and implemented under the Gaussian noise assumption. In the UWB-MA systems, however, the sum of the multiple access interference (MAI) and channel noise can be more adequately modelled as impulsive interference [2]. Thus, the selective rake receiver (SRR) optimized for Gaussian environment, which will be called the Gaussian optimized (GO) SRR in this paper, could experience severe performance degradation in the UWB-MA systems in impulsive channels.

In this paper, we obtain and evaluate the performance of the optimum and reduced-complexity SRRs under impulsive interference.

II. UWB-MA system

2.1 Transmitter

We assume that the users employ binary phase shift keyed (BPSK) modulation in which the transmitted signals consist of a low duty-cycle sequence of a large number of UWB pulses. The duration T_q of a unit energy UWB pulse $q(t)$ is assumed to be a very small portion of the frame time (or pulse repetition period) T_f. Since we focus on SRRs under impulsive noise in this paper, we are not concerned with the shape of the UWB pulses.

The signal of the k-th user for $0 \le t \le N_s T_f$ is one of the two equi-probable signals $s_0^{(k)}(t)$ and $s_1^{(k)}(t)$, where

$$s_i^{(k)}(t) = \theta b_i^{(k)} \sum_{j=0}^{N_s-1} q(t - jT_f - c_j^{(k)} T_c), \quad i = 0, 1. \tag{1}$$

In (1), N_s is the number of the UWB pulses that are modulated by a given symbol, $T_s = N_s T_f$ is the symbol duration, θ is the signal strength, $b_i^{(k)}$ is the binary data bit of equiprobable $+1$ and -1 transmitted by the k-th user, $\{c_j^{(k)}\}_{j=0}^{N_s-1}$ is the time-hopping sequence of the k-th user with period N_c (i.e., $0 \le c_j^{(k)} \le N_h$ and $c_{j+nN_c}^{(k)} = c_j^{(k)}$ for all integers j and n with N_h an integer, with $c_j^{(k)}$ representing the 'location' in the j-th frame of the signal of the k-th user), and T_c is the chip duration.

The frame time T_f is chosen to be sufficiently large ($T_f > N_h T_c + T_c$) to reduce the intersymbol and intrasymbol interference caused by the delay spread, where the difference $T_f - (N_h + 1) T_c$ is called the guard interval.

2.2 Channel

We employ the channel model in [3] accepted by the IEEE 802.15 Study Group 3a based on indoor channel measurements in the 2-8 GHz frequency band. The fading is assumed to be independent for each cluster, and independent for each ray in a cluster.

The impulse response of the channel is then given by

$$h(t)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}U_{m,n}\delta(t-T_m-T_{m,n}), \quad (2)$$

where M is the number of clusters, N is the number of multipath components (rays) in a cluster, $U_{m,n}$ is the multipath channel coefficient of the n-th ray of the m-th cluster, T_m is the arrival time of the m-th cluster, $T_{m,n}$ is the arrival time of the n-th ray measured from the beginning of the m-th cluster, and $\delta(\cdot)$ is the impulse function.

2.3 Receiver

Assume that the desired user is $k=1$. The received signal $r(t)$ can then be written as

$$r(t)=s_i^{(1)}(t)*h(t)+w(t)+n(t) \quad (3)$$

where $*$ is the convolution operation, $w(t)$ represents the MAI caused by the other users, $n(t)$ denotes the channel noise.

The received signal $r(t)$ contains MN resolvable multipath components, all of which can be ideally exploited by the all rake receiver (ARR) [4]. Since the number of resolvable multipath components increases with the spreading bandwidth, the number of matched filters required for the ARR could be inordinately large for UWB channels.

We thus consider the SRR [5], which selects and combines the strongest L multipath channel coefficients among the MN resolvable multipath components. Each of the L paths selected is correlated with the first user's pulse, and passed through a matched filter.

Assuming that the multipath channel coefficients $\{U_{m,n}\}$ and the arrival times $\{T_m\}$ and $\{T_{m,n}\}$ are estimated perfectly, the output of the matched filter corresponding to the l-th finger of the SRR at the j-th frame is given by

$$\begin{aligned} R_{jl} &= \int_{jT_f}^{(j+1)T_f} r(t)q(t-c_j^{(1)}T_c-t_{jl})dt, \\ &= u_{jl}\theta b_i^{(1)}+N_{jl}, \end{aligned} \quad (4)$$

where t_{jl} is the estimated value of $\{T_m+T_{m,n}\}$, u_{jl} is the estimated value of U_{jl}, and

$$N_{jl}=\int_{jT_f}^{(j+1)T_f}(w(t)+n(t))q(t-c_j^{(1)}T_c-t_{jl})dt \quad (5)$$

is the noise component of the correlator output.

When the system makes a hard decision per frame, the SRR is to choose between

$$H_0: R_{jl}=-u_{jl}\theta+N_{jl} \quad (6)$$

and

$$H_1: R_{jl}=u_{jl}\theta+N_{jl}, \quad (7)$$

for $l=0,1,\ldots,L-1$ at the j-th frame. Here, H_i is the hypothesis that $s_i^{(1)}(t)$ is transmitted for $i=0,1$.

Ⅲ. Rake receivers in impulsive environment

3.1 Decision rules

Evaluating the likelihood ratio with the hypotheses (6) and (7) and adopting the ML decision rule in the j-th frame, the decision region of the hypothesis H_i is

$$D_i^{ML}=\{\underline{r}: p_{\underline{R}}(\underline{r}|s_i^{(1)},\theta)\geq p_{\underline{R}}(\underline{r}|s_k^{(1)},\theta), \ \forall i\neq k\}, \quad (8)$$

where $p_{\underline{R}}(\underline{r}|s_i^{(1)},\theta)$ represents the conditional pdf of $\underline{R}=(R_{j0},R_{j1},\cdots,R_{j,L-1})$ given that $s_i^{(1)}(t)$ is transmitted in the j-th frame and the value of the signal strength is θ, and $\underline{r}=(r_{j0},r_{j1},\ldots,r_{j,L-1})$. Using the suboptimum ML (S-ML) decision rule [6], the probability of error is minimized if the decision region of the hypothesis H_i is

$$D_i^{S-ML}=\left\{\underline{r}: \frac{d}{d\theta}p_{\underline{R}}(\underline{r}|s_i^{(1)},\theta)\Big|_{\theta=0}\geq \frac{d}{d\theta}p_{\underline{R}}(\underline{r}|s_k^{(1)},\theta)\Big|_{\theta=0}, \quad \forall i\neq k\right\} \quad (9)$$

when the signal strength approaches zero. A rationale for using the S-ML decision rule is that it is desired to design a SRR which has optimum performance at low signal to noise ratio (SNR) since the UWB-MA systems are low power communication systems.

3.2 Decision rules in S αS environment

We model $\{N_{jl}\}_{l=0}^{L-1}$ with the independent and identically distributed (i.i.d.) symmetric alpha stable (S α S) distribution [2], [7]. The zero mean (S αS) pdf is given by

$$f_{N_{jl}}(x)=\begin{cases} \dfrac{1}{\pi\gamma^{1/\alpha}}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k!}\Gamma(\alpha k+1) \cdot \sin\left(\frac{k\alpha\pi}{2}\right)\left(\frac{|x|}{\gamma^{1/\alpha}}\right)^{-\alpha k-1}, & \text{for } 0<\alpha\leq 1, \\ \dfrac{1}{\pi\alpha\gamma^{1/\alpha}}\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^{k}}{(2k)!} \cdot \Gamma\left(\frac{2k+1}{\alpha}\right)\left(\frac{x}{\gamma^{1/\alpha}}\right)^{2k}, & \text{for } 1\leq\alpha\leq 2, \end{cases} \quad (10)$$

where γ is related to the spread of the SαS pdf, and α $(0 < \alpha \leq 2)$ is related to the heaviness of the tails of the SαS pdf. A small value of α indicates severe impulsiveness while a value close to 2 indicates a more Gaussian type of behaviors.

The two infinite series in (10) become a Cauchy pdf

$$f_{N_{jl}}(x) = \frac{\gamma}{\pi(x^2 + \gamma^2)} \tag{11}$$

when $\alpha = 1$, and the second infinite series in (10) produces a Gaussian pdf

$$f_{N_{jl}}(x) = \frac{1}{2\sqrt{\gamma\pi}} \exp\left(-\frac{x^2}{4\gamma}\right). \tag{12}$$

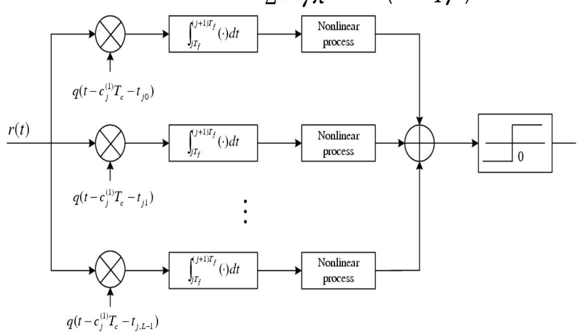

Fig. 1: The rake receiver for the desired user designed for impulsive environment (When the nonlinear processes are absent, the receiver is for Gaussian environment).

when $\alpha = 2$. It is also known that a closed form expression of (10) does not exist except when $\alpha = 1$ or 2. Under the Gaussian noise $\alpha = 2$, we can show that

$$\sum_{l=0}^{L-1} u_{jl} r_{jl} \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{13}$$

using (8) or (9), for which the SRRs are shown in Fig. 1. The matched filter outputs in Fig. 1 are individually weighted and then combined by a linear combiner called the maximal ratio combining (MRC), the optimum diversity scheme [8] providing the maximum output SNR in the Gaussian environment.

When $\alpha = 1$, on the other hand, we have the optimum decision rule

$$\sum_{l=0}^{L-1} \ln\left(\frac{(r_{jl} + u_{jl}\theta)^2 + \gamma^2}{(r_{jl} - u_{jl}\theta)^2 + \gamma^2}\right) \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{14}$$

using (8) and the suboptimum decision rule

$$\sum_{l=0}^{L-1} \frac{u_{jl} r_{jl}}{r_{jl}^2 + \gamma^2} \underset{H_0}{\overset{H_1}{\gtrless}} 0 \tag{15}$$

from (9), for which the SRRs are also shown in Fig. 1.

The SRR based on (14), called the Cauchy optimized (CO) SRR, requires an estimate of the signal strength θ while the SRR based on (15), called the Cauchy sub-optimized (CS) SRR, does not. This implies that the CS SRR requires less structural complexity than the CO SRR.

Unlike the GO SRR based on (13), the CO and CS SRRs for impulsive noise employs a nonlinear process at each finger before the observations are summed. The nonlinear process in Fig. 1 is $\ln\left(\frac{(r_{jl} + u_{jl}\theta)^2 + \gamma^2}{(r_{jl} - u_{jl}\theta)^2 + \gamma^2}\right)$ and $\frac{u_{jl} r_{jl}}{r_{jl}^2 + \gamma^2}$ for the CO and CS SRRs, respectively, as is evident from (14) and (15).

The nonlinear process reduces the influence of very large amplitudes, resulting in the performance improvement in impulsive environment as is reported in robust signal detection theory [9], [10].

Ⅳ. Numerical results

Since the standard SNR becomes meaningless for an SαS noise when $\alpha < 2$ due to an infinite variance, we employ the geometric SNR (G-SNR) [11] to indicate the relative strength between the information-bearing signal and SαS noise. The G-SNR is defined as

$$G-SNR = \frac{1}{2C_g}\left(\frac{\theta}{S_0}\right)^2, \tag{16}$$

where

$$S_0 = \frac{(C_g \gamma)^{1/\alpha}}{C_g} \tag{17}$$

and

$$C_g = \exp\left\{\lim_{s\to\infty}\left(\sum_{z=1}^{s}\frac{1}{z} - \ln s\right)\right\} \simeq 1.78. \tag{18}$$

The G-SNR becomes the standard SNR when $\alpha = 2$.

Let us assume that the fading is sufficiently slow so that a large number of bits are transmitted over essentially the same channel [4]. The Monte-Carlo simulations are set up based on the channel model 1 described in [3], and the bit error rate (BER) is obtained from 2.5×10^6 Monte-Carlo runs, where the noise samples $\{N_{jl}\}_{l=0}^{L-1}$ are generated using

$$\gamma^{\frac{1}{\alpha}} \frac{\sin(\alpha)}{(\cos A)^{1/\alpha}}\left(\frac{\cos[(1-\alpha)A]}{B}\right)^{\frac{1-\alpha}{\alpha}}. \tag{19}$$

In (19), A is uniform on $(-\pi/2, \pi/2)$ and B is exponential with mean 1 [7]. Without loss of generality, we have assumed $\gamma = 1$.

Figs. 2-3 show the performance characteristics of the GO, CO, and CS SRRs in various SαS environment. It is observed that the CO and CS SRRs possess almost the same performance characteristics especially when the G-SNR is small. The CO and CS SRRs uniformly outperform the GO SRR except when $\alpha=2$, which becomes clearer as the impulsiveness gets higher (α gets smaller) as is evident in Fig. 2, for example.

It is noteworthy that the GO SRR performs worse as the number L of the fingers of the SRR increases when the interference is impulsive, while the CO and CS SRRs perform better as the number L increases irrespective of the interference distribution (Fig. 3).

Again, the results obtained and shown here are not unexpected from the signal detection point of view: it is well-known [9], [10] that an observation with a very large magnitude in impulsive environment should be considered not as a signal plus noise but as noise only. The nonlinear processes in Fig. 1 attenuate the influence of observations with very large magnitudes, thereby protecting the CO and CS SRRs from using the observations inappropriately. On the other hand, the GO SRR favors any observation with a larger magnitude, resulting in poor performance in non-Gaussian interference.

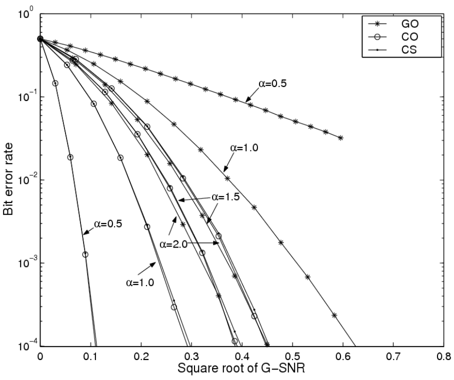

Fig. 2: Performance comparison of the GO, CO, and CS SRRs with $L=6$ when $N_s=100$ in the SαS environment with $\alpha=0.5,\ 1.0,\ 1.5$, and 2.0.

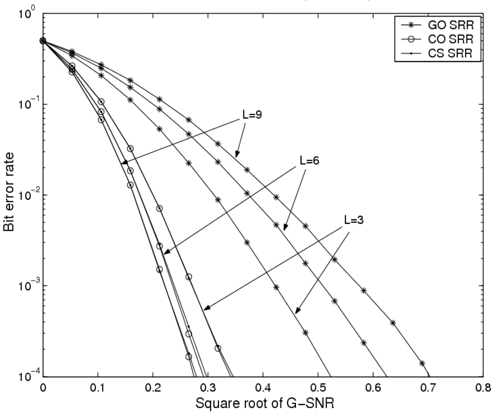

Fig. 3: Performance comparison of the GO, CO, and CS SRR with $L=3,\ 6$, and 9 when $N_s=100$ in the Cauchy environment.

V. Conclusion

Under impulsive circumstances, we have derived the optimum and suboptimum SRRs based on the ML and S-ML decision rules, respectively. We have compared various SRRs in the SαS environment. In impulsive environment, the CO and CS SRRs generally outperform the GO SRR. As the number of fingers increases in impulsive environment, the performance of the CO and CS SRRs improves while that of the GO SRR degrades.

It is also observed from simulation results that, when the noise is impulsive, the SRRs designed for impulsive noise offer performance improvement over those optimized for Gaussian environment. The suboptimum SRR is observed to exhibit almost the same performance as the optimum SRR.

Acknowledgment

This research was supported by the Ministry of Science and Technology (MOST) under the National Research Laboratory (NRL) Program of Korea Science and Engineering Foundation (KOSEF), for which the authors would like to express their thanks.

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