한국소음진동공학회 2005년도 추계학술대회논문집, pp.844~847.

변위응답의 측정으로부터 변형률응답을 예측하는 방법의 특성

Characteristics of the Method to Predict Strain Responses from the Measurements of Displacement Responses

이건명† ·고재흥*

Gun-Myung Lee and Jae-Heung Ko

Key Words : Strain Response Prediction(변형률응답 예측), Displacement(변위), Transformation Matrix(변환 행렬.

ABSTRACT

A method to predict the strain responses from the measurements of displacement responses is considered. The method uses a transformation matrix which is composed of a displacement modal matrix and a strain modal matrix. The method can predict strains at points where displacements are not measured as well as at displacement measuring points. One of the drawbacks of the strain prediction method is that the displacement responses must be measured at many points on a structure simultaneously. This difficulty can be overcome by measuring the FRFs between displacements at a reference point and other point in sequence with a two channel measuring equipment. This procedure is based on the assumption that the characteristics of excitation applied to the structure do not vary with time.

1. Introduction

Modern mechanical structures should meet the design requirements of compactness and light weight. Also those structures should be proven to have sufficient dynamic and fatigue strengths. To assure dynamic and fatigue strengths it is necessary to measure the dynamic strain distribution on structures. However, strain measurements with conventional strain gauges are not always possible and they are also expensive since the gauges are not reusable and cannot be moved from point to point when they have been attached to structures. Some methods have been developed to predict the dynamic strain distribution from displacement measurements on structures. Okubo and Yamaguchi [1] predicted the dynamic strain distribution under operating condition using the transformation matrix from displacements to strains. In the method proposed by Sehlstedt [2], results from hybrid modal analysis are transformed from the displacement space to the strain space by use of finite difference schemes. Inversely, techniques to determine displacements at points of a vibrating body from measured strains have been developed [3,4]. These techniques can be applied for active control of smart composite structures with embedded fiber optic strain sensors and for end point control of flexible manipulators.

In this paper, the method to predict the strain distribution using the transformation matrix is considered. Characteristics and limitations of the method are investigated and a procedure to overcome the limitations is proposed.

† 경상대학교 항공기부품기술연구소, 기계항공공학부
E-mail : gmlee@gsnu.ac.kr
Tel : (055) 751-5313, Fax : (055) 757-5622

* 경상대학교 기계항공공학부

2. Theory

The displacements at points on a vibrating structure, $\{u(t)\}$, can be expressed as a linear combination of vibrational modes as follows:

$$\{u(t)\} = [\Phi]\{q(t)\} \tag{1}$$

where $[\Phi]$ is the modal matrix whose columns represent the mode shapes of vibrtional modes and $\{q(t)\}$ modal coordinates. The strains at points on a structure, $\{\varepsilon(t)\}$, can be expressed by the spatial differentiation of the displacement distribution as follows:

$$\begin{aligned}\{\varepsilon(t)\} &= D([\Phi])\{q(t)\} \\ &= [\Psi]\{q(t)\}\end{aligned} \tag{2}$$

where D represents a linear spatial differential operator and $[\Psi]$ is the strain modal matrix whose columns represent the strain mode shapes. From Eq. (1), $\{q(t)\}$ is obtained as

$$\{q(t)\} = [\Phi]^{-1}\{u(t)\} \tag{3}$$

Substituting the above equation into Eq. (2),

$$\{\varepsilon(t)\} = [\Psi][\Phi]^{-1}\{u(t)\} \tag{4}$$

Using the notation,

$$[T]=[\Psi][\Phi]^{-1} \tag{5}$$

we obtain the following equation

$$\{\varepsilon(t)\}=[T]\{u(t)\} \tag{6}$$

In the above equation, $[T]$ is the transformation matrix which converts displacements to strains. The equation means that the strain distribution can be obtained from the displacement measurements if the transformation matrix is known. Eq. (6) can be written in detail as follows:

$$\begin{Bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \vdots \\ \varepsilon_n(t) \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1n} \\ T_{21} & T_{22} & \cdots & T_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ T_{n1} & T_{n2} & \cdots & T_{nn} \end{bmatrix} \begin{Bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_n(t) \end{Bmatrix} \tag{7}$$

When the number of measuring points on a structure is n, and the number of the considered modes is m, the size of matrix $[\Phi]$ becomes n x m. If n $\neq$ m, the pseudo-inverse matrix, $[\Phi]^-$, can be used instead of $[\Phi]^{-1}$ in Eq. (5).

$$[\Phi]^- = ([\Phi]^T[\Phi])^{-1}[\Phi]^T \tag{8}$$

As Eq. (5) shows, the transformation matrix is composed of the displacement modal matrix and the strain modal matrix of a structure. The modal matrices can be obtained analytically, or numerically by using a finite element method, or experimentally. To obtain the modal matrices experimentally, displacement frequency response functions(DFRFs) between applied forces and displacement responses and strain frequency response functions(SFRFs) between applied forces and strain responses should be measured through modal testing. Then the displacement modal matrix is obtained by the modal analysis of the.DFRFs, and the strain modal matrix by the modal analysis of the SFRFs. A DFRF is expressed for viscous damping as follows:

$$H_{jk}(\omega)=\sum_r \frac{{}_r\phi_j\,{}_r\phi_k}{\omega_r^2-\omega^2+i2\zeta_r\omega_r\omega} \tag{9}$$

Similarly, a SFRF is expressed as follows [5]:

$$S_{jk}(\omega)=\sum_r \frac{{}_r\psi_j\,{}_r\phi_k}{\omega_r^2-\omega^2+i2\zeta_r\omega_r\omega} \tag{10}$$

where ${}_r\psi_j$ and ${}_r\phi_k$ represent the mode shape component of strain mode r at point j and the mode shape component of displacement mode r at point k, respectively. Since SFRFs take similar forms as DFRFs, modal analysis routines which extract mode shape components from DFRFs can be also used to extract strain mode shape components from SFRFs.

3. Application

The strain prediction method using the transformation matrix was applied to a cantilever beam. The size of the beam was 300mm x 30mm x 2mm and the density was 7857 kg/m^3 . As Fig. 1 shows, there are 5 measuring points with equal spacing on the beam.

To obtain the transformation matrix, the displacement modal matrix and the strain modal matrix of the beam were obtained analytically. The DFRF of the beam can be obtained from the solution of the equation of motion when subjected to harmonic excitation and is given as follows [6]:

$$H_{jk}(\omega)=\frac{1}{\rho AL}\sum_r \frac{Y_r(x_j)Y_r(x_k)}{\omega_r^2-\omega^2+i2\zeta_r\omega_r\omega} \tag{11}$$

where ρ, A and L represent the density, the cross-sectional area and the length of the beam, respectively, and $Y_r(x)$ is given as follows:

$$Y_r(x)=\cos\beta_r x-\cosh\beta_r x + \frac{\sin\beta_r L-\sinh\beta_r L}{\cos\beta_r L+\cosh\beta_r L}(\sin\beta_r x-\sinh\beta_r x) \tag{12}$$

where β_r is related to the eigenvalue of mode r. Comparing Eq. (11) with the general expression of a DFRF, Eq. (9), it is found that

$${}_r\phi_j=\frac{1}{\sqrt{\rho AL}}Y_r(x_j) \tag{13}$$

Since the strain on the surface of a thin beam with thickness $2h$ is obtained by the relation,

$$\varepsilon(x)=h\frac{\partial^2 u(x)}{\partial x^2} \tag{14}$$

the strain mode shape component is expressed as follows:

$${}_r\psi_j=\frac{h}{\sqrt{\rho AL}}\ddot{Y}_r(x_j) \tag{15}$$

From Eq. (5) with mode shape components expressed in Eqs. (13) and (15), the transformation matrix is obtained.

Considering five lower order modes the transformation matrix calculated from Eq. (5) becomes

[T] =

$$\begin{bmatrix} -1.0568 & 0.6297 & -0.1861 & 0.0786 & -0.0201 \\ 0.5916 & -0.9376 & 0.5735 & -0.1349 & 0.0292 \\ -0.1559 & 0.5716 & -0.9282 & 0.5366 & -0.0737 \\ 0.0790 & -0.1599 & 0.5753 & -0.8519 & 0.3870 \\ 8.7205e-9 & 1.2227e-8 & -9.1773e-9 & -8.3791e-9 & 6.8744e-9 \end{bmatrix} \quad (16)$$

Examining the above transformation matrix, it is found that the elements of the last row are very small compared to other elements. As a result the strain at point 5 becomes very small compared to those at other points and this agrees with the fact that the strain at the end of a cantilever beam is zero.

The strain prediction method was verified using simulation data for the above cantilever beam subject to a normally distributed random force. The random force had frequency components between 0 and 500 Hz, and the Nyquist frequency of the sampled force signal was 2000 Hz. The force was applied at point 2 on the beam and the strain response was predicted at point 4. Each of five modes had a damping ratio of 1%. As explained in Eq. (7), the displacement responses at all measuring points are needed to predict the strain response at one point. The displacement responses were calculated through modal analysis. That is, the response of each mode due to the applied force was calculated by solving a second order ordinary differential equation and then the displacement responses were obtained by the superposition of the considered modes as follows:

$$u(x,t) = \sum_{r=1}^{m} Y_r(x) q_r(t) \quad (17)$$

The strain response at point 4 was calculated using Eqs. (14) and (17), and this strain response agreed exactly with the response predicted by the explained method as Fig. 2 shows.

One of the drawbacks of the above strain prediction method is that the displacement responses must be measured at many points on a structure simultaneously. Therefore, if the number of measuring points is limited by the measuring equipments available, the method cannot be applied to the cases where many measuring points are needed. To overcome this difficulty the following procedure is proposed. From Eq. (7) the strain response at point i can be written in detail as follows:

$$\varepsilon_i(t) = T_{i1}u_1(t) + T_{i2}u_2(t) + \cdots + T_{in}u_n(t) \quad (18)$$

Taking the Fourier transform of the above equation, we obtain

$$\mathrm{E}_i(\omega) = T_{i1}U_1(\omega) + T_{i2}U_2(\omega) + \cdots + T_{in}U_n(\omega) \quad (19)$$

Dividing both sides of the above equation by one of $U_j(\omega)$'s, for example, $U_1(\omega)$, we obtain

$$\frac{\mathrm{E}_i(\omega)}{U_1(\omega)} = T_{i1} + T_{i2}\frac{U_2(\omega)}{U_1(\omega)} + \cdots + T_{in}\frac{U_n(\omega)}{U_1(\omega)} = H(\omega) \quad (20)$$

That is,

$$\mathrm{E}_i(\omega) = H(\omega) \bullet U_1(\omega) \quad (21)$$

In measuring $H(\omega)$, the FRF between the displacement response at point 1 and the strain response at point i, it is necessary to measure FRFs between the displacement responses at point 1 and all the other points simultaneously. However, if we assume that the characteristics of excitation applied to the structure under consideration do not vary, these FRFs would not vary with time. Therefore it is possible to measure each FRF in sequence using only two channels of a measuring equipment. Thus this procedure eliminates the necessity of a measuring equipment with many channels. Once $\mathrm{E}_i(\omega)$ is obtained by Eq. (21), the strain response at the point can be obtained by its inverse Fourier transform.

To evaluate the above procedure the same system in Fig. 1 was considered. A random force was applied at point 2 and the strain response was predicted at point 4. In Eq. (20) each $U_j(\varpi)/U_1(\varpi)$ was calculated repeatedly from separate simulations and averaged. The predicted strain at point 4 showed strong high frequency noises and consequently there were much differences between the calculated and predicted responses as Fig. 3 shows. The reason can be attributed to the fact that the division, $U_j(\varpi)/U_1(\varpi)$ can cause large errors at frequencies where the magnitude of $U_1(\varpi)$ is small. To solve this problem the following procedure was followed. Since it is usual except special cases such as nodes that $U_j(\varpi)$ is also small at frequencies where $U_1(\varpi)$ is small, the ratio $U_j(\varpi)/U_1(\varpi)$ was neglected at frequencies where $U_j(\varpi)$ is smaller than a pre-determined value, for example, α times its largest value. This procedure results in neglect of the term including $U_j(\varpi)$ in Eq. (19) when $U_j(\varpi)$ is small, which is rational. Following this procedure, the accuracy of the predicted strains was much improved as Fig. 4 shows. In this case averaging was also performed and the number of averaging was 100. The used value of α was 0.002. The accuracy of the predicted strain did not change much with the value of α.

4. Conclusion

A method to predict the strain responses from the measurements of displacement responses is considered in this paper. The method uses a transformation matrix which is composed of a displacement modal matrix and a strain modal matrix. The method can predict strains at points where displacements are not measured as well as at displacement measuring points. Using the prediction method accurate strain responses were predicted for a cantilever beam subject to a random force. One of the drawbacks of the above strain prediction method is that the displacement responses must be measured at many points on a structure simultaneously. This difficulty can be overcome by measuring the FRFs between displacements at a reference point and other point in sequence with a two channel measuring equipment. This procedure is based on the assumption that the characteristics of excitation applied to the structure under consideration do not vary with time. The procedure was verified using simulation data for the same cantilever beam.

Acknowledgements

This work was supported by grant no. RTI04-01-03 from the Regional Technology Innovation Program of the Ministry of Commerce, Industry and Energy(MOCIE).

References

(1) Okubo, N. and Yamaguchi, K., 1995, Prediction of dynamic strain distribution under operating condition by use of modal analysis, Proceedings of the 13th IMAC, pp. 91-96.

(2) Sehlstedt, N., 2001, Calculating the dynamic strain tensor field using modal analysis and numerical differentiation, Journal of Sound and Vibration 244, pp. 407-430.

(3) Pisoni, A. C., Santolini, C., Hauf, D. E. and Dubowsky, S., 1995, Displacements in a vibrating body by strain gauge measurements, Proceedings of the 13th IMAC, pp. 119-125.

(4) Foss, G. C. and Haugse, E. D., 1995, Using modal test results to develop strain to displacement transformations, Proceedings of the 13th IMAC, pp. 112-118.

(5) Bernasconi, O. and Ewins, D. J., 1989, Modal strain/stress fields, International Journal of Analytical and Experimental Modal Analysis 4 , pp. 68-76.

(6) Meirovitch, L., 1986, Elements of Vibration Analysis, McGraw-Hill Book Company.

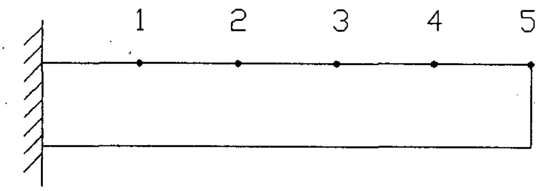

Fig. 1. A cantilever beam with five measuring points.

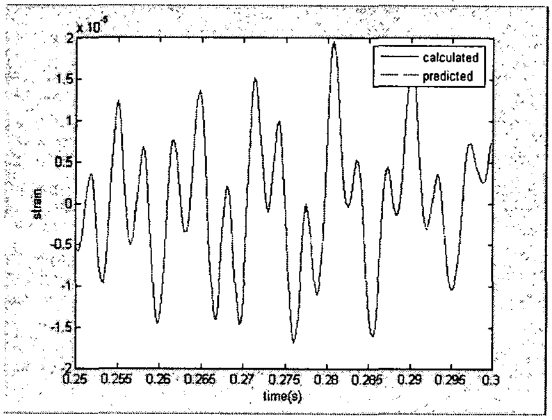

Fig. 2. Comparison of the calculated and predicted strains when the displacements are measured simultaneously.

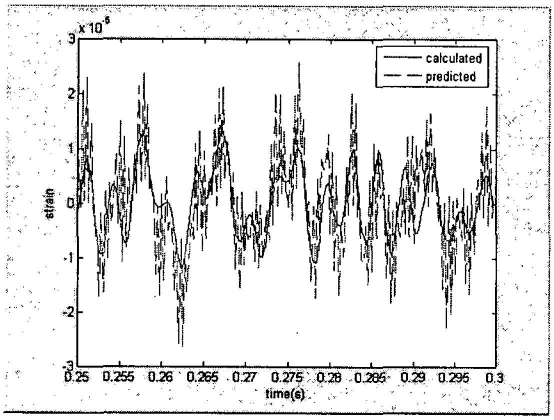

Fig. 3. Comparison of the calculated and predicted strains when pairs of displacements are measured in sequence.