한국소음진동공학회 2005년도 추계학술대회논문집, pp. 271 ~274.

# Order Tracking without a Tacho Signal

- A new Method and Its Applicability for Rotating Machinery Diagnostics

Thorkild Find Pedersen*, Hans Konstantin-Hansen*

**Key Words :** Order Tracking, Tacho Signal, Machinery Diagnostics

**ABSTRACT**

For noise and vibration order-analysis on rotating machinery, it is compulsory to measure the RPM of the rotating part of the machine. Normally the RPM is measured using dedicated tacho-probes. In this paper we describe a new method that in real-time synthesizes a tacho signal from the measured noise or vibration signal thus eliminating the tacho probe. The method strengths and weaknesses are evaluated on practical signals.

## 1. Introduction

Typically the fundamental frequency of rotating machines is measured using dedicated sensors like proximity probes or photo sensors requiring direct access to the rotating part of the machine. Often rotating parts are not easily accessible and, even if they are, then mounting and setting up the tacho probe is a problem in itself. It is therefore of interest to investigate the possibility of extracting the fundamental frequency from other sources, e.g., the sound and vibration signals already measured for other analysis purposes.

In [1] the authors introduced the standard Bayesian approach of periodic component detection to non-stationary RPM estimation of an automotive engine. In the following we will review some of this theory and show some results of using this estimated RPM value as a reference for resampled order analysis.

The objective of the Bayesian analysis is to compute the posterior probability of the event that the fundamental frequency is $\omega_0$, i.e., the conditional probability $p(\omega_0|\mathbf{d},I)$ where $I$ represents the prior knowledge in the estimation model, and **d** represents the measured signal. In order to cope with non-stationary signals, we segment the signal into overlapping records and compute the posterior probability for each record.

The method is tested on a vibration signal measured with an accelerometer mounted on the engine block in a passenger car and also on the acoustic signal measured with a microphone 1m above the same engine block. The derived fundamental frequencies from both signals are compared that obtained with an optical tacho probe mounted on the cam shaft.

* Bruel & Kjaer Sound & Vibration Measurement A/S
E-mail : tfpedersen@bksv.com
Tel : (000) 000-0000, Fax : (000) 000-0000

## 2. Fundamental Frequency Estimation

The motivation for using the Bayesian statistical framework to estimate the fundamental frequency is that it allows for simple inclusion of prior knowledge in the estimation process. The impulsive nature of rotating and reciprocating machines results in the presence of higher harmonics of the fundamental frequency in the observed signals. E.g., it is well known that the dominant harmonic orders for a 4-cylinder engine is the sequence $K = \{2, 4, \dots\}$ and for a 5-cylinder engine $K = \{2.5, 5, \dots\}$. Within the Bayesian framework this prior knowledge is easily included to improve the performance of the estimator.

In the signal processing literature, the problem of detecting and tracking periodic signals is often defined as estimating the amplitude and phase of a number of harmonic components, i.e.

$$d_m(t) = a_{m,0} + \sum_{k=1}^{K} \left( a_{m,k} \cos(\omega_k t) + b_{m,k} \sin(\omega_k t) \right) + e_m(t) \quad \text{.........(1)}$$

The noise-term $e_m(t)$ is assumed zero mean, white and Gaussian with variance, $\sigma^2$. The frequencies in the harmonic sequence are defined as orders $\alpha_k$ of the fundamental frequency, $\omega_0$, i.e., $\omega_k = \alpha_k \omega_0$.

For a segment of samples Eq. (1) can be reformulated as the linear problem

$$\mathbf{d} = \mathbf{G}\mathbf{b} + \mathbf{e} \tag{2}$$

where the column vector $\mathbf{d}$ is the observed or measured data, $\mathbf{G}$ is a matrix with basis vectors as columns corresponding to the sines and cosines, $\mathbf{b}$ is a column vector with the associated amplitudes. $\mathbf{e}$ is the noise contribution:

$$\begin{aligned} \mathbf{d} &= \left[d(t_0) \ \cdots \ d(t_{N-1})\right]^T \\ \mathbf{G} &= \left[\mathbf{1} \ \ \cos\left(\mathbf{t}\cdot\Omega^T\right) \ \ \sin\left(\mathbf{t}\cdot\Omega^T\right)\right] \\ \mathbf{b} &= \left[a_0 \ \cdots \ a_K \ \ b_1 \ \cdots \ b_K\right]^T \\ \mathbf{t} &= \left[t_0 \ \cdots \ t_{N-1}\right]^T \\ \Omega &= \left[\omega_1 \ \cdots \ \omega_K\right]^T \end{aligned} \tag{3}$$

### 2.1 Bayesian Formulation

Bayes theorem states that the joint posterior probability density function (PDF) of the parameters in a model equals the likelihood of the data multiplied by the prior knowledge of the parameters divided by the evidence of the data. Mathematically this is expressed as:

$$p(\Theta \mid \mathbf{d}) = \frac{p(\mathbf{d} \mid \Theta)\, p(\Theta)}{p(\mathbf{d})}$$

where $\Theta$ and $\mathbf{d}$ respectively represents the model parameters and the data.

Using Bayes theorem the posterior distribution of $\omega_0$ conditioned on the measurement data is given by:

$$p(\omega_0, \{\sigma, \mathbf{b}\} \mid \mathbf{d}) = \frac{p(\mathbf{d} \mid \omega_0, \{\sigma, \mathbf{b}\})\, p(\omega_0, \{\sigma, \mathbf{b}\})}{p(\mathbf{d})} \tag{4}$$

To find the distribution of the observations conditioned only on $\omega_0$, the nuisance parameters, $\{\sigma, \mathbf{b}\}$, must be eliminated by marginalization of the likelihood $p(\mathbf{d} \mid \omega_0, \{\sigma, \mathbf{b}\})$ which simplifies Eq. (4) into [2]:

$$p(\omega_0 \mid \mathbf{d}) \propto p(\mathbf{d} \mid \omega_0)\, p(\omega_0) \tag{5}$$

The marginalization of the likelihood depends on the prior distributions chose n for the nuisance parameters. In [2, 3] an analytical solution is derived when the amplitudes are assigned uniform priors and Jeffreys prior is used for the noise variance:

$$p(\mathbf{d}_m \mid \omega_0) \propto \frac{\left(\mathbf{d}^T\mathbf{d} - \mathbf{d}^T\mathbf{G}\left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{d}\right)^{\frac{-(N-2K-1)}{2}}}{\sqrt{\left|\mathbf{G}^T\mathbf{G}\right|}} \tag{6}$$

(2)

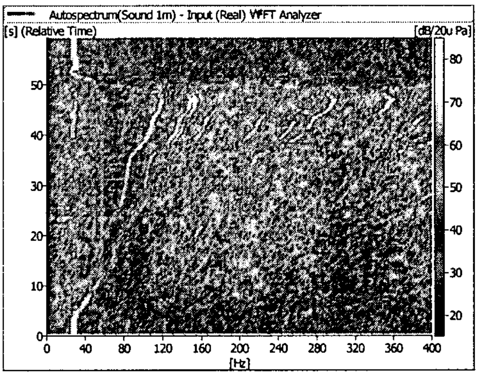
## 3. Evaluation

The measured acoustic signal is shown in Figure.1 The dominating harmonics in the signal changes as the RPM increase as is illustrated in Figure 2. At low RPM the 2.5th order is clearly dominating but as the RPM increases the 2nd, 2.6th and 5th orders increase in amplitude why these four harmonics are used for estimating the fundamental frequency from the acoustic signal. For the vibration signal only the 1st and 2nd harmonics where found sufficient as the signal to noise ration is much better (not shown). The estimated RPM profiles are shown in Figure 3.

To examine the effect of estimation error of the RPM value, a resampled order analysis of the vibration signal is performed with the B&K PULSE multianalyzer for each of the three tacho references. The result of this is shown in Figure 4. for the optical tacho probe, in Figure 5. for the estimated RPM from the acoustic signal and in Figure 6. for the estimated RPM from the vibration signal. For the acoustic reference the estimation error is observed as a smearing of the order lines, whereas for the vibration reference is is difficult to see noticeable differences to that of the optical reference.

## 4. Conclusion

We have shown that with the Bayesian approach it is possible to accurately estimate the fundamental frequency of an automotive engine from both acoustic and vibration signals. It was found that for order analysis the vibration signal was a better reference.



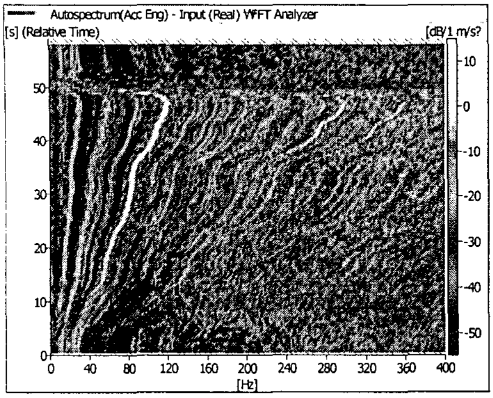
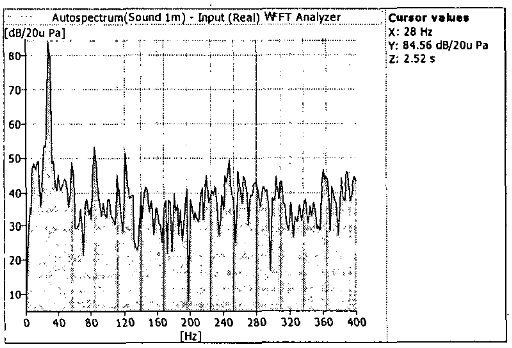



**Figure 1** Spectrogram of microphone signal (upper) and vibration signal(lower)



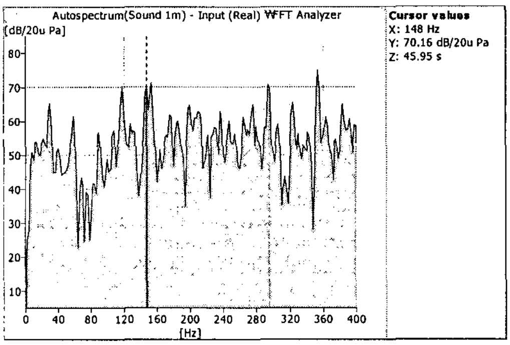
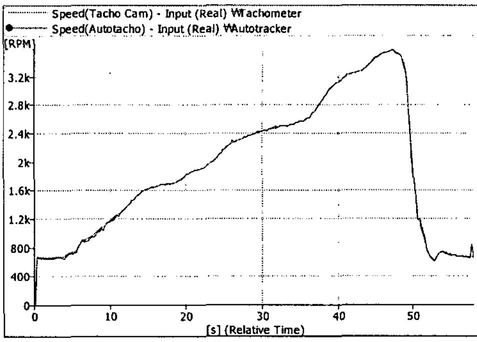




**Figure 2** Frequency content of Acoustic signal at low RPM (upper) and max RPM (lower). The harmonic cursors are placed on the expected orders of fundamental (2.5, 5, 7.5, etc)



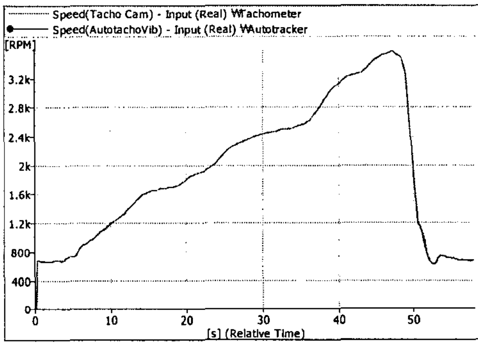




**Figure 3** Estimated tacho profile from acoustic signal (upper) and from vibration signal (lower)

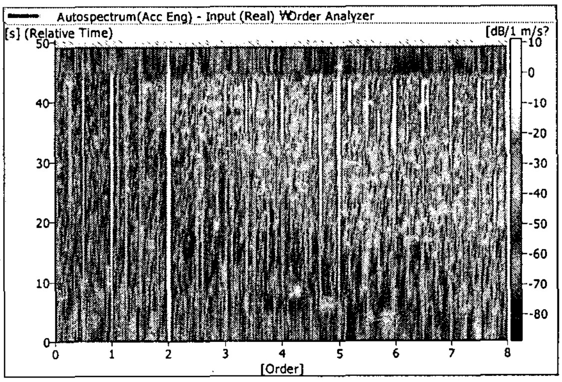



**Figure 4** Resampled order spectrum of vibration signal w/ optical tacho reference

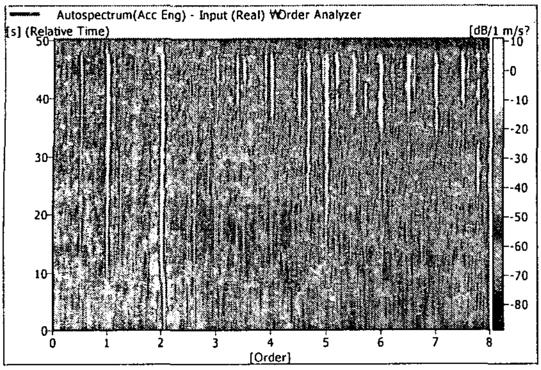



**Figure 5** Resampled order spectrum of vibration signal w/ acoustic tacho reference

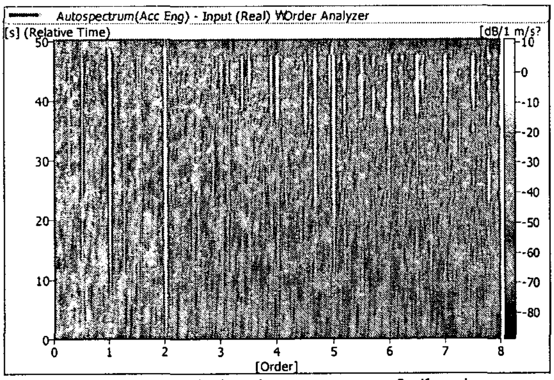



**Figure 6** Resampled order spectrum of vibration signal w/ vibration tacho reference

# Reference


[1] Thorkild Find Pedersen, Henrik Herlufsen, Hans Konstantin Hansen, *Order Tracking in Vibro-acoustic Measurements: A Novel Approach Eliminating the Tacho Probe*, SAE 2005 Noise and Vibration Conference and Exhibition, May 2005

[2] Joseph J.K. Ò Ruanaidh and William J. Fitzgerald, *Numerical Bayesian Methods Applied to Signal Processing*, Statistics and Computing, Springer, 1998.

[3] G. L. Bretthorst, *Bayesian Spectrum Analysis and Parameter Estimation*, vol. 48 of *Estimation*, vol. 48 of Lecture Notes in Statistics, Springer-Velag, 1988