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반응표면법을 이용한 헬리컬기어 치형수정의 최적화

Optimization of the Tooth Surface in the Helical Gears Using a Response Surface Method

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ABSTRACT

Optimum design of the tooth surface for the reduction of transmission error is very difficult to determine analytically due to nonlinearity of transmission error under the several load condition. The design of tooth surface that can give a low noise under the various load condition is very important. Therefore, this study proposes the method to determine the optimal lead curve and robust design of the tooth surface by using the response surface method. To do so, the design variables are selected by a screening experiment. Then the fitted regression model is built with the check of the usefulness of the model. The model with constraints is solved to obtain the optimum values for the lead curve and the robust design for the tooth surface.

Nomenclature

K_b : the influence function of bending deflection
K_c : the influence function of contact deformation
$e(x)$: tooth error
$p(x)$: the load distribution of tooth
L : contact line
W : transmitted load
Δ : transmission error
β_g : base helix angle
v_o : contact deformation
F_0 : Test statistic
F : F-distribution
x_1, x_2, x_3 : the coded variables

1. Introduction

A tooth profile in helical gear system has a great effect on gear noise. The design of the tooth profile is essential to guarantee the quietness of gear system under the several load condition. The tooth surface in the helical gear is composed of the profile in the tooth height direction and lead in the lead direction. Previous research mainly focused on the design of the profile in the tooth height direction and the research on the selection of the lead curve is very few. Moreover, the design of tooth surface that can give a low noise under the various load condition is very important. However, research on the robust design for the tooth surface is also very few. Although the gears do not have manufacturing errors, the contact pattern in a loaded gear due to the twisting moment and bending moment is different from that in an unloaded gear. Actually, the gear comes into partial contact because of the shaft misalignment, bearing clearance, and deformation of the shaft and housing. Therefore, the original lead in the teeth should be modified so that the gear does not come into partial contact, thus reducing noise and vibration. This work is to design optimally the lead curve in the helical gear system under the various load condition and robustly the tooth surface by using response surface method.

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2. The Transmission Error Analysis

It is reported that gear noise in the helical gears is highly related of transmission error. In this study, it is calculated for the evaluation of gear noise. It results from tooth deformation and the tooth error. Tooth deformation consists of bending deflection and contact deformation. Transmission error analysis needs to calculate bending deflection, contact deformation, and tooth error.

2.1 Bending deflection

Bending deflection is calculated by the influence function of bending deflection. The influence function of deflection uses the approximate equation suggested by Umezawa [2]. It is expressed by the common characteristics of deflection, which is calculated by finite element analysis. In FEA, the standard gear model with tooth foundation as shown in Fig.1 is used.

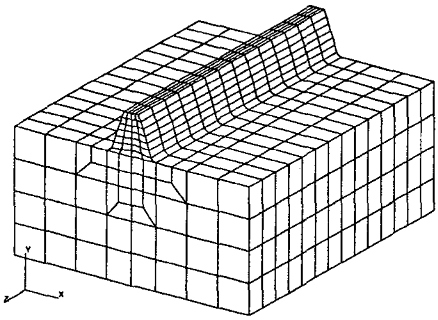


Fig. 1 Finite element model

2.2 Contact deformation

Contact deformation has a considerable contribution to deflection as well as bending deflection. It has the nonlinear characteristics, which both deformation and contact width are changed by the load. In order to obtain the influence function of contact deformation, the contact forces along the contact line are calculated under the constant contact deformation v_o, using the equation proposed by Weber [3]. Then the influence function of contact deformation $K_c(x)$ is given by

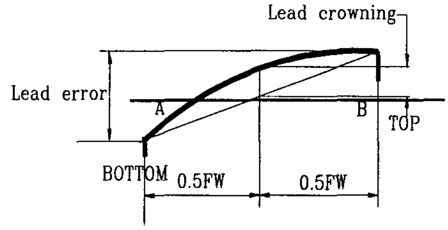
$$K_c(x) = \frac{v_o}{p_c(x)} \quad (1)$$

2.3 Modeling of the tooth surface

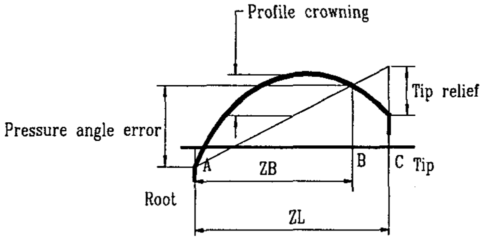
The first interest of this work is the optimal design of the lead curve and it consists of lead error and lead crowning as shown in Fig.2(a). Lead error defines as a linear equation from the bottom of the lead curve to the top of the lead curve and lead crowning defines as a quadratic equation from the bottom of the lead curve to the top of the lead curve based on the input data at the center of the face width. Profile consists of tip (root) relief, pressure angle error, and profile crowning as shown in Fig.2(b). Tip relief is modeled as a linear equation in the extent of tip modification and pressure angle error defines as a linear equation from the root of profile to the position starting the tip modification. Profile crowning defines as a quadratic equation from the root of the profile to the position starting the tip modification based on the input data at the center of the profile.

2.4 Calculation of the transmission error

Transmission error Δ is calculated by bending deflection, contact deformation and the tooth error. To do so, the following load deformation equation along the contact line is used.



(a) Lead



(b) Profile

Fig. 2 Modeling of the tooth surface

$$\Delta = \int_{-L/2}^{L/2} K_b(x,\xi)p(\xi)d\xi + K_c(x)P(x) + e(x) \quad (2)$$

The transmitted load W_j of tooth pair j is given by

$$W_j = \int_{-L/2}^{L/2} p(\xi)d\xi \quad (3)$$

Under the condition that the sum of the transmitted load of each tooth pair in meshing is the whole transmitted load W, the following equation is obtained

$$W = \sum_j^n W_j \cos\beta_g \quad (4)$$

Solving the nonlinear simultaneous equation (2), (3) and (4), transmission error are calculated.

3. Optimization of the Lead Curve

Optimum design of the lead curve for the reduction of transmission error is very difficult to determine analytically due to nonlinearity of transmission error under the several load condition. Therefore, this study proposes the method to determine the optimum lead curve by combining computer simulation, design of experiment, and response surface method. The object function for this purpose is the transmission error. Gear specification used in this analysis is shown in Table 1.

Table 1 Gear specification

	Pinion	Gear
Normal module	2.25	
Normal pressure angle (deg)	17.5°	
Center distance (mm)	127	
Whole depth (mm)	6.6	
Helix angle (deg)	28°	
Number of teeth	48	50
Outside diameter (mm)	130.36	135.26
Pitch diameter (mm)	122.32	127.41
Amount of add. Mod. (mm)	1.17	1.07

Table 2 Central composite design and experimental results (α=1)

Run	x_1	x_2	x_3	y_i
1	-1	-1	-1	0.2345
2	1	-1	-1	0.3656
3	-1	1	-1	0.6659
4	1	1	-1	0.3804
5	-1	-1	1	0.6659
6	1	-1	1	0.3804
7	-1	1	1	1.135
8	1	1	1	0.3627
9	-α	0	0	0.6659
10	α	0	0	0.3804
11	0	-α	0	0.1618
12	0	α	0	0.5841
13	0	0	-α	0.1618
14	0	0	-α	0.5841
15	0	0	0	0.3851

As the candidate design variables, the lead error, the lead crowning, and torque are selected. In order to eliminate the unimportant ones, the effect of transmission error analysis on the candidate ones for a screening experiment has been investigated using an analysis of variance. As a result, the lead error has no great effect on transmission error. Therefore, the design variables are torque, the lead crowning for a driving gear, and the lead crowning for a driven gear. After the design variables have been selected at the region of the optimum, the second-order model is used as follows

$$y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j + \varepsilon \qquad (5)$$

The central composite design(CCD) for fitting a second-order model is introduced. The CCD consists of a 2^k factorial, 2k axial runs, and n_c center runs. The model can be written in matrix notation as

$$y = X\beta + \varepsilon \qquad (6)$$

Then, the least squares estimator of β is given by

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad (7)$$

The region of exploration for fitting the second-order model is (5,15) kgf·m of torque, (7,21) μm of the lead crowning for a driving gear, and (7,21) μm of the lead crowning for a driven gear. To simplify the calculations, the independent variables will be coded to the usual (-1,1) interval. Here, the coded variable x_1 denotes torque and the coded variable x_2 denotes the lead crowning for the driving gear and the coded variable x_3 denotes the lead crowning for the driven gear.

Then, the fitted regression model is given by

$$\hat{y} = 0.3762 - 0.14988x_1 + 0.132x_2 + 0.132x_3 + 0.1491x_1^2 - 0.0011x_2^2 - 0.0011x_3^2$$
$$- 0.1129x_1x_2 - 0.1129x_1x_3 + 0.006x_2x_3 \qquad (8)$$

In order to measure the usefulness of the model, an analysis of variance and the coefficient of multiple determination are used. Since F_o exceeds $F_{0.01,3,11}$ as shown in Table 3, at least one of the regressor variables x_1, x_2, and x_3 contributes significantly to the model. The coefficient of multiple determination R^2 is 0.9635. It means that the model explains about 96.35% of the variability observed.

Table 3 Analysis of Variance for Significance of Regression in Multiple Regression

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Fo	F(3,11;0.01)
Regression	0.851	3	0.28337	96.71	6.22
Error or residual	0.0322	11	0.00293		
Total	0.8823	14			

Since the usefulness of the model is confirmed, the optimum problem can be defined as the minimization of the Equation (8) subject to

$$-1 \le x_1 \le 1, -1 \le x_2 \le 1, -1 \le x_3 \le 1.$$

The multivariable constrained function is solved, using Box method. As a result, minimum transmission error 0.1891 is obtained at torque of 5 kgf·m and the lead crowning of 7μm for the driving and driven gear. Although the model has the high coefficient of multiple determination, the existence of another local minimum leaves the future research for the application of the response surface method in this problem.

4. Robust Design of the Tooth Profile

The region of exploration for fitting the second-order model is (4,10) kgf·m of torque, (6,18) μm of the tip relief, and (7,21) μm of the lead crowning for a driven gear. Regression models corresponding to torque 4,7, and 10kgf·m are built in a similar manner as shown in Table 4. Here, the coded variable x_1 denotes the tip relief and the coded variable x_2 denotes the lead crowning for the driven gear. Since Fo of each regression model exceeds F(2,6;0.01) =10.9 and the coefficient of multiple determination is more than 0.9122, the usefulness of the models is confirmed.

Because all regression models should keep low transmission error at all torques, a equation that add three equations becomes the object function as follows

$$\hat{y} = 0.736 + 0.0594x_1 + 0.1981x_2 + 0.037x_1^2 + 0.14312x_2^2 - 0.0169x_1x_2 \quad (9)$$

Then, robust design of the tooth surface results in the minimization problem of Equation (9) subject to

$$-1 \le x_1 \le 1, -1 \le x_2 \le 1.$$

Table 4 Regression model corresponding to each torque

Torque	Regression model	Fo	R^2
4 kgf·m	$\hat{y} = 0.2811 + 0.0355x_1 + 0.1921x_2 +$ $0.0131x_1^2 + 0.0347x_2^2 + 0.022x_1x_2$	1090.8	0.9973
7 kgf·m	$\hat{y} = 0.125 + 0.0176x_1 + 0.0473x_2 +$ $0.0021x_1^2 + 0.1186x_2^2 + 0.0223x_1x_2$	31.18	0.9122
10 kgf·m	$\hat{y} = 0.3299 + 0.0063x_1 - 0.0413x_2 +$ $0.0218x_1^2 - 0.1018x_2^2 - 0.0612x_1x_2$	57.36	0.9503

The minimum of the constrained multivariable function is obtained at the tip relief 11.43 μm and lead crowing 9.09 μm by Box's method. The possible values in manufacturing are the tip relief 11 μm and lead crowing 9 μm. However, some problems are very hard to find the useful fitted response model.

5. Conclusion

This study developed the program to analyze the transmission error in order to predict the gear noise. Using the analysis program, the method to determine the optimum design of lead curve by the response surface method was proposed. Robust design of the tooth surface under the several torques by response surface method was also proposed. Multivariable constrained function of the fitted response model with constraint was solved to find the optimum design of the tooth surface.

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