Convergence Analysis of CMADR Acceleration for the Method of Characteristics

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1. Introduction

As the nuclear reactor core becomes more complex, heterogeneous, and geometrically irregular, the method of characteristics (MOC)[1] is gaining its wide use in the neutron transport calculations. However, the long computer times require good acceleration methods. In our previous paper, the concept of coarse-mesh angular dependent rebalance (CMADR)[2,3] acceleration was described and applied to the MOC calculations. The method is based on angular dependent rebalance factors defined on the coarse-mesh boundaries; a coarse-mesh consists of several fine meshes that may be (1) heterogeneous and (2) of mixed geometries with irregular or unstructured mesh shapes. In addition, (3) the coarse-mesh boundaries may not coincide with the structural interfaces of the problem and can be chosen artificially for convenience. The CMADR acceleration method on the MOC scheme that enables the very desirable features (1), (2), and (3) above is new in the neutron transport literature to the best of the authors' knowledge. In this paper, we analyze the convergence of CMADR acceleration for MOC calculation in *x-y-z* (infinite) geometry by using Fourier analysis.

2. Coarse-Mesh Angular Dependent Rebalance (CMADR) Method

In MOC calculations in *x-y-z* (infinite) geometry, for fine mesh *i* with flat source approximation, the outgoing angular flux along ray *r* and the average angular flux are given as follows:

$$\psi^{r,l+1/2}_{m,n,i+1/2} = \exp\left(-\frac{\sigma_i L^r_{m,n,i}}{\sin\theta_n}\right)\psi^{r,l+1/2}_{m,n,i-1/2} + \frac{q^l_{m,n,i}}{\sigma_i}\left(1-\exp\left(-\frac{\sigma_i L^r_{m,n,i}}{\sin\theta_n}\right)\right), \quad (1)$$

$$\overline{\psi}^{l+1/2}_{m,n,i} = \frac{q^l_{m,n,i}}{\sigma_i} + \frac{\sin\theta_n}{\sigma_i A_i}\sum_{r\in i}\delta^r_m\left(\psi^{r,l+1/2}_{m,n,i-1/2} - \psi^{r,l+1/2}_{m,n,i+1/2}\right), \quad (2)$$

where *l* is the iteration index, *m* and *n* are azimuthal and polar angle indices respectively, θ_n is *n*-th polar angle, $L^r_{m,n,i}$ is the projected track length of *r*-th ray on mesh *i* in (*m*,*n*) direction, A_i is the area of mesh *i*, and δ^r_m is the ray spacing of ray *r* in *m*-th azimuthal angle.

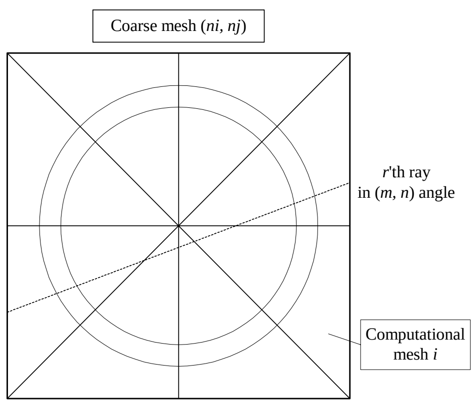

Figure 1. A coarse mesh and computational meshes

The angular dependant rebalance factors are defined on the coarse mesh boundaries *t*:

$$f_{x,m,n,t_x} = \frac{\psi^{r,l+1}_{m,n,t_x}}{\psi^{r,l+1/2}_{m,n,t_x}} \cong f^{\gamma}_{x,t_x}, \quad f_{x,m,n,t_x} = \frac{\psi^{r,l+1}_{m,n,t_x}}{\psi^{r,l+1/2}_{m,n,t_x}} \cong f^{\gamma}_{x,t_x}, \quad (3)$$

where direction (*m*,*n*) is in quadrant γ.

Using the MOC equation and the rebalance factors defined in Eq. (3), the rebalance equations are obtained as follows:

$$f^{\gamma}_{x,t_x} = AX^{l+1/2,\gamma} f^{\gamma}_{x,u_x} + BX^{l+1/2,\gamma} f^{\gamma}_{y,u_y} + CX^{l+1,\gamma}, \quad (4)$$
$$f^{\gamma}_{y,t_y} = AY^{l+1/2,\gamma} f^{\gamma}_{x,u_x} + BY^{l+1/2,\gamma} f^{\gamma}_{y,u_y} + CY^{l+1,\gamma}.$$

In addition to Eq. (4) the update equations are obtained as follows:

$$\overline{\phi}^{l+1,\gamma}_{m,n,i} = DM^{l+1/2,\gamma} f^{\gamma}_{x,u_x} + EM^{l+1/2,\gamma} f^{\gamma}_{y,u_y} + FM^{l+1,\gamma}, \quad (5)$$
$$\overline{\phi}^{l+1}_{m,n,i} = \sum_{\gamma}\overline{\phi}^{l+1,\gamma}_{m,n,i}.$$

The CMADR equations [Eqs. (4) and (5)] resemble DP_0 transport equations and we can solve these equations by transport-like sweep or the Krylov subspace method[4]. Also, the basic coefficients can be calculated and stored before the iteration. Moreover, if modular ray tracing is used in calculation, only several coefficients are stored according to cell types. In this paper we use the CRX code[5,6,7] for MOC, which uses modular ray tracing and BiCGSTAB method to reduce computing time to solve the CMADR equations.

We applied the method to a test problem. The test problem is a homogeneous medium with vacuum boundaries but the source whose density is 1.0 $cm^{-3}sec^{-1}$ is located at the inner square only as shown in Fig. 2. The problem consists of 16x16 coarse meshes and a coarse mesh contains 24 computational meshes. The radii of circles are 0.45*cm* and 0.35*cm*. Scattering ratio is 0.999, angles are (8,4), and the number of rays is 50. Convergence criteria for high- and low-order calculations are 10^{-5}. Table I shows that CMADR is about 36 times faster in the number of iterations and 11 times faster in computing time than the original CRX code.

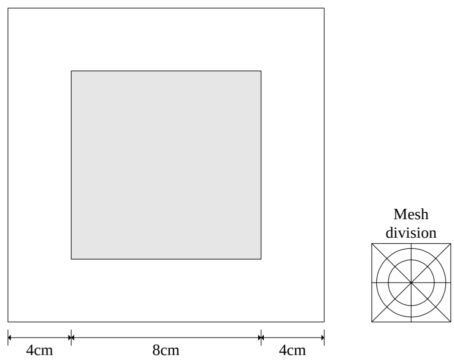

Figure 2. Configuration of test problem

Table I. Results of test problem

	CRX	CRX-CMADR	Speedup
Number of iterations	332	9	36.89
Computing time (sec)	1551.67	134.41	11.54

3. Fourier analysis

The Fourier analysis is the most popular technique that analyzes iterative schemes and it can apply only to linear methods. But Cefus and Larsen[8,9] successfully applied this technique to the analysis of CMR and PDO iterative schemes through linearization.

In slab geometry, it was shown numerically and Fourier analytically [2] that CMADR is unconditionally stable for various transport discretization schemes. Our numerical experience indicates that CMADR on MOC scheme in two-dimensional problems is also unconditionally stable. Thus in this section we investigate the convergence of the CMADR method on MOC in *x-y* geometry theoretically by the Fourier analysis following the Cefus and Larsen's approach.

The test problem is a model problem of the infinite homogeneous medium containing uniform source with square fine meshes. The scattering ratio is 0.999 and the number of rays per direction is 4. Fig. 3 shows the results of Fourier analysis for two cases of coarseness *p* for the MOC calculation with one angle per quadrant (the angle is (45^o,45^o)). Fig. 4 shows the results of Fourier analysis for two angular sets. The spectral radii are always less than unity (and very small) regardless of the mesh size. The maximum spectral radius occurs at the zero limit of the mesh size. We ascertain from the results (and this can be confirmed in the one-dimensional case) that the maximum spectral radius is bounded above by c/3 with regard to the increasing angular orders.

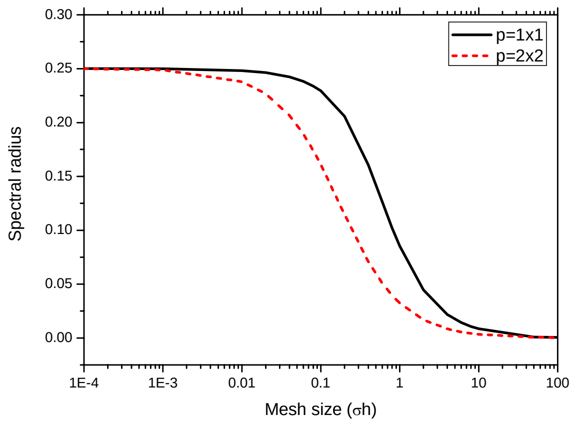

Figure 3. Results of Fourier analysis for two coarseness

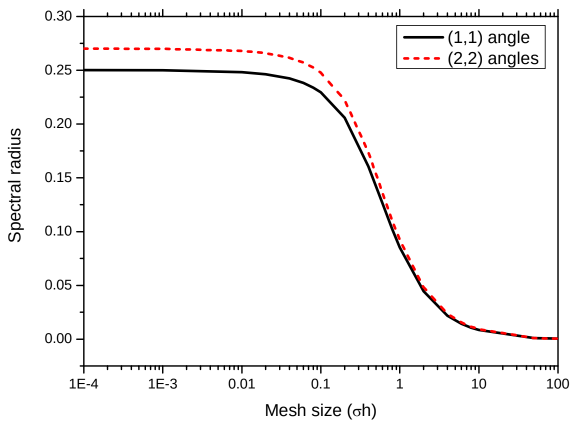

Figure 4. Results of Fourier analysis for two angular sets

4. Conclusions

In this paper, the MOC transport calculation was accelerated by the coarse-mesh angular dependent rebalance (CMADR) method. The CMADR method is based on the ADR factor concept, in which the rebalance factors are angular dependent and defined only on the coarse-mesh boundaries. The coarse mesh can be overlayed on a collection of fine meshes that may be heterogeneous and of mixed geometries with irregular or unstructured mesh shapes. This is possible due to the capability of the MOC. Furthermore, the coarse-mesh boundaries may or may not coincide with the structural interfaces of the problem and can be chosen flexibly for the convenience of analysis. The CMADR method on MOC was tested successfully on a test problem and the results showed that it is very effective in reducing the number of iterations and computing time. We also performed Fourier convergence analysis on a model problem of *x-y-z* (infinite) geometry. The results of the Fourier analysis indicate that CMADR acceleration on MOC is unconditionally stable with small spectral radius.

ACKNOWLEDGMENT

The work was supported in part by the Ministry of Science and Technology of Korea through the National Research Laboratory (NRL) Program.

REFERENCES

[1] J. R. Askew, "A Characteristic Formulation of the Neutron Transport Equation in Complicated Geometries," AEEW-M 1108 (1972).

[2] Y. R. Park and N. Z. Cho, "Coarse-Mesh Angular Dependent Rebalance Acceleration of the Discrete Ordinates Transport Calculations," *Nucl. Sci. Eng.* **148**, 355 (2004).

[3] Y. R. Park and N. Z. Cho, "The MOC Neutron Transport Calculations Accelerated by Coarse-Mesh Angular Dependent Rebalance," *Proc. the Korean Nuclear Society Autumn Meeting*, October 28-29, Yongpyong, Korea, 81, 2004.

[4] C. T. Kelly, *Iterative Methods for Linear and Nonlinear Equations*, SIAM Philadelphia, 1995.

[5] N. Z. Cho and S. G. Hong, "CRX : A Transport Theory Code for Cell and Assembly Calculations Based on Characteristic Method," *Proc. Int. Conf. on the Physics of Reactors*, Sep. 16-20, Mito, Japan, Vol. 1A, 250, 1996.

[6] S. G. Hong and N. Z. Cho, "CRX : A Code for Rectangular and Hexagonal Lattices Based on the Method of Characteristics," *Ann. of Nucl. Energy*, **25**, 547 (1998).

[7] G. S. Lee, N. Z. Cho and S. G. Hong, "Acceleration and Parallelization of the Method of Characteristics for Lattice and Whole-core Heterogeneous Calculation," *ANS Int. Topl. Mtg. on Advances in Reactor Physics and Mathematics and Computation into the Next Millenium (PHYSOR2000)*, II.C-1, Pittsburgh, PA, May 2000.

[8] G. R. Cefus and E. W. Larsen, "Stability Analysis of Coarse-Mesh Rebalance," *Nucl. Sci. Eng.*, **105**, 31 (1990).

[9] G. R. Cefus and E. W. Larsen, "Stability Analysis of the Quasidiffusion and Second Moment Methods for Iteratively Solving Discrete Ordinates Problems," *Transport Theory Stat. Phys*., **18**, 493 (1990).