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SPECTRAL ELEMENT DYNAMIC ANALYSIS OF THE PIPELINE CONVEYING INTERNAL UNSTEADY FLOW

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ABSTRACT

In this paper, a spectral element model is developed for the uniform straight pipelines conveying internal unsteady fluid. The spectral element matrix is formulated by using the exact frequency-domain solutions of the pipe-dynamics equations. The spectral element dynamic analyses are then conducted to evaluate the accuracy of the present spectral element model and to investigate the vibration characteristics and internal fluid transients of an example pipeline system.

I. INTRODUCTION

There have been extensive studies on the modeling and analysis of the flow-induced vibrations of pipeline systems over the past half-century: an extensive review on this subject can be found in Païdoussis and Li [1]. In most existing pipe-dynamics theories, the structural vibration of pipeline itself has been the main concern, neglecting the transient dynamics of internal fluid which should be coupled with the structural vibration of pipeline. To account for the effects of the coupling between the pipeline vibration and unsteady internal flow, Lee *et al.* [2] derived a set of coupled pipe-dynamic equations for the axial, radial, and transverse vibrations of pipeline as well as for the transients of unsteady internal fluid pressure and velocity. The coupled pipe-dynamic equations were further generalized by including the circumferential strain effect caused by the internal fluid pressure [3] and, later on, by including the radial shell vibration and initial axial tension [4].

In the literature, the FFT (fast Fourier transforms)-based dynamic stiffness method is often named *spectral element method* (SEM) [6-7]. Because the *exact* dynamic stiffness matrix is formulated from the *exact* dynamic shape functions which satisfy the governing equations of motion, it represents the dynamic behavior of a structural element *exactly*. Thus, the SEM is often justifiably referred to as an *exact* solution method [5-7].

The purposes of the present paper are (1) to develop a spectral element model for the axial and transverse vibrations of a pipeline conveying internal unsteady fluid, and (2) to conduct spectral element analysis to investigate the structural dynamic characteristics and the internal fluid transients of an example pipeline problem.

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II. PIPE-DYNAMICS EQUATIONS

We consider a straight pipeline subject to a small amplitude vibration. The equations of motion for a pipeline and the relevant boundary conditions can be derived from the Hamilton's principle. The kinetic energy T and the potential energy U for the pipeline system are given by

$$T = \frac{m_p}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dx$$
$$U = \frac{EA_p}{2}\int_0^L \left\{\frac{T_o}{EA_p} + \left(u' + \frac{1}{2}u'^2 + \frac{1}{2}w'^2\right)\right\}^2 dx \quad (1)$$
$$+ \frac{EI_p}{2}\int_0^L w''^2 dx$$

δW is the virtual work done by the flow-induced forces acting on the pipe wall and by the resultant forces and moments applied at the boundaries.

Introducing Eqs. (1) and δW into Hamilton's principle and integrating by parts gives

$$EI_p w'''' + \left(pA - T_o + m_w c^2\right)w'' + p'Aw'$$
$$+ m_w\left(2c\dot{w}' + \dot{c}w' + cc'w'\right) + m\ddot{w} = 0 \quad (2)$$
$$\left(EA_p + T_o\right)u'' - m_p\ddot{u} + m_w g_Y w' + m_w \frac{f}{2D}c^2 = 0$$

By applying the Newton's law of motion to the control volume (fluid element), the equations of fluid can be obtained as

$$c'' - \frac{1}{a^2}\dot{c}c' - \frac{1}{a^2}c\dot{c}' - \frac{1}{a^2}\ddot{c} - \frac{f}{a^2 D}c\dot{c}$$
$$- \frac{g_Y}{a^2}\dot{w}' - 2\nu\dot{u}'' - \frac{1}{a^2}\dot{c}\dot{u}' - \frac{1}{a^2}c\ddot{u}' = 0$$

$$\dot{p}A + a^2 m_w (c' - 2\nu\dot{u}') = 0 \quad (3)$$

Equations (2) and (3) represent a set of coupled nonlinear pipe-dynamics equations for the pipeline conveying internal unsteady fluid. To linearlize the pipe-dynamics equations, the fluid velocity and pressure are assumed as

$$c(x,t) = c_o + c_d(x,t), \quad p(x,t) = p_o + p_d(x,t) \quad (4)$$

where $c_d(x, t)$ and $p_d(x, t)$ represent the small perturbations with respect to constant steady-state values c_o and p_o, respectively. Accordingly one may assume that $c_d < c_o$ and $p_d < p_o$. Substituting Eq. (4) into Eq. (2) and Eq. (3) and neglecting small nonlinear terms a set of linearized pipe-dynamics equations as follows:

$$\begin{aligned}
&EI_p w'''' + (p_o A - T_o + m_w c_o^2) w'' + 2m_w c_o \dot{w}' + m\ddot{w} = 0 \\
&(EA_p + T_o) u'' - m_p \ddot{u} + m_w g_Y w' + m_w \frac{f}{D} c_o c_d + m_w \frac{f}{2D} c_o^2 = 0 \\
&c_d'' - \frac{c_o}{a^2}\dot{c}_d' - \frac{f}{a^2 D} c_o \dot{c}_d - \frac{\ddot{c}}{a^2} - \frac{g_Y}{a^2}\dot{w}' - 2\nu\dot{u}'' - \frac{c_o}{a^2}\ddot{u}' = 0 \\
&\dot{p}_d A + a^2 m_w (c_d' - 2\nu\dot{u}') = 0
\end{aligned} \quad (5)$$

III. FORMULATION OF SPECTRAL ELEMENT MODEL

The general solutions of Eq. (5) can be assumed in the spectral forms as

$$\begin{aligned}
w(x,t) &= \sum_{n=1}^{N} W_n(x) e^{i\omega_n t}, \quad u(x,t) = \sum_{n=1}^{N} U_n(x) e^{i\omega_n t} \\
c_d(x,t) &= \sum_{n=1}^{N} C_n(x) e^{i\omega_n t}, \quad p_d(x,t) = \sum_{n=1}^{N} P_n(x) e^{i\omega_n t}
\end{aligned} \quad (6)$$

where $W_n(x)$, $U_n(x)$, $C_n(x)$ and $P_n(x)$ are the spatially dependent spectral components of $w(x,t)$, $u(x,t)$, $c_d(x,t)$ and $p_d(x,t)$, respectively. Substituting Eq. (6) into Eq. (5) yields

$$\begin{aligned}
&a_1 W'''' + a_2 W'' + a_3 i\omega W' - a_4 \omega^2 W = 0 \\
&b_1 U'' - b_2 \omega^2 U + b_3 W' + b_4 C + b_5 = 0 \\
&C'' + c_1 i\omega C' + (c_2 i\omega - c_3 \omega^2) C \\
&\quad - c_4 i\omega W' + c_5 i\omega U'' - c_1 \omega^2 U' = 0 \\
&i\omega P + d_1 (C' + c_5 i\omega U') = 0
\end{aligned} \quad (7)$$

where the following definitions are used:

$$\begin{aligned}
&a_1 = EI_p, a_2 = p_o A - T_o + m_w c_o^2, a_3 = 2m_w c_o, a_4 = m \\
&b_1 = EA_p + T_o, b_2 = -m_p, b_3 = m_w g_Y, b_4 = m_w \frac{f}{D} c_o \\
&b_5 = m_w \frac{f}{2D} c_o^2, c_1 = -\frac{c_o}{a^2}, c_2 = -\frac{f}{a^2 D} c_o, c_3 = -\frac{1}{a^2} \\
&c_4 = -\frac{g_Y}{a^2}, c_5 = -2\nu, d_1 = \rho_w a^2
\end{aligned} \quad (8)$$

The general solutions with removing nonhomogeneus terns, are assumed as

$$W(x) = \overline{W} e^{ikx}, U(x) = \overline{U} e^{ikx}, C(x) = \overline{C} e^{ikx}, P(x) = \overline{P} e^{ikx} \quad (9)$$

where k is the wavenumber. Substituting Eq. (9) into Eq. (7) gives an eigenvalue problem from which one can obtain two dispersion equations. The first dispersion equation provides the wavenumbers (k_1, k_2, k_3, k_4) for the beam bending vibration modes, whereas the second dispersion equation provides the wavenumbers (k_5, k_6, k_7, k_8) for the axial vibration-fluid velocity coupling modes. By using the eight wavenumbers, the general solutions can be expressed as following forms:

$$\begin{aligned}
W(x) &= \sum_{j=1}^{4} \overline{W}_j e^{ik_j x} = [\mathbf{e}_w(x)]\{\phi_w\} \\
U(x) &= \sum_{j=1}^{4} \overline{U}_j e^{ik_{j+4} x} = [\mathbf{e}_{uc}(x)]\{\phi_{uc}\} \\
C(x) &= \sum_{j=1}^{4} \alpha_j \overline{U}_j e^{ik_{j+4} x} = [\mathbf{e}_{uc}(x)][\mathbf{D}_{uc}]\{\phi_{uc}\}
\end{aligned} \quad (10)$$

and $\{\varphi_w\}$ and $\{\varphi_{uc}\}$ are constant vectors to be eliminated later on. Now, consider a finite pipeline element of length l. The spectral nodal degrees of freedom (simply, spectral nodal DOFs) are defined by

$$\begin{aligned}
&W(0) = W_1, \ \Theta(0) = \Theta_1, \ U(0) = U_1, \ C(0) = C_1, \ P(0) = P_1 \\
&W(l) = W_2, \ \Theta(l) = \Theta_2, \ U(l) = U_2, \ C(l) = C_2, \ P(l) = P_2
\end{aligned} \quad (11)$$

where $\Theta(x) = W'(x)$ denotes the slope. Substituting Eq. (10) into Eq. (11) gives the relationships between the spectral nodal DOFs vectors and the constants vectors as follows:

$$\{\mathbf{d}_w\} = [\mathbf{H}_w(\omega)]\{\phi_w\}, \ \{\mathbf{d}_{uc}\} = [\mathbf{H}_{uc}(\omega)]\{\phi_{uc}\} \quad (12)$$

where

$$\begin{aligned}
&\{\mathbf{d}_w\} = \{W_1 \ \theta_1 \ W_2 \ \theta_2\}^T, \ \{\mathbf{d}_{uc}\} = \{U_1 \ U_2 \ C_1 \ C_2\}^T \\
&[\mathbf{H}_w(\omega)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ ik_1 & ik_2 & ik_3 & ik_4 \\ e_1 & e_2 & e_3 & e_4 \\ ik_1 e_1 & ik_2 e_2 & ik_3 e_3 & ik_4 e_4 \end{bmatrix} \\
&[\mathbf{H}_{uc}(\omega)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ e_5 & e_6 & e_7 & e_8 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_1 e_5 & \alpha_2 e_6 & \alpha_3 e_7 & \alpha_4 e_8 \end{bmatrix}
\end{aligned} \quad (13)$$

with $\quad e_j = e^{ik_j l} \quad (j = 1, 2, 3, \ldots, 8)$ (14)

From Eqs. (10) and (12), one may obtain

$$\begin{aligned} W(x) &= [N_w(x;\omega)]\{d_w\} \\ U(x) &= [N_u(x;\omega)]\{d_{uc}\} \\ C(x;) &= [N_c(x;\omega)]\{d_{uc}\} \end{aligned} \tag{15}$$

where

$$\begin{aligned} [N_w(x;\omega)] &= [e_w(x)][H_w(\omega)]^{-1} \\ [N_u(x;\omega)] &= [e_{uc}(x)][H_{uc}(\omega)]^{-1} \\ [N_c(x;\omega)] &= [e_{uc}(x)][D_{uc}(\omega)][H_{uc}(\omega)]^{-1} \end{aligned} \tag{16}$$

The variational approach can be used to formulate the spectral element matrix by using the displacements and fluid fields given by Eq. (15). Substituting Eq. (16) into the weak form of Eq. (12) and taking some manipulation gives an equation in the form as

$$[S(\omega)]\{d\} = \{f\} \tag{17}$$

where $\{d\}$ and $\{f\}$ are the spectral nodal DOFs vector and the spectral nodal forces vector, respectively, and $[S(\omega)]$ is the spectral element matrix defined in the form of

$$[S(\omega)] = \begin{bmatrix} s_{11} & 0 \\ s_{21} & s_{22} \end{bmatrix} \tag{18}$$

Assembling the spectral element equation (17) and then applying the appropriate boundary conditions will provide a global system dynamic equation.

IV. NUMERICAL RESULTS AND DISCUSSIONS

A uniform straight pipeline simply supported at both ends is considered as an illustrative example problem. The pipeline has the length L = 6000*mm*, the internal diameter D = 32.12 *mm*, the thickness t = 1.4 *mm*, the Young's modulus E = 117*GPa*, and the mass density of pipeline m_p = 0.0515 *kg/m*. The mass density of fluid is m_w = 1.318 *kg/m*. It is assumed that the pipeline is subject to the constant axial tension T_0 = 82 *N*.

The high accuracy of the present spectral element model is verified first by comparing the eigenfrequencies obtained by using the present SEM with those obtained by the conventional FEM and also with the exact analytical results from Blevins [8].

Table 1 compares the eigenfrequencies of the lowest four transverse displacement (bending) modes, the first axial displacement mode, and the first fluid mode obtained by the present SEM, the FEM, and the exact theoretical result from Blevins [8]. The FEM results are obtained by increasing the number of finite elements from 10 to 100, as shown in Table 1. When the (steady-state) flow velocity is c_0 = 0 *m/s*, the eigenfrequencies obtained by the present SEM are found to be identical to the exact ones by Blevins [8]. It can be also observed from Table 1 that the FEM results converge to the SEM results at all flow velocities as the number of finite elements is increased. For the present example problem, more than fifty finite elements should be used in FEM to achieve the same high accuracy of the fifth eigenfrequency by SEM.

Table 1. Eigenfrequencies(Hz) of the pipeline obtained by the present SEM, FEM and the exact theory [8]

Fluid Velocity (*m/s*)	Method	N	$\omega_1^{(w)}$	$\omega_2^{(w)}$	$\omega_3^{(w)}$	$\omega_4^{(w)}$	$\omega_6^{(c)}$	$\omega_{12}^{(u)}$
0	Exact[25]	.	1.47	5.89	13.26	23.57	51.98	150.73
	SEM	1	1.47	5.89	13.26	23.57	51.98	150.73
	FEM	10	1.47	5.89	13.27	23.61	52.03	157.39
		20	1.47	5.89	13.26	23.58	51.99	150.77
		50	1.47	5.89	13.26	23.58	51.98	150.74
		100	1.47	5.89	13.26	23.57	51.98	150.74
10	SEM	1	1.37	5.81	13.18	23.50	52.00+0.47*i*	150.74+0.02*i*
	FEM	10	1.37	5.81	13.19	23.54	52.03+0.47*i*	150.89+0.02*i*
		20	1.37	5.81	13.18	23.50	51.99+0.47*i*	150.77+0.02*i*
		50	1.37	5.81	13.18	23.50	51.98+0.47*i*	150.74+0.02*i*
		100	1.37	5.81	13.18	23.50	51.98+0.47*i*	150.74+0.02*i*
28.65	SEM	1	0.00	5.18	12.59	22.93	52.00+1.18*i*	150.74+0.04*i*
	FEM	10	0.00	5.18	12.60	22.97	52.02+1.18*i*	150.89+0.04*i*
		20	0.00	5.18	12.59	22.93	51.98+1.18*i*	150.77+0.04*i*
		50	0.00	5.18	12.59	22.93	51.97+1.18*i*	150.74+0.04*i*
		100	0.00	5.18	12.59	22.93	51.97+1.18*i*	150.74+0.04*i*

Note : N = number of finite elements; (w) = transverse displacement mode; (u) = axial displacement mode; (c) = fluid mode

It can be also observed from Table 1 that the real parts of eigenfrequencies (*i.e.*, natural frequencies) are reduced in magnitude as the fluid velocity is increased. The first natural frequency becomes zero at c_0 = 28.65 *m/s* at which the divergence instability occurs as discussed in the following.

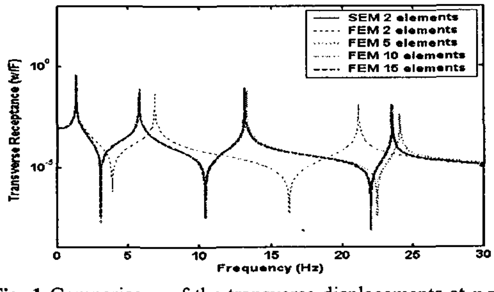

Fig. 1 Comparisons of the transverse displacements at $x = L/5$ in the frequency domain obtained by the present SEM and FEM

Figure 1 compares the dynamic responses of the transverse displacement in the frequency and time domains obtained by the present SEM and the FEM. It is assumed that the fluid velocity is c_0 = 10*m/s*. To excite the pipeline, a point load $f(t)$ of magnitude 1 *kN* is applied for 0.001 seconds at $x = L/5$. The dynamic responses are then measured at the excitation point, *i.e.*, $x = L/5$. It is certain from Fig. 1 that the dynamic responses obtained by the FEM converge to the SEM results as the number of finite elements used in the FEM is increased. Thus, both the results shown in both Table 1 and Fig. 1 prove the high accuracy of the present spectral element model.

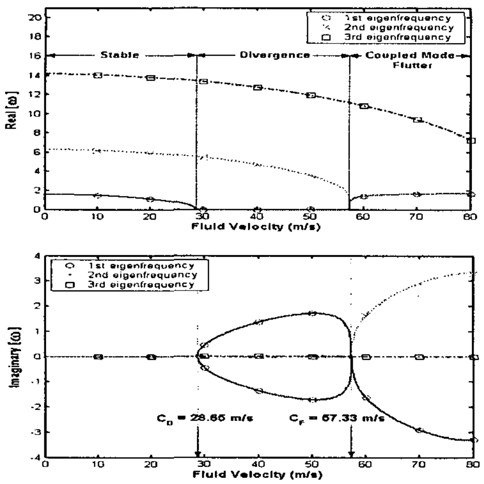

Fig. 2 Fluid velocity dependence of the lowest three eigenfrequencies of pipeline

Figure 2 shows the fluid velocity dependence of the real and imaginary parts of the lowest three eigenfrequencies. The divergence instability may occur when the imaginary part of an eigenfrequency is negative and the real part is zero, whereas the flutter instability may occur when the imaginary part is negative, but the real part is not zero. For the present example problem, Fig. 2 shows that the divergence instability occurs in the first bending mode at c_0 = 28.65*m/s* (*i.e.*, divergence velocity, c_D), and the flutter instability occurs in the coupled mode of the first and second bending modes at c_0 = 57.33*m/s* (*i.e.*, flutter velocity, c_F).

When the fluid velocity at the inlet is controlled by c_0 = $10(1 + 0.001\sin 104\pi t)$ *m/s*, the axial displacement, perturbed fluid velocity are shown in Fig. 3. Figure 3 shows that all responses tend to increase with time. This is because the excitation frequency of the inlet fluid velocity 52π almost coincides with the first natural frequency of the fluid when c_0 = 10 *m/s* (see Table 1) so that a resonance phenomenon occurs.

V. CONCLUSIONS

In this study, a spectral element model is developed for the straight pipelines conveying internal unsteady fluid. The high accuracy of the spectral element model is then proved by comparing the eigenfrequencies obtained by the present SEM and the conventional FEM. Finally, the spectral element analysis is conducted to investigate the stability and forced vibration responses of an example pipeline conveying internal unsteady fluid.

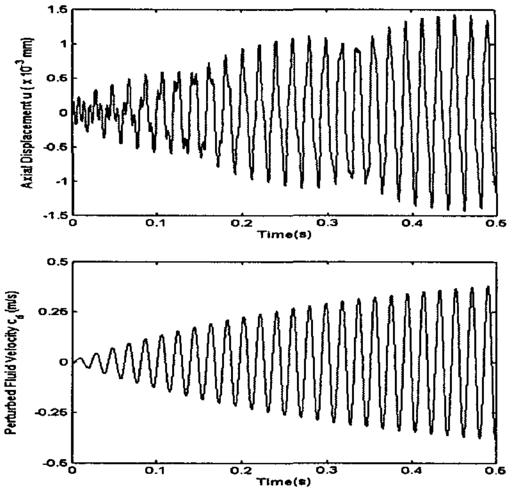

Fig. 3 The axial displacement and perturbed fluid velocity at $x = L/2$ when $c_0 = 10(1+0.001\sin 102\pi t)$ *m/s*

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