

Development of an AFEN Method Code in Cylindrical (r,θ,z) Geometry for Design Analysis of Pebble Bed Modular Reactors

Nam Zin Cho, Joo Hee Lee, and Jaejun Lee

Korea Advanced Institute of Science and Technology, Department of Nuclear and Quantum Engineering
373-1 Kusong-dong, Yusong-gu, Daejeon, Korea, nzcho@kaist.ac.kr

Do Sam Kim

Korea Institute of Nuclear Safety, 19 Kusong-dong, Yusong-gu, Daejeon, Korea

1. Introduction

There is growing interest in developing pebble bed modular reactors (PBMRs)[1] as a candidate of very high temperature gas-cooled reactors (VHTRs). A typical pebble bed reactor core houses a multitude of graphite balls which are cycled continuously through the core. Until now, most existing methods of nuclear design analysis for this type of reactors are based on old finite-difference solvers or on statistical methods.

There is strong desire of making available high fidelity nodal codes in cylindrical (r,θ,z) geometry. Recently, Kim and Cho[2,3] extended the analytic function expansion nodal (AFEN) method developed quite extensively in Cartesian (x,y,z) geometry and in hexagonal-z geometry to treat the cylindrical geometry in (r,z) coordinates.

This paper presents further work[4] to extend the AFEN method to the full three-dimensional cylindrical (r,θ,z) geometry. The AFEN methodology in this geometry as in hexagonal geometry is “robust” (e.g., no occurrence of singularity), due to the unique feature of the AFEN method that it does not use the transverse integration[5]. The usual nodal methods based on transverse integration lead, however, to an impasse[6].

2. Basic Theory and Method

The AFEN formulation in the (r,θ,z) coordinates system starts from the following two-group diffusion equations in a homogenized node (see Fig. 1) :

$$-\nabla^2 \vec{\phi}(r, \theta, z) + [A] \vec{\phi}(r, \theta, z) = 0, \quad (1)$$

where

$$[A] = [D]^{-1} \left([\Sigma] - \frac{1}{k_{eff}} [\chi][\nu\Sigma_f] \right).$$

All the notations are standard. The equations can be decoupled as follows:

$$\frac{\partial^2 \xi_\mu}{\partial r^2} + \frac{1}{r} \frac{\partial \xi_\mu}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \xi_\mu}{\partial \theta^2} + \frac{\partial^2 \xi_\mu}{\partial z^2} - \lambda_\mu \xi_\mu = 0, \quad (2)$$

where

$$\vec{\xi} = (\xi_1, \xi_2)^T = [R]^{-1} \vec{\phi}, \quad [R] = [e_1, e_2],$$

$\lambda_\mu, e_\mu (\mu = 1, 2)$: eigenvalues and corresponding eigenvectors of [A]

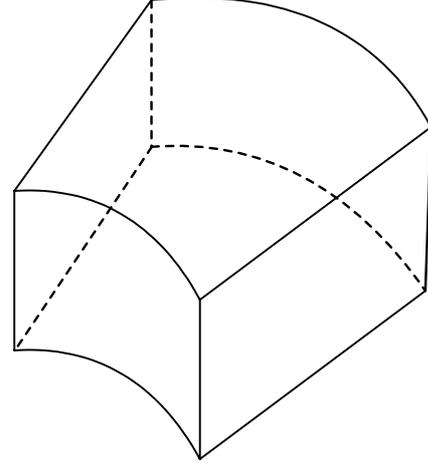


Fig. 1 A node in (r,θ,z) coordinates system

A general solution to Eq.(2) can be represented in terms of analytic basis functions that can be obtained using the method of separation of variables. For practical implementation, we choose the modal solution ξ_μ of a node expressed in a finite number of terms:

$$\begin{aligned} \xi_\mu(r, \theta, z) = & a_{\mu 0} + a_{\mu 1} S(\kappa_\mu z) + a_{\mu 2} C(\kappa_\mu z) \\ & + a_{\mu 3} F(\kappa_\mu r; 0) + a_{\mu 4} G(\kappa_\mu r; 0) \\ & + [a_{\mu 5} \sin(\theta) + a_{\mu 6} \cos(\theta)] \\ & \times [a_{\mu 7} F(\kappa_\mu r; 1) + a_{\mu 8} G(\kappa_\mu r; 1)] \\ & + a_{\mu 9} z F(\kappa_\mu r; 0) + a_{\mu 10} z G(\kappa_\mu r; 0) \\ & + [a_{\mu 11} S(\kappa_\mu z) + a_{\mu 12} C(\kappa_\mu z)] \ln(\kappa_\mu r), \quad \mu = 1, 2, \end{aligned} \quad (3)$$

where $\kappa_\mu = \sqrt{|\lambda_\mu|}$, and

	$\lambda_\mu < 0$	$\lambda_\mu > 0$
$S(x)$	$\sin(x)$	$\sinh(x)$
$C(x)$	$\cos(x)$	$\cosh(x)$
$F(x; m)$	$J_m(x)$	$I_m(x)$
$G(x; m)$	$Y_m(x)$	$K_m(x)$

Note that each term in Eq.(3) is an analytic solution of Eq.(2). The thirteen coefficients in Eq.(3) are made to correspond to the thirteen nodal unknowns for a node : i) one node average flux, ii) six surface average fluxes, and iii) six interface flux moments. Alternatively, the thirteen nodal unknowns can be chosen as: i) one

node average flux, and ii) twelve half-interface average fluxes (two half-interface average fluxes for each of the six surfaces).

In an innermost node, the inner radial surface degenerates (disappears) into the z-axis and thus less nodal unknowns are necessary. In addition, usually the innermost nodes are smaller in size. Moreover, some terms in Eq.(3) become singular at $r=0$. Therefore, for such a node the six (i.e., $a_{\mu 4}$, $a_{\mu 8} \times 2$, $a_{\mu 10}$, $a_{\mu 11}$, and $a_{\mu 12}$) terms in Eq.(3) are excluded. The remaining seven coefficients are made to correspond to : i) one node average flux, ii) two half-interface average fluxes on the outer radial surface, and iii) four surface average fluxes on the other surfaces.

3. Implementation in the TOPS Code

After the coefficients in Eq.(3) are expressed in terms of the nodal unknowns, we build as many solvable nodal coupling equations as the number of these nodal unknowns to be determined. The nodal coupling equations in AFEN typically consist of the nodal balance equation, the interface current continuity equation, and the interface current moment continuity equation (or two half-interface current continuity equations).

Development of a computer code called TOPS is in progress following the method described in Section 2. The details and numerical results will be presented at the meeting.

ACKNOWLEDGMENT

This work was supported in part by the Ministry of Science and Technology of Korea through the Nuclear Hydrogen Production Development and Demonstration (NHPDD) Program coordinated by Korea Atomic Energy Research Institute.

REFERENCES

- [1] A. C. Kadak, D. A. Petti, *et al.*, *Modular Pebble Bed Reactor Project University Research Consortium Annual Report*, INEEL/EXT-2000-01034, Idaho National Engineering & Environmental Laboratory, (2000).
- [2] D. S. Kim, N. Z. Cho, "The AFEN Nodal Method in R-Z Coordinates for Analysis of PBMR Cores with Truncated Cone Shape," *Trans. Am. Nucl. Soc.*, **87**, 415 (2002).
- [3] D. S. Kim, N. Z. Cho, "The Analytic Function Expansion Nodal (AFEN) Method with Half-Interface Averaged Fluxes in Mixed Geometry Nodes for Analysis of Pebble Bed Modular Reactor (PBMR) Cores," *Journal of Nuclear Science and Technology*, **40**, 291 (2003).
- [4] N. Z. Cho, "Development of an AFEN Method Computer Code for Design of Pebble Bed Type Reactors," presented at the 2nd Year Program Kick-Off Meeting, March 31-April 1, 2005.

[5] N. Z. Cho, "Fundamentals and Recent Developments of Reactor Physics Methods," Invited Critical Review Paper in *Nuclear Engineering and Technology*, **37**, 25 (2005).

[6] A. M. Ougouag and W. K. Terry, "Analytic Solution of the Neutron Diffusion Equation in Three-Dimensional Cylindrical Geometry for Application in a Nodal Method," *Proceedings of the Nuclear Mathematical and Computational Sciences: A Century in Review - A Century Anew (M&C 2003)*, Gatlinburg, TN, April 6-10, 2003.