DECISION SUPPORT SYSTEM FOR SUBURBAN STATION REHABILITATION

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ABSTRACT : Every public agency governing infrastructure has to plan effectively for rehabilitation of existing facilities within the constraints of the capital program. Numerous technical, social, political, financial, and management constraints govern the decision to rehabilitate a facility. However, without a systematic procedure for selecting facilities for rehabilitation, within the prevailing constraints, it is possible that the funds available for rehabilitation might be suboptimized. Therefore, a decision support system that assists the user in selecting facilities for rehabilitation while considering the technical, social, financial, and political and management constraints will be useful in the decision-making process. This paper compares the Analytical Hierarchy Process (AHP) with the Swing Weight method used to prioritize functional criteria for suburban station rehabilitation. This paper also contains a brief discussion about the relevance of the Multi Attribute utility theory in developing a decision model for the problem at hand. The results of this paper provides the user with a decision support system that would prioritize the stations in order of their weights obtained by a systematic evaluation of various criteria and sub-criteria involved in the decision making process

Key words : decision support system, suburban station, analytical hierarchy process, swing weigh method, multi attribute utility theory

1. INTRODUCTION

Although a great amount of deterioration problem are present in all major U.S. city's infrastructure facilities, more and more of federal investment for infrastructure rehabilitation is declining. As compared to other sectors of the construction industry, many urban infrastructure management agencies are missing opportunities to enhance infrastructure performance (Hastak and Abu-Mallouh). The Federal Infrastructure Strategy developed by the U.S. Advisory Commission on Intergovernmental Relations suggests that in the future: (1) the federal role in supporting public infrastructure will continue to decline; and (2) it is more likely that maintaining, operating, and refining existing infrastructure systems will plays more major role than new large capital programs (U.S. Army Corps of Engineers (USACE) 1995, Griffis 1996). Therefore, it is important develop decision support system to assist decision maker in selecting facilities for rehabilitation.

Although several in-house procedures exist for station rehabilitation planning, all lack one or more important considerations as follows: (1) systematic consideration of all the important criteria influencing the rehabilitation planning; and (2) optimization of rehabilitation planning regarding the capital budget and existing constraints. The focus of this paper is the rehabilitation needs of subway stations in a city. The purpose of this paper is to model a decision support system to optimize the number of stations that can be accommodated within a given capital program while incorporating numerous technical, social, political, financial and management constraints. This paper compares the AHP with the Swing Weighting method used to prioritize functional criteria for suburban station rehabilitation. This paper also discusses briefly the relevance of the Multi Attribute utility theory in developing a decision model for the problem at hand. The results of this paper may provide the user with a decision support provides the user with a decision support system that would prioritize the stations in order of their weight obtained by a systematic evaluation of various criteria and sub-criteria involved in the decision making process.

2. FUNCTIONAL CRITERIA FOR SUBURBAN STATION REHABILITATION

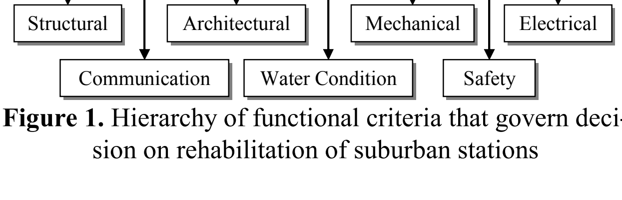

Figure 1. Hierarchy of functional criteria that govern decision on rehabilitation of suburban stations

According to the study by Hastak and Abu-Mallouh (2001), seven functional criteria for suburban station rehabilitation have been identified to determine the physical condition and rehabilitation needs of each station (refer to Figure 1).

However, for easy of developing the hierarchy for the model and calculating priority vectors, the physical condition and rehabilitation needs of each station are analyzed using only three functional criteria: (i) structural; (ii) mechanical; and (iii) electrical. The three functional criteria are further divided into four sub-criteria as shown in Figure 2.

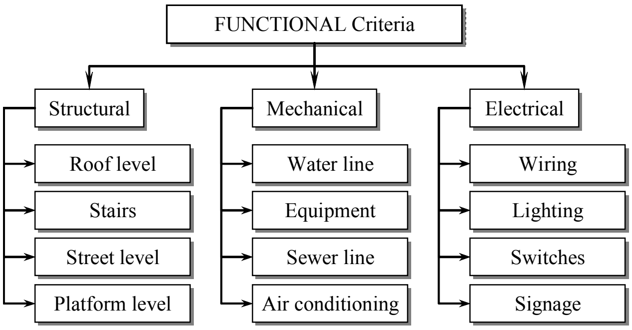

Figure 2. Hierarchy of functional criteria and sub-criteria

3. AHP ANALYSIS

The AHP developed by Professor Thomas L. Saaty in the 1970's was used to evaluate the hierarchy of criteria with each level and to determine each criterion's relative importance by establishing weight (or priority) among the criteria and sub-criteria (Saaty 1982). Additionally, the actual physical state of the station is evaluated per criterion through visual inspection by field engineer/consultant using a pre-determined scale as shown in Table 1.

Table 1. Field evaluation scale for functional sub-criteria

Condition	Rating	Description
Good	1	No damage; station in good condition; no repairs required
Fair	2	Minor damage; station in fair condition; minor repairs required
Moderate	3	Moderate defects; 10% deteriorated; moderate repairs required
Poor	4	Poor Condition; 15% deteriorated; repairs required
Severe	5	Severe Condition; 25% deterioration; repairs required

(Source: Hastak and Abu-Mallouh 2001, Abu-Mallouh 1999)

The inspection is conducted at the lowest level in the hierarchy (i.e., at the sub-criteria level). The combination of weights (or priorities) and the results of physical inspection provide a weighted score for each station called the "total station score." The station scores can be arranged in descending order to obtain a ranking where a station with higher ranking is a better candidate for rehabilitation. The essential criteria for selection of fundamental objectives and their attributes are

1. The set of objectives, as represented by the fundamental hierarchy should be complete; it should include all relevant aspects of a decision
2. The set of objectives should be as small as possible
3. Each objective should differentiate the available alternatives. If all the alternatives are equal regarding a particular objective, then the objective will not be of any help in making the decision.
4. The set of fundamental objectives should not be redundant.
5. The objectives should not be closely related.
6. The set of objectives should decomposable. In other words, the decision maker should be able to think about each objective easily without having to consider others
7. Means and fundamental objectives should be distinguished.
8. Attribute scales must be operational. They should provide an easy way to measure performance of the alternatives or the outcomes on the fundamental objectives.

The three functional criteria (structural = X, mechanical = Y, and electrical = Z) were evaluated by a project engineer by using AHP comparison scale shown in Table 2.

Table 2. AHP comparison scale

Degree of Importance	Definition
1	Equal importance of elements
3	Weak importance of one element over the other
5	Strong importance of one element over the other
7	Demonstrated or Very strong importance of one element over the other
9	Absolute importance of one element over the other
2,4,6,8	Intermediate values between two adjacent degrees of importance

Table 3. Summary of pair wise comparison of functional criteria

Pair wise Comparison	X	Y	Z	Local weights
X	1	2	7	0.63
Y	0.5	1	2	0.26
Z	0.14	0.5	1	0.11
			SUM =	1

Table 3 suggests that the structural features receive a weight of 0.63, the mechanical aspects receive weight of 0.26, and the electrical aspects receive a weight of 0.11. In other words, the percentage of importance is distributed in the following manner.

- Structural: 63.1%
- Mechanical: 26.1%
- Electrical: 10.8%
- Total: 100%

In addition, the summaries of pair wise comparison of structural, mechanical, and electrical sub-criteria are shown in Tables 4 to 6. For information on the calculation of priority vectors, refer to appendix I.

Table 4. Summary of pair wise comparison of structural sub-criteria

Pair wise Comparison	X1	X2	X3	X4	Local Weights
X1	1	3	6	7	0.60
X2	0.33	1	2	3	0.21
X3	0.17	0.5	1	1	0.10
X4	0.14	0.33	1	1	0.08
				SUM =	1.00

Note: X1 = roof level, X2 = stairs, X3 = street level, X4 = platform level

Table 5. Summary of pair wise comparison of mechanical sub-criteria

Pair wise Comparison	Y1	Y2	Y3	Y4	Local Weights
Y1	1	0.33	2	3	0.20
Y2	3.03	1.00	7	8	0.61
Y3	0.50	0.14	1	4	0.13
Y4	0.33	0.13	0.25	1	0.06
				SUM =	1.00

Note: Y1 = water line, Y2 = equipment, Y3 = sewer line, Y4 = air conditioning

After the AHP analysis of the function criteria and sub-criteria is conducted, the total station score is obtained by a combination of the available AHP evaluation and a field evaluation based on the scale shown in Table 1. The summary of the evaluation and an example for a station score is shown in Table 7.

Table 6. Summary of pair wise comparison of electrical sub-criteria

Pair wise Comparison	Z1	Z2	Z3	Z4	Local Weights
Z1	1	0.5	5	1.00	0.24
Z2	2	1	7	5	0.54
Z3	0.20	0.14	1	0.5	0.06
Z4	1.00	0.20	2	1	0.15
				SUM =	1.00

Note: Z1 = wiring, Z2 = lighting, Z3 = switches, Z4 = signage

The field scores of Table 7 will vary for different stations based on the sate of deterioration of each sub-criterion in consideration. However, the field evaluation and subsequent calculation is conducted for one station example and the total station score was found to be 3.37. This process can be repeated for all the stations in consideration and they can be ranked based on the descending values of their scores. The stations that feature on top of the ranking will have more potential for rehabilitation, evaluated in a systematic way.

To test the level of consistency of the user in evaluating, the Consistency Ration (CR) was calculated for each of the three pair wise comparison matrices. The method for calculation of CR is explained in Appendix II and its CR is as follows:

- Structural sub-criteria: CR – 0.0058
- Mechanical sub-criteria: CR – 0.0636
- Electrical sub-criteria: CR – 0.0458

The preferred value for CR should be less than or equal to 0.1. In all three cases, this was satisfied.

Table 7. Summary of functional criteria evaluation and field evaluation for one station example evaluation

Criteria	Local weight	Sub Criteria	Local weight	Global weight	Field Score	Weighted Field Score	Total Station Score
X	0.63	X1	0.60	0.38	2	0.76	
		X2	0.21	0.14	2	0.27	
		X3	0.10	0.06	3	0.18	
		X4	0.08	0.05	4	0.21	1.43
Y	0.26	Y1	0.20	0.05	7	0.36	
		Y2	0.61	0.16	6	0.95	
		Y3	0.13	0.04	5	0.18	
		Y4	0.06	0.02	5	0.08	1.57
Z	0.11	Z1	0.24	0.03	2	0.05	
		Z2	0.54	0.06	4	0.23	
		Z3	0.06	0.01	8	0.06	
		Z4	0.15	0.02	2	0.03	0.37
				1.00			3.37

3. SWING WEIGHTING METHOD

This method requires a thought experiment in which the decision maker compares individual attributes directly by imagining (typically) hypothetical outcomes (Clemen 1996). To assess swing weights, the first step is to create a swing weight assessment table like the one in Table 8. The first row indicates the worst possible outcome, or the outcome that is at the worst level on each of the attributes.

In the case of structural criteria, it would be (i) broken platform with cracks (danger of falling), (ii) leaking roof with rain water flowing in, (iii) congested traffic with entry/exit point blocked, (iv) and broken and missing handrails on stairs with a danger of falling (refer to Table 8).

In the case of mechanical features of a station, it would be (i) leaking water pipes, mis-directed flow, no water, (ii) defunct equipment, failing signals, train delays, (iii) blocked sewers, no sanitation, unusable restrooms, and (iv) defunct HVAC system, uncomfortable waiting times (refer to Table 9).

Table 8. Swing weight assessment table for structural sub-criteria

Attribute swung from worst to best	Consequence to compare	Rank	Rate	Weight
(Benchmark)	Broken platform with cracks (danger of falling) Leaking roof (rain water flowing in) Congested with traffic (entry/exit point blocked) Broken and missing handrails (danger of falling)	5	0	0
Platform level	Functionally sound platform, neatly maintained Leaking roof (rain water flowing in) Congested with traffic (entry/exit point blocked) Broken and missing handrails (danger of falling)	4	15	0.06
Roof level	Broken platform with cracks (danger of falling) No repairs required, No leaks, Proper day lighting Congested with traffic (entry/exit point blocked) Broken and missing handrails (danger of falling)	1	100	0.43
Street level	Broken platform with cracks (danger of falling) Leaking roof (rain water flowing in) Unhindered Traffic flow, Entry/Exit defined well Broken and missing handrails (danger of falling)	3	40	0.17
Stairs	Broken platform with cracks (danger of falling) Leaking roof (rain water flowing in) Congested with traffic (entry/exit point blocked) Non-slippery, no cracks, handrails in place	2	80	0.34
Total			235	1.00

Table 9. Swing weigh assessment table for mechanical sub-criteria

Attribute swung from worst to best	Consequence to compare	Rank	Rate	Weight
(Benchmark)	Leaking water pipes, mis-directed flow, no water Defunct equipment, failing signals, train delays Blocked sewers, No sanitation, unusable restrooms Defunct HVAC system, uncomfortable waiting times	5	0	0
Water line	Sealed pipes, no leaks, even flow, water at all times Defunct equipment, failing signals, train delays Blocked sewers, No sanitation, unusable restrooms Defunct HVAC system, uncomfortable waiting times	4	40	0.17
Equipment	Leaking water pipes, mis-directed flow, no water Functional equipment, no repairs or replacement reqd. Blocked sewers, No sanitation, unusable restrooms Defunct HVAC system, uncomfortable waiting times	1	100	0.41
Sewer line	Leaking water pipes, mis-directed flow, no water Defunct equipment, failing signals, train delays New sewer lines, no blocks, good sanitation/restroom Defunct HVAC system, uncomfortable waiting times	3	40	0.17
Air Conditioning	Leaking water pipes, mis-directed flow, no water Defunct equipment, failing signals, train delays Blocked sewers, No sanitation, unusable restrooms Good Air conditioning, comfort zone temp for interiors	2	60	0.25
Total			240	1.00

Table 10. Swing weigh assessment table for electrical sub-criteria

Attribute swung from worst to best	Consequence to compare	Rank	Rate	Weight
(Benchmark)	Visible wiring, danger of short circuit, repairs needed No lighting, night use affected, mugging, uninviting interiors Defunct switches, signal delays, bad control systems No proper signage, entry/exit undefined, lack of info.	5	0	0
Wiring	Concealed wiring, no dangers, no repair/replacement No lighting, night use affected, mugging, uninviting interiors Defunct switches, signal delays, bad control systems No proper signage, entry/exit undefined, lack of info.	4	60	0.27
Lighting	Visible wiring, danger of short circuit, repairs needed Functional lights, night use unaffected, safe interiors Defunct switches, signal delays, bad control systems No proper signage, entry/exit undefined, hindered flow	1	100	0.45
Switches	Visible wiring, danger of short circuit, repairs needed No lighting, night use affected, mugging, uninviting interiors New switch and control system, no signal delays No proper signage, entry/exit undefined, lack of info.	3	10	0.05
Signage	Visible wiring, danger of short circuit, repairs needed No lighting, night use affected, mugging, uninviting interiors Defunct switches, signal delays, bad control systems Good signage for both night/day use, ADA satisfied	2	50	0.23
Total			220	1.00

Finally, in the case of electrical criteria that govern proper functioning of a station, it would be (i) visible wiring, danger of short circuit, immediately needed repairs, (ii) no lighting, night use affected, mugging, uninviting interiors, (iii) defunct switches, signal delays, bad control systems, and (iv) no proper signage, undefined entry/exit, lack of information. With table constructed, the assessment can begin. The first step is to rank order the outcomes. There are five hypothetical stations to compare, and it is safe to assume that the benchmark station – the one that is worst on all the objectives – will rank fifth (worst) overall. The other must be compared to determine which ranks first (best), second, third, and fourth.

The next step is to fill in the "Rate" column in the table. As shown in Tables 8 to 10, two of the ratings are predetermined; the rating for the benchmark station is 0 and the rating for the top-ranked station is 100. The rating for the other three stations must fall between 0 and 100. The comparison is fairly straightforward to make.

If we think about it in terms of percentage, considering the increase in satisfaction that results from swinging roof conditions as 100%, what percentage of that increase do we get by swinging condition of the stairs from worst to best? The weights are the normalized ratings. The weights are added up to 1. For example, the weight for wiring in Table 10 is calculated as 60 / (60 + 100 + 10 + 50) = 0.27. Likewise, the weight for lighting, switches, and signage in Table is 100 / 220 = 0.45, 10 / 220 = 0.05, and 50 / 220 = 0.23, respectively.

4. RESULTS AND DISCUSSION OF EVALUATION USING TWO METHODS

The same person (a project engineer) evaluated functional criteria and sub-criteria. It turns out that the AHP and Swing weight method give similar results in terms of an overall order of preference. However, the percentage (weights) varies quite a bit and the rankings change for the second, third, and fourth preference in the case of mechanical and electrical criteria. The preference for the best choice remains same in both the evaluation methods (refer to Table 11).

Table 11. Comparison of two methods

	(1)	(2)
Roof level (X1)	60%	43%
Stairs (X2)	21%	34%
Street Level (X3)	10%	17%
Platform level (X4)	8%	6%
Total	100%	100%
Water line (Y1)	20%	17%
Equipment (Y2)	61%	41%
Sewer line (Y3)	13%	17%
Air conditioning (Y4)	6%	25%
Total	100%	100%
Wiring (Z1)	24%	27%
Lighting (Z2)	54%	45%
Switches (Z3)	6%	5%
Signage (Z4)	15%	23%
Total	100%	100%

Note: (1) = AHP evaluation of criteria and (2) = swing weigth assessment of criteria

5. MULTI ATTRIBUTE UTILITY THEORY

The essential problem in multiobjective decision-making is deciding how best to trade-off increased value on one objective for lower value on another. Making these tradeoffs is a subjective matter and requires the decision maker's judgment (Clemen 1996). In this section, both functional and sociopolitical criteria for rehabilitation of suburban stations were considered. Let us suppose we have narrowed down our choice to three stations A, B, and C.

Table 12. Functional and sociopolitical criteria for three stations

	Station A	Station B	Station C
Functional criteria	9	6	5
Sociopolitical criteria	5	7	8

Station A scores high on functional criteria while Station C scores high on sociopolitical criteria for rehabilitation needs. If these numbers are translated into reality, it would mean that Station A suffers maximum physical deterioration and is in need for physical repair/rehabilitation. Station C is probably not well-used in spite of being functionally efficient due to certain social issues like unsafe neighborhood, bad site location, distance from bus transit, distance from parking lot, etc. Station B, however, fits into the middle regarding both criteria. The utilities for the three stations are shown in Table 12. As long as the objectives have natural numerical attributes, it is a straightforward matter to scale those attributes so that the utility of the best is 1, the utility of the worst is 0, and the intermediate alternatives have scores that reflect the relative distance between the best and worst. A general formula for the intermediate alternatives is as follows;

$$U_i(x) = \frac{x - \text{Worst Value}}{\text{Best Value} - \text{Worst Value}} \quad (1)$$

For the utility of Station B for the functional criteria,

$$U_F(6) = \frac{6-5}{9-5} = 0.25$$

Likewise, the utility of Station B for the sociopolitical criteria is

$$U_S(7) = \frac{7-5}{8-5} = 0.67$$

The utilities for three stations are summarized in Table 13.

Table 13. Utilities for three stations on two attributes

	Station A	Station B	Station C
Functional criteria	1	0.25	0
Sociopolitical criteria	0	0.67	1

Suppose the weight for functional (k_F) = 0.5 and sociopolitical (k_S) = 0.5 should be weighted equally.

$$\begin{aligned} &\text{U (Functional, Sociopolitical)} \\ &= k_F U_F(\text{Functional}) + k_S U_S(\text{Sociopolitical}) \end{aligned} \quad (2)$$

Thus, the weighted utilities would be

	Rank
U (Station A) = 0.50 (1.00) + 0.50 (0.00) = 0.50	1
U (Station B) = 0.50 (0.25) + 0.50 (0.67) = 0.46	3
U (Station C) = 0.50 (0.00) + 0.50 (1.00) = 0.50	1

Station A and C came out with exactly the same overall utility because the way that functional and sociopolitical criteria are traded off against each other.

Suppose that functional should be twice as important as sociopolitical. To model this, let k_F = 0.67 and k_S = 0.33. Thus, the weighted utilities would be

	Rank
U (Station A) = 0.67 (1.00) + 0.33 (0.00) = 0.67	1
U (Station B) = 0.67 (0.25) + 0.33 (0.67) = 0.39	2
U (Station C) = 0.67 (0.00) + 0.33 (1.00) = 0.33	3

It turns out that Station A has a greater priority compared to both Stations B and C. This method gives us a clear idea about assigning utilities for intangible or unquantifiable items when the "Expected Monetary Value" approach cannot be used to rank criteria used in decision making. It is evident that when the weights for the criteria are altered, it has a direct correlation to the ranking of the stations.

6. CONCLUSIONS

The AHP and Swing weight method can be used in cases where a decision has to be made based on a set of criteria and sub-criteria that would influence the expected outcome. This paper demonstrates that these two methods are not very different each other in terms of eliciting values from the user for comparing variables involved in the decision support model. A project engineer conducted the evaluation in either case. The results turned out to be similar. Also, it was interesting to observe that the user was able to display a good level of consistency in his evaluation process.

The Multi Attribute Utility theory can be used in situations where a decision has to be made between two or more choices that are compared with each other based on the same set of attributes. In this paper, the Multi Attribute Utility theory was used to solve a slightly different case. While the AHP and the swing weighting method were used to rank the functional criteria in order of preference, the Multi Attribute Utility theory was used to rank three stations based on their utility values obtained from the evaluation of the different attributes involved. It turns out that the assigning of utilities for the choices has a major role to play in the final rank obtained by them.

It should be noted that the results of this paper were limited to a large extent by certain constraints that allowed only a portion of the decision support system to be evaluated and analyzed. The actual rehabilitation of sub-urban stations could have numerous criteria and sub-criteria in the hierarchical model that have to be systematically evaluated and analyzed. However, this paper has attempted to look into one such aspect (functional criteria) in detail. There is scope for further research in this paper. The integration of Group Decision Model (GDM) could be a probable addendum to this study. Each member that does the evaluation can be assigned a weight based on some attributes like technical knowledge, experience, station knowledge, etc. and the weights can be combined with the station scores to obtain a "weight station score." This would remove the subjectively involved in the evaluation by individual group members.

Overall, this study was an effort to understand the various theories and tools available for developing "Decision Support Systems" to solve real-life situation in suburban station rehabilitation.

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APPENDIX: CALCULATION OF PRIORITY VECTORS FOR FUNCTIONAL SUB-CRITERIA

The following pair wise comparison matrix represents one of structural sub-criteria. We call this matrix as "*A*" matrix here.

$$A = \begin{bmatrix} 1 & 3 & 6 & 7 \\ 1/3 & 1 & 2 & 3 \\ 1/6 & 1/2 & 1 & 1 \\ 1/7 & 1/3 & 1 & 1 \end{bmatrix}$$

To normalize the weights, compute the sum of each column and then divide each column by the corresponding sum. Using an overbar to denote normalization, we get:

$$\overline{A} = \begin{bmatrix} 0.609 & 0.621 & 0.600 & 0.583 \\ 0.203 & 0.207 & 0.200 & 0.250 \\ 0.101 & 0.103 & 0.100 & 0.083 \\ 0.087 & 0.069 & 0.100 & 0.083 \end{bmatrix}$$

The sum of the rows is the column vector "*w*" which when averaged by the sample size of four columns gives the column vector of priorities. Divide by 4 to get column vector of priorities.

$$w = \begin{bmatrix} 0.603 & 0.215 & 0.097 & 0.085 \end{bmatrix}$$

Using the same technique, the priority vectors of mechanical and electrical sub-criteria were calculated.

APPENDIX II: CALCULATION OF CONSISTENCY RATIO (CR) FOR FUNCTIONAL SUB-CRITERIA

Saaty defines the Consistency Index (CI) as $(\lambda - n) / (n - 1)$. Where λ = the average value of the consistency vector and n = the number of potential decision being compared. The ratio of CI to the average Random index (RI) for the same order matrix is called the CR. A CR of 0.1 or less is considered acceptable. Here are some R.I. values for a given matrix dimension n:

n	2	3	4	5	6	7	8	9	10
R.I	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

Therefore, CR = CI / RI

Step1: Multiply the vector of priorities "*w*" by original matrix "*A*" to get the product.

$$A = \begin{bmatrix} 1 & 3 & 6 & 7 \\ 1/3 & 1 & 2 & 3 \\ 1/6 & 1/2 & 1 & 1 \\ 1/7 & 1/3 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0.603 \\ 0.215 \\ 0.097 \\ 0.085 \end{bmatrix} = \begin{bmatrix} 2.422 \\ 0.865 \\ 0.390 \\ 0.340 \end{bmatrix}$$

Step 2: Divide the products by the weight to get the consistency vector and then take its average to get λ.

$$\lambda = \frac{\frac{2.422}{0.603} + \frac{0.865}{0.215} + \frac{0.390}{0.097} + \frac{0.340}{0.085}}{4} = 4.016$$

Calculate CI and CR

$$CI = \frac{4.016 - 4}{4 - 1} = 0.0053, CR = \frac{CI}{RI} = \frac{0.0053}{0.9} = 0.0058$$

Therefore, this matrix is consistent enough to use. Using the same technique, the CR of mechanical and electrical sub-criteria was calculated.