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Carrier Frequency Offset Estimation Using ESPRIT for the Interleaved OFDMA Uplink Systems

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Abstract

In this paper, a carrier frequency offset (CFO) estimator is proposed for the interleaved OFDMA uplink systems. It is based on the estimation of signal parameters via rotational invariance technique (ESPRIT). Compared with the Cao's estimator, the proposed estimator has low computational complexity. Simulation results demonstrate that the proposed estimator performs better than Cao's estimator at the relatively low SNR region. Hence, the proposed estimator is more applicable to the practical environments than the Cao's estimator.

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) has obtained much attention recently and has been proposed for the uplink of wireless communication systems [1]. Since the one-tap channel equalizer is available in the frequency domain, OFDMA proffers an attractive solution to high-rate multiple access.

In the uplink of an OFDMA system, the frequency domain signals from all users should be orthogonal. Otherwise, intercarrier interference (ICI) and multiple access interference (MAI) occur, that results in severe performance degradation. Therefore, multiuser carrier frequency offset (CFO) estimation problem is a essential task in designing OFDMA uplink receivers [2].

Cao [3] gives a blind CFO estimator using the inherent signal structure of the interleaved OFDMA uplink, and multiple signal classification (MUSIC). Without pilot symbols, it offers a very high estimation performance. However, the computational complexity is very high, since it has to perform numerous matrix multiplication and addition to operate MUSIC. Besides, at the relatively low SNR region, the peak values often disappear inducing the dominant error terms. That results in severe performance degradation of estimation.

In this thesis, a new CFO estimator is proposed for the interleaved OFDMA uplink systems. It is based on the estimation of signal parameters via rotational invariance technique (ESPRIT). In comparison with [3], the proposed estimator has two major advantages. Those are the lower complexity, and better performance at the relatively low SNR region. The next section introduces the signal model for the interleaved OFDMA uplink systems. The proposed estimator is described in Section 3, and the comparisons between the two estimators are also given in Section 3. The simulation results are described in Section 4, and the conclusions are presented in Section 5.

II. SIGNAL MODEL

It is assumed that the total N subcarriers are divided into Q subchannels in an interleaved type. Each subchannel has $P=N/Q$ subcarriers. For the sake of convenience, it is assumed that each user has only one subchannel. After CP is removed, the received signal of an OFDMA block of the kth user in the time domain is given by

$$\begin{aligned} r^{(k)}(n) &= \sum_{p=0}^{P-1} D^{(k)}(p) H_s^{(k)}(p) e^{j2\pi n(pQ+q^{(k)}+\varepsilon^{(k)})/N} \\ &= e^{j2\pi n w^{(k)}/P} \sum_{p=0}^{P-1} D^{(k)}(p) H_s^{(k)}(p) e^{j2\pi np/P}, \end{aligned} \quad (1)$$

where $D^{(k)}(p)$ is a set of P data stream of the kth user, and $H^{(k)}(l)$ is the channel frequency response of the kth user and $H_s^{(k)}(p)$ is the samples from $H^{(k)}(l)$ at $l = pQ + q^{(k)}$. $w^{(k)} = (q^{(k)} + \varepsilon^{(k)})/Q$, and $q^{(k)}$ is the subchannel index of the kth user, and $\varepsilon^{(k)}$ is the *normalized* CFO of the kth user. From (1), it can be known that the received signal has a periodicity with period $P=N/Q$. That is,

$$r^{(k)}(n+mP) = e^{j2\pi m w^{(k)}/P} r^{(k)}(n). \qquad (2)$$

Thus, the N received samples in (1) can be arranged into a $Q \times P$ matrix like following:

$$\mathbf{\Gamma}^{(k)} = \vartheta^{(k)}\left[\beta^{(k)} \circ \left(\lambda^{(k)}\mathbf{W}\right)\right], \qquad (3)$$

where $\circ$ means Schur product [4], $\mathbf{W}$ is a $P \times P$ IFFT matrix,

$$\vartheta^{(k)} = \begin{bmatrix} 1 & e^{j2\pi w^{(k)}} & \cdots & e^{j2\pi(Q-1)w^{(k)}} \end{bmatrix}^T,$$

$$\beta^{(k)} = \begin{bmatrix} 1 & e^{j2\pi w^{(k)}/P} & \cdots & e^{j2\pi(P-1)w^{(k)}/P} \end{bmatrix}^T, \text{ and}$$

$$\lambda^{(k)} = \left[D^{(k)}(0)H_s^{(k)}(0) \;\; D^{(k)}(1)H_s^{(k)}(1) \;\cdots\; D^{(k)}(P-1)H_s^{(k)}(P-1)\right].$$

The received signal at the base station is the superposition of the signals from all users, as

$$r(n) = \sum_{k=1}^{K} r^{(k)}(n). \qquad (4)$$

From (2) to (4), the N received samples at the base station can be arranged into a $Q \times P$ matrix like following:

$$\mathbf{\Gamma} = \sum_{k=1}^{K} \mathbf{\Gamma}^{(k)} = \mathbf{\Theta}\left[\mathbf{B} \circ \left(\mathbf{\Lambda W}\right)\right] = \mathbf{\Theta S}, \qquad (5)$$

where $\mathbf{\Theta} = \left[\vartheta^{(0)}, \vartheta^{(1)}, \cdots, \vartheta^{(K-1)}\right]$,

$\mathbf{S} = \left[\mathbf{B} \circ \mathbf{\Lambda W}\right]$,

$\mathbf{B} = \left[\left\{\beta^{(0)}\right\}^T, \left\{\beta^{(1)}\right\}^T, \cdots, \left\{\beta^{(K-1)}\right\}^T\right]^T$, and

$\mathbf{\Lambda} = \left[\left\{\lambda^{(0)}\right\}^T, \left\{\lambda^{(1)}\right\}^T, \cdots \left\{\lambda^{(K-1)}\right\}^T\right]^T$.

In equation (5), $\mathbf{S}$ is a $K \times P$ matrix and its kth row is $\left[r^{(k)}(0), r^{(k)}(1), \cdots r^{(k)}(P-1)\right]$. In addition, $\mathbf{\Theta}$ is a $Q \times K$ matrix formed by

$$\mathbf{\Theta} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j2\pi w^{(1)}} & e^{j2\pi w^{(2)}} & \cdots & e^{j2\pi w^{(K)}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j2\pi(Q-1)w^{(1)}} & e^{j2\pi(Q-1)w^{(2)}} & \cdots & e^{j2\pi(Q-1)w^{(K)}} \end{bmatrix}. \qquad (6)$$

III. CFO ESTIMATION

A. Proposed CFO estimator using ESPRIT

We omit the mathematical derivation here, due to the limitation of pages. Let vector $\mathbf{r}_l$ denote the lth column of $\mathbf{\Gamma}$ in (5). Firstly, formulate the covariance matrix $\hat{\mathbf{R}} = (1/P)\sum_{l=1}^{P} \mathbf{r}_l \mathbf{r}_l^H$. Then, find K eigenvectors of $\hat{\mathbf{R}}$ corresponding to the K largest eigenvalues of $\hat{\mathbf{R}}$. Let $\mathbf{E}$ denote the set of K eigenvectors. $\mathbf{E}_x$ is defined as the first *(Q-1)* rows of $\mathbf{E}$, and $\mathbf{E}_y$ is defined as the last *(Q-1)* rows of $\mathbf{E}$. A matrix $\mathbf{F}$ is obtained by

$$\mathbf{F} = \mathbf{E}_x^{+}\mathbf{E}_y, \qquad (7)$$

where $(\cdot)^+$ denotes the pseudo-inverse.

Now the set of $\lambda^{(k)}$, $k = 1, 2, \cdots, K$ is evaluated from the set of K eigenvalues of $\mathbf{F}$. $\hat{w}^{(k)}$ is obtained from

$$\hat{w}^{(k)} = \frac{angle(\lambda^{(k)})}{2\pi}. \qquad (8)$$

In the assumption of $\left|\varepsilon^{(k)}\right| < 0.5$, the range of $w^{(k)}$ is $\left((q^{(k)} - 0.5)/Q, \;\; (q^{(k)} + 0.5)/Q\right)$. Since each user lies in its own range, one-to-one mapping is possible between $w^{(k)}$ and $\varepsilon^{(k)}$ [3]. Hence, $\hat{\varepsilon}^{(k)}$ is computed by

$$\hat{\varepsilon}^{(k)} = Q\hat{w}^{(k)} - q^{(k)}. \qquad (9)$$

B. Comparisons with Cao's CFO estimator

Since $\hat{\mathbf{R}}$ is a square matrix, the Cao's estimator requires $O\left(Q^3\right)$ multiplications for the singular value decomposition (SVD) of $\hat{\mathbf{R}}$ [5]. This is the same as in the proposed estimator, which requires $O\left(Q^3\right)$ for the eigendecomposition of $\hat{\mathbf{R}}$. In addition, the order of finding K peaks in the Cao's estimator is $O\left(2\alpha Q(Q-K)\right)$, where α is the number of iterations to find K peaks. On the other hand, the proposed estimator requires $O\left(K^3\right)$ multiplications to compute K eigenvalues from $\mathbf{F}$. Generally, α has to be very large number to prevent the error floor at the high SNR region. Thus, the Cao's estimator is much more complex than the proposed one.

Now the cumulative distribution functions (CDF) of two estimators are compared with each other. In Cao's estimator, if both $|\varepsilon^{(k)}|$ and $|\varepsilon^{(k+1)}|$ are relatively large and the sign of CFO for the *k*th and the *(k+1)*th user are plus and minus, respectively, $\varepsilon^{(k)} - \varepsilon^{(k+1)} \approx 1$ due to noise. This means that one peak disappears and the ambiguity for the CFO estimation occurs. It results in severe performance degradation of the Cao's estimator.

On the other hand, the proposed estimator always extracts the *K* eigenvalues, since $\mathbf{F}$ has full rank. That is the reason why the proposed estimator outperforms the Cao's at the relatively low SNR region. In Figure 1, the CDFs of the estimation error $\Delta\varepsilon = \left\{ {}^{\exists}k; k = \{1, \cdots K\} \mid \left|\varepsilon^{(k)} - \hat{\varepsilon}^{(k)}\right| \right\}$ is shown. 'Previous' indicated Cao's estimator and 'Proposed' denotes the proposed estimator. The CDFs of the proposed estimators converge to 1 faster than the CDFs of the previous estimators, which means that the proposed estimator has small errors in estimated CFOs than the previous estimator.

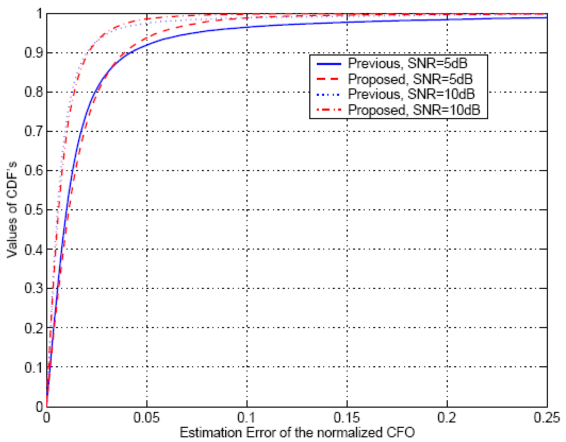

Figure 1. CDFs of the estimation error at *Q=8* and *K=4*.

IV. SIMULATION RESULTS

A. System parameters

The total number of subcarriers in the considered OFDMA uplink system is *N=1024*. The subcarriers are assigned to users in an interleaved type as in Section 2. Without losing generality, all experiments are performed in the assumption that all subchannels are consecutive. The total number of subchannels is *Q=8* or *Q=16*, and the corresponding number of subcarriers in a subchannel is *P=128* or *P=64*, respectively. The channel response of each user has the same length $L^{(k)} = 10$. Their power delay profiles are

$$E\left[\left|h^{(k)}(n)\right|^2\right] = p^{(k)} e^{(-n/5)}, \quad n = 0,1,\cdots,9, \tag{10}$$

where $p^{(k)}$ is a normalizing constant, used to set the channel power to be unity.

SNR per each user is assumed to be equal. In each following figure, the horizontal axis represents the SNR per each user and the vertical axis represents the averaged MSE of all users.

B. System performance

Figure 2 shows the performance improvement of the proposed estimator using multiple OFDMA blocks. *N=1024*, *Q=8*, *P=128*, and *K=2*. We generate the CFOs being random variables uniformly distributed in [-0.45, 0.45], and mutually independent among users. Use of two OFDMA blocks results in 3dB gain compared with one block case. Similarly, use of four blocks induces 3dB gain compared with two block case.

Figure 3 illustrates the variations of the estimation performances of the proposed estimator corresponding to the number of users. The number of subchannels is *Q=8*. As shown in the figure, the MSEs are increased with the growth of the number of users. The performance of the proposed estimator is affected by the difference between the CFOs of adjacent users, although that is not severe problem compared with the case of Cao's estimator. Hence, it can be known intuitively that the increase of users gives rise to the degradation of the estimation performance.

Figure 4 shows the MSE performance variation due to the maximum value of CFO and SNR. The proposed estimator outperforms the previous one at the relatively low SNR. When the maximum value of generated CFO is $\varepsilon_{\max} = 0.35$, the proposed estimator is superior to the Cao's estimator at the SNR region below than 12dB. Even at the SNR region above 12dB, the previous estimator has only about 1dB gain. However, when $\varepsilon_{\max} = 0.45$, the proposed estimator outperforms Cao's estimator from 3dB to 30dB. This implies that the lager $\varepsilon_{\max}$ is, the wider the range in which the proposed estimator outperforms the other is. Thus, in the circumstances which ε_{max} is relatively large and the SNR is relatively low, the proposed estimator have two merits compared with the previous estimator. Those are much lower computational complexity and the superior estimation performance.

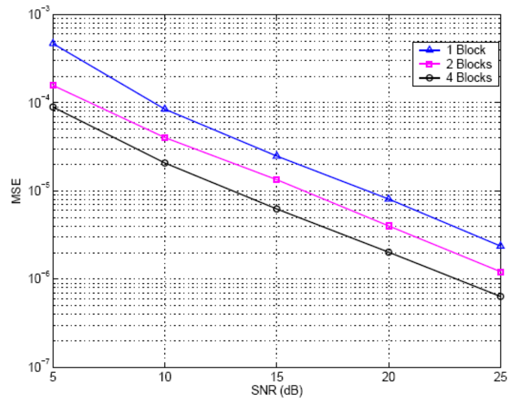

Figure 2. Improvement of estimation performance using multiple OFDMA blocks.

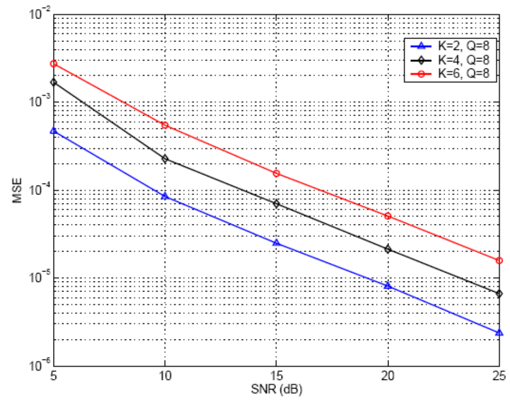

Figure 3. Variation of estimation performance corresponding to the number of users.

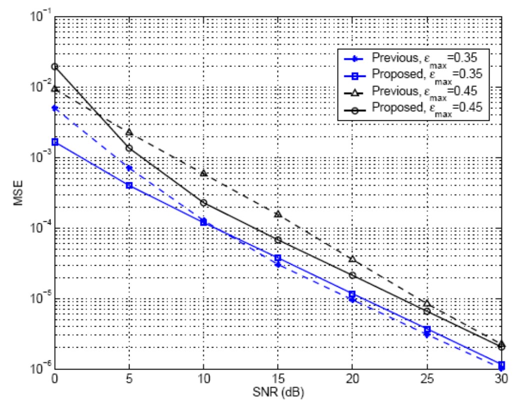

Figure 4. Variation of MSE due to the maximum value of CFO and SNR.

V. CONCLUSIONS

In this paper, a CFO estimator using ESPRIT for the interleaved OFDMA uplink systems is proposed. Since it exploits the periodic property of the inherent signal structure of the interleaved OFDMA uplink system, CFOs can be estimated without pilot symbols. It was numerically shown that the proposed estimator has lower computational complexity than the Cao's. In addition, the proposed estimator performs better than Cao's at the relatively low SNR region. Hence, the proposed estimator is more applicable to the practical environments than the previous estimator. Simulation results show that the performance of the proposed CFO estimator can be improved by using the several number of OFDMA blocks. It is also shown that the proposed estimator outperforms the previous one at the relatively low SNR region.

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