Study on the preamble based OFDM timing synchronization

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Abstract

Preamble based synchronization result in performanc e degradation due to the high time variation in the power delay profile of the channel in fast varying channel. Since most of the timing metric that exploit the preamble is not based on theoretical back ground, it is not easy to find out the reason. In this paper, the behavior of the timing metric in the multipath fading channel is explained.

Key words: *orthogonal frequency-division multiplexing (OFDM), preamble, timing estimation, synchronization*

1. Introduction

In packet oriented application, the preamble based synchronization methods (data aided method) are often employed [1]-[5], [10]. The most popular data-aided algorithms is the method proposed by Schmidle and Cox [2]. The algorithm transmits identical patterns at the transmitter; based on the correlation between these patterns at the receiver, synchronization parameters are estimated. Schmidle's estimator provides simple and robust estimates for the symbol timing and frequency offset. However, the timing metric of his method has a plateau, which causes a large estimation variance in the timing estimation.

To overcome the uncertainty, Minn proposes special training symbol pattern [1], [3]. The timing metric plateau is eliminated and, hence, timing estimation variance is reduced. However, the performance of the estimation is still unsatisfactory since the side lobe of the timing metric result in additional timing estimation variance.

Further improved methods are proposed by Park [4] and Guangliang Ren's (GR) [5]. Both methods have the impulse shape timing metric, which admits them to achieve a more accurate timing estimation.

In this paper, the behavior of the timing metric that affect to the performance of the estimator is explained in the multipath fading channel.

2. System description

The general case samples of complex-valued baseband OFDM symbol can be described as

$$x(d)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}c_n e^{j(2\pi/N)dn} \qquad -N_g \le d < N-1 . \ (1)$$

where c_n is complex valued information symbol, N is the number of subcarriers, and N_g is the number of the samples of the guard interval.

At the receiver, timing offset is modeled as a delay in the received signal, and the frequency offset is modeled as a phase distortion of the received data. The received signal with two uncertainties is expressed as

$$r(d)=e\,\mathrm{xp}(j\phi)\exp(j2\pi dv/N)y(d-n_\varepsilon)+w_1(d) \ (2)$$

$$y(d)=\sum_{m=0}^{L-1}h_m s(d-\tau_m)+w_1(d) \qquad (3)$$

where ϕ is an arbitrary phase factor, v is carrier frequency offset normalized by subcarrier spacing, n_ε is the unknown arrival time of a symbol, h_m is the channel impulse response with path delay τ_m, L is channel memory, and $w_1(d)$ is zero mean complex Gaussian noise processe.

3. Timing synchronization

3.1 Schmidle and Cox (S&C) method: S&C's preamble has the following form:

$$P_{S\&C}=[C_{N/2}\ C_{N/2}]$$

(4)

It has 2 identical halves of PN sequence in time order. $C_{N/2}$ represents the samples of length $N/2$ and is generated IDFT of PN sequence .

In S&C method the estimated timing is obtained at the instant that maximizes the following timing metric $M(d)$.

$$M_{SC}(d)=\frac{|P_1(d)|^2}{R_1^2(d)} \qquad (5)$$

where $$P_1(d)=\sum_{m=0}^{N/2-1} r^*(d+m)r(d+m+N/2)$$ (6)

$$R_1(d)=\sum_{m=0}^{N/2-1} |r(d+m+N/2)|^2 \quad (7)$$

P_1 is First half and second half correlation and R_1 is second half energy.

As can be seen from () that the difference between $M_{SC}(d)$ and $M_{SC}(d+1)$ is very small. The $P_1(d)$ can be written

$$P_1(d)=r^*(d+1)r(d+1+N/2)+r^*(d+2)r(d+2+N/2) + ...+r^*(d+N/2-1)r(d+N-1) \quad (8)$$

The only difference is the first and term. The timin g metric of Schmidle's method has a plateau due to the cyclic prefix, which causes a large estimation v ariance in the timing estimation.

In the mutipath fading channel, each identical part of the S&C method is equally affected equally due to the cyclic prefix. And there is no degradation in performance in the fast varying channel.

3.2 Guangliang Ren (GR) method [5]:: GR's preamble has the following form.

$$P_{GR}=[\ C_{N/2}\quad C_{N/2}\ ] \quad (9)$$

It is generated from IDFT of CAZAC sequence to make constant envelop preamble. The samples in the preamble satisfy the following condition.

$$x_i = x_{i+N/2}=C\cdot e^{\theta_i},\quad i=0,...,N/2-1 \qquad \|x_k\|=C,\quad k=0,...,N-1 \quad (10)$$

where C is the constant amplitude and θ_i is the phase of each sample of CAZAC sequence.

To make full use of the advantage of constant envelop preamble in the transmission, GR introduces a PN sequence weighting factor into the preamble.

The new preamble is defined as

$$x_k' = s_k x_k,\qquad k=0,1,...,N-1 \quad (11)$$

where s_k is the PN sequence weighting factor of the k-th sample of the preamble and the value of the PN sequence is $+1$ or -1. The timing metric based on the preamble is expressed as

$$M_{GR}(d)=\frac{|P_1(d)|^2}{R_1^2(d)} \quad (12)$$

where

$$P_1(d)=\sum_{k=0}^{N/2-1} s_k s_{k+N/2} r^*(d+k)\cdot r(d+k+N/2) \quad (13)$$

$$R_1(d)=\sum_{m=0}^{N/2-1} |r(d+m+N/2)|^2 \quad (14)$$

At the correct timing, the weighting factor is removed by multiplying the preamble by the corresponding PN sequence in (8). Then, the two identical parts in the preamble is fully correlated in AWGN channel.

However the performance of GR method is degraded in multipath fading channel with high Doppler frequency. The effect of the fast varying channel to GR method can be approximately described using linear model of the received signal.

Substituting (3) into $P_1(d)$, the product term of the received preamble is expressed as

$$\begin{aligned} P_1(d) &= \sum_{k=0}^{N/2-1}\left(s_k' r^*(d+k)\cdot r(d+k+N/2)\right) \\ &= \sum_{k=0}^{N/2-1} s_k'\left(\left[\sum_{m'=0}^{L-1} h_m a_0(k,m)b_0(k,m))\right]^*\left[\sum_{m=0}^{L-1} h_m a_1(k,m)b_1(k,m)\right]\right)+w_2(d) \\ &= \sum_{k=0}^{N/2-1} C^2 s_k'\left(\left[\sum_{m'=0}^{L-1} h_m' a_0(k,m)\right]^*\left[(\sum_{m=0}^{L-1} h_m' a_1(k,m)\right]\right)+w_2(d) \\ &= \sum_{k=0}^{N/2-1}\left(C^2 s_k'\left[\sum_{\substack{m'=0\\ m=m'}}^{L-1} |h_m|^2 s'(d+k-m)\right]\right)+ \\ &\quad \sum_{k=0}^{N/2-1} C^2 s_k'\left[\sum_{m=0}^{L-1} h_m' a_0(k,m))\right]^*\left(\left[\sum_{\substack{m'=0\\ m\neq m'}}^{L-1} h_m' a_1(k,m))\right]\right)+w_2(d) \end{aligned} \quad (15)$$

where

$$w_2(d)=\left[\sum_{m'=0}^{L-1} h(m)a_0(k,m)b_0(k,m)\right]^* w(d+k+N/2)+ \quad (16)$$
$$w^*(d+k)\left[\sum_{m=0}^{L-1} h(m)a_1(k,m)b_1(k,m)\right]+w^*(d+k)w(d+k+N/2),$$

$s_k s_{k+N/2}=s_k'$, $s_{d+k-m}=a_0(k,m)$, $s_{d+k-m+N/2}=a_1(k,m)$, $x_{d+k-m}=b_0(k,m)$, $x_{d+k-m+N/2}=b_1(k,m)$, and $h'(m)=h(m)e^{\theta_{d+k-m}}$.

The first term in (10) can be assumed as cross-correlation between transmitted PN sequence s_k' and received signal of s_k' in the multipath fading channel with channel impulse response of $|h(d)|^2$ without additional white Gaussian noise (AWGN). Then first term can be expressed by using the minimum variance unbiased estimator (MVUE) for the linear model of system identification [11. pp. 90-94]. (10) can be written as

$$P_1(d)=C_1|\hat{h}(d)|^2+ \sum_{k=0}^{N/2-1} C^2 s_k'\left(\left[\sum_{m'=0}^{L-1} h'(m)a_0(k,m)\right]^*\left[\sum_{\substack{m=0\\ m\neq m'}}^{L-1} h'(m)a_1(k,m)\right]\right)+w_2(d) \quad (17)$$

Where $$C_1=C^2\cdot\frac{1}{2N}\sum_{k=0}^{N/2-1}(s'(k))^2 \quad (18)$$

(12) means that $P_1(d)$ estimates the energy of channel impulse response. However, GR method does not contain the numbers of N large enough. Together with interference term, the second term in (12), it keeps $P_1(d)$ from reliable estimation [11. pp. 90-94].

Substituting (12) into (7), timing estimation of GR method implies the first sample of channel path with largest channel tab energy. It performs well under static channel condition, which the first channel tap is dominant and the coherence time is much longer than the OFDM burst duration. However, the scheme becomes sensitive to time-variant nature of the channel (ex. Fast varying channel that the first channel tap is not always dominant with high probability). Simulation results also prove this.

3.3 Park method [5]: The form of the time-domain preamble proposed by Park is as follows:

$$P_{Park} = [C_{N/4} \;\; D_{N/4} \;\; C^*_{N/4} \;\; D^*_{N/4}] \quad (19)$$

$C_{N/4}$ represents samples of length $N/4$ generated by IFFT of a PN sequence. $D_{N/4}$ is designed to be symmetric with $C_{N/4}$. The Park's timing estimator finds the starting point of the symbol at the maximum point of the timing metric given by

$$M_{Park}(d) = \frac{|P_1(d)|^2}{R_1^2(d)}. \quad (20)$$

where
$$P_1(d) = \sum_{k=0}^{N/2-1} r(d-k)\cdot r(d+k) \quad (21)$$

$$R_1(d) = \sum_{k=0}^{N/2-1} |r(d+k)|^2 \quad (22)$$

There are $N/2$ different pairs of product between two adjacent values.

Let's assume that timing metric has its peak value at the correct symbol timing and the values are almost zero at all the other positions in each channel path. Substituting (3) into $P_2(d)$, the product term is approximated as

$$\begin{aligned}
P_2(d) &= \sum_{k=0}^{N/2-1} r(d-k)\cdot r(d+k+1)\\
&= \sum_{k=0}^{N/2-1}\left(\left[\sum_{m=0}^{L-1} h_m x_{d-k-m} + w(d-k)\right]\left[\sum_{m'=0}^{L-1} h_m x_{d+k+1-m'} + w(d+k+1)\right]\right)\\
&= \sum_{k=0}^{N/2-1}\left(\left(\sum_{m=0}^{L-1} h_m x_{d-k-m}\right)\left(\sum_{m'=0}^{L-1} h_{m'} x_{d+k+1-m'}\right) + w_3(d,k)\right)\\
&\approx \sum_{k=0}^{N/2-1}\left(\sum_{m=0}^{L-1} h^2_m x_{d-k-m}x_{d+k+1-m} + \sum_{m'=0}^{L-1}\sum_{\substack{m=0\\(m'\neq m)}}^{L-1} h_m h_{m'} x_{d-k-m} x_{d+k+1-m'} + w_3(d,k)\right)\\
&\approx \sum_{k=0}^{N/2-1}\left(\sum_{m=0}^{L-1} h^2_m x_{d-k-m}x_{d+k+1-m} + w_3(d,k)\right)\\
&= \sum_{m=0}^{L-1} h^2_m \sum_{k=0}^{N/2-1} x_{d-k-m}x_{d+k+1-m} + \sum_{k=0}^{N/2-1} w_3(d,k) \quad (23)
\end{aligned}$$

where

$$w_3(d,k) = w(d-k)\sum_{m'=0}^{L-1} h_m x_{d-k-m'} + w(d+k+1) \sum_{m=0}^{L-1} h_m x_{d-k-m} + w(d-k)*w(d+k+1) \quad (24)$$

The effect of the frequency offset is not considered in the derivation as the output $M_{pro}(d)$ is independent of the frequency offset. Let the first sample index of each channel path in time domain

$$t_i = [t_0\, t_1\, t_2 \ldots t_{L-1}] \quad (25)$$

Let the correlation output at correct timing of each channel path

$$C_2 = \sum_{k=0}^{N/2-1} x_{t_i-k-m} x_{t_i+k+1-m}. \quad (26)$$

Then (20) can be expressed as

$$P_2(d) = \begin{cases} \approx C_2 h_i^2 + w_4(d,k) & if\ d = t_i \\ \approx w_4(d,k) & elsewhere \end{cases} \quad (27)$$

where
$$w_4(d,k) = \sum_{k=0}^{N/2-1} w_3(d,k). \quad (28)$$

As can be seen in (22), the proposed method has many subpeaks that are approximately proportional to the squared channel impulse response. Substituting (22) into (15), the estimated timing that maximizes $M_{Pro}(d)$ is the starting point of the channel path that has the channel tap with largest gain. However, the method itself is not good estimator since the estimated timing is varying according to the time-variant nature of channel response.

3.4 Minn method [5]: The form of the time-domain preamble considered in this paper is as follows:

$$P_{\min} = [-C_{N/4} \;\; +C_{N/4} \;\; -C_{N/4} \;\; -C_{N/4}]$$

(29)

The Park's timing estimator finds the starting point of the symbol at the maximum point of the timing metric given by

$$\Lambda_\varepsilon(d) = \left(\frac{L}{L-1}\frac{|P(d)|}{E(d)}\right)^2 \quad (30)$$

$$P(d) = \sum_{k=0}^{L-2} b(k) \sum_{m=0}^{M-1} r^*(d+kM+m)\cdot r(d+(k+1)M+m) \quad (31)$$

$$E(d) = \sum_{i=0}^{M-1}\sum_{k=0}^{L-1} |r(d+i+kM)|^2 \quad (32)$$

$$b(k) = p(k)p(k+1) \quad (33)$$

where $\{p(k): k = 0,1,\ldots,L-1\}$ The sign of the repeated parts of the training symbol (Training symbol pattern) and L is the number of the identical part.

The method is basically same idea as Park. The transmitted phase factor (PN sequence in Park method and training symbol pattern in Minn method) is recovered in both methods. Then correlation is performed between the identical part in the preamble. It is expected that the same problem as Park method is aroused in the extended manner.

4. Simulation Results

OFDM system with 2048 subcarriers and 256 cyclic prefix is considered. The Rayleigh fading channel has an exponential power delay profile and the ratio of the first

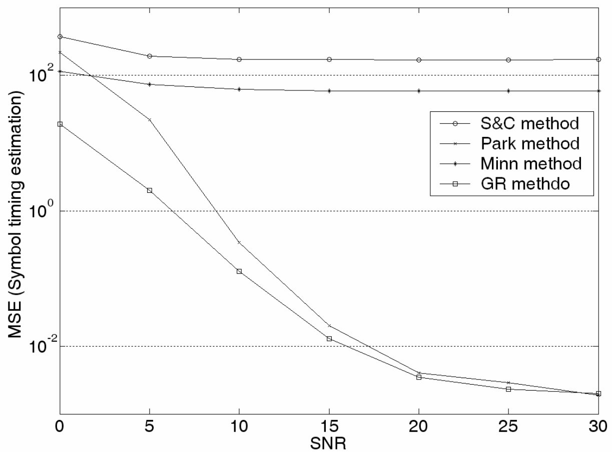

Fig. 5. MSE of the timing offset versus SNR in ISI channel.

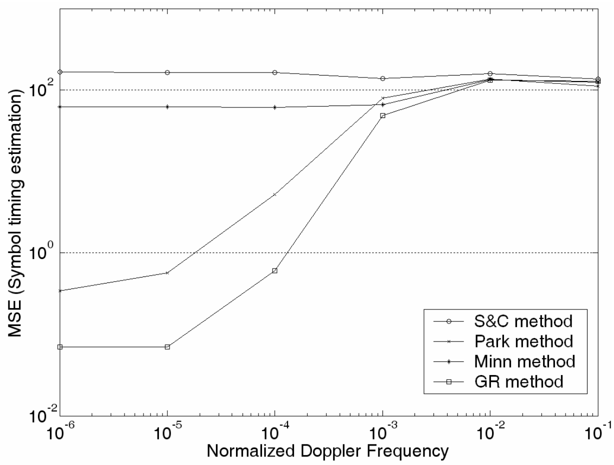

Fig. 6. MSE of the timing offset versus Normalized Doppler frequency under SNR 10 dB in Rayleigh fading channel

tab to the last Rayleigh fading tap is set to 20 dB. All tab have equal tap spacing with 8 samples and 17 tabs are used.

The static ISI channel has fixed tap gains and the tap gain powers are the same as those of the Rayleigh fading channel. The preamble is transmitted followed by one OFDM symbol in the simulation. Unless state otherwise 10000 simulation runs will be applied.

The performance is evaluated by MSE (Samples^2) versus SNR in ISI channel and versus normalized Doppler frequency at SNR 10 dB in Rayleigh fading channel. Fig. 5 and Fig. 6 show MSE of the estimated timing of S&C method (with 90% maximum point averaging), Park method, and GR method.

5. Discussion

Synchronization problem that is identified in conventional GR method is not only its problem. Preamble based synchronization method that contain non-identical part encounter the same problem since each part is affected differently in multipath fading channel. Let $\Theta(d)$ the general output of timing metric. Let $\Theta_m(d)$ each multipath component. It is assumed that each path gain has Rayleigh distribution and they are iid (independently and identically distributed). Then, $\Theta(d)$ is expressed as the summation of $\Theta_m(d),\ m=1,...,L-1$.

$$\Theta(d)=\Theta_1(d)+\Theta_2(d)+....+\Theta_{L-1}(d)=\sum_{m=1}^{L-1}\Theta_m(d) \quad (34)$$

In the view of linear algebra, the peak value of $\Theta(d)$ point the dominant component of $\Theta_m(d),\ \ m=1,...,L-1$ not the first one.

In general, the timing metric output does not necessarily point to the desired point and it varies greatly in fast varying channel.

References

[1] H. Minn, V. K. Bhargava, and K. B. Letaief, “A novel timing estimation method for OFDM systems,” *IEEE Trans. Commun.,* vol. 2, pp. 822 – 839, July 2003.

[2] T. M. Schmidl and D. C. Cox, “Robust frequency and timing synchronization for OFDM,” *IEEE Trans. Commun.*, vol. 45, pp. 1613–1621, Dec. 1997.

[3] H. Minn, M. Zeng, and V. K. Bhargava, "On timing offset estimation for OFDM systems," *IEEE Commun. Lett.*, vol. 4, pp. 242-244, July 2000.

[4] Byungjoon Park, Hyunsoo Cheon, Changeon Kang, and Daesik Hong, “A novel timing estimation method for OFDM systems,” *IEEE Commun. Lett.*, vol. 7, pp. 239 – 241, May 2003.

[5] G. Ren, Y. Chang, and H. Zhang, “Synchronization method based on a new constant envelop preamble for OFDM systems,” *IEEE Trans. Broadcast.* vol. 51, pp. 139-143, Mar. 2005.

[6] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications,* Artech House Publishers, London, U.S.A 2000.

[7] R. L. Frank and S. A. Zadoff, “Phase shift pulse codes with good periodic correlation properties,” *IRE Trans. Inform. Theory*, pp. 381–382, Oct. 1962.

[8] J-J van de Beek, M. sandell, and P. O. Borjesson, “ML estimation of time and frequency offset in OFDM systems,” *IEEE Trans. Signal Processing*, vol. 45 pp. 1800-1805, July 1997.

[9] M. speth, F. Classen, and H. Meyr, “ Frame synchronization of OFDM system over frequency selective fading channels,” in *Proc. Vehicular Technol. Cof.,* Phoenix, AZ, Mayc1997, pp. 1807-1811

[10] P. H. Moose, “A technique for orthogonal frequency division multiplexing frequency offset correction,” IEEE Trans Commun., vol. 42, pp.2908-2914, Oct. 1994.

[11] Steven M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory.* Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1993.

[12] C. Williams, M.A Beach, S. McLaughlin, "Robust OFDM timing synchronization," Electron. Lett., vol. 41, pp.751- 752, June. 2005