FLEXIBLE OPTIMIZATION MODEL FOR LINEAR SCHEDULING PROBLEMS

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ABSTRACT : For linear projects, it has long been known that resource utilization is important in improving work efficiency. However, most existing scheduling techniques cannot satisfy the need for solving such issues. This paper presents an optimization model for solving linear scheduling problems involving resource assignment tasks. The proposed model adopts constraint programming (CP) as the searching algorithm for model formulation, and the proposed model is designed to optimize project total cost. Additionally, the concept of outsourcing resources is introduced here to improve project performance.

Key words : Linear projects, Constraint programming, Outsourcing resources

1. INTRODUCTION

Effective resource utilization for linear projects, including high-rise buildings and bridges, is attracting growing attention in construction industry. Linear projects consist of several similar activities in units or sections, and the production rate for each type of activity is tremendously influenced by major resources. Therefore, it is important to arrange the usage of limited major resources efficiently for achieving project goals.

Numerous techniques for handling the such problems have been developed in recent optimization decades. Several studies have adopted mathematical programming, such as linear programming and integral programming. Halpin [1] developed a graphical interface to assist in using linear programming techniques for solving linear scheduling problems. Mattila [2] presented an integer programming model for leveling the resources of activities in a repetitive project that was scheduled using the linear scheduling method.

Furthermore, some significant studies have been performed using dynamic programming. Selinger [3] employed a dynamic programming approach to minimize project duration, and it is the first work to adopt dynamic programming approach. More research have been done by applying dynamic programming to minimize either the total cost or the project duration by integrating cost, time categories, or heuristic rules [4,5,6,7].

Given rapid development of computer techniques, more researchers have solved the increasing complexity of construction projects by using artificial intelligence techniques, such as genetic algorithm. For example, Leu and Hwang [8] addressed a GA-based resource-constrained repetitive scheduling model.

To enhance the efficiency and accuracy of the solutions of scheduling problems, constraint programming (CP) was recently applied as a new technique for handling such problems. El-Bibany [9] described the computational basis of a constraint-management methodology as applied to planning and scheduling. Moreover, Lottaz et al. [10] constructed constraint-based support for collaboration in design and construction, and provided a simple implementation of least commitment decision strategies. Additionally, Chan and Hu [11] applied CP to production scheduling for precast plants, and implemented several problem-derived constraints to improve resource utilization. However, literature review demonstrated that the CP method has not been adopted to repetitive scheduling problems for optimization purposes.

This study develops a optimization model for repetitive projects, for example, minimizing project total cost. Furthermore, the concept of outsourcing resources deriving from temporary subcontracting is also introduced, and is integrated with the proposed model for implementing in current construction.

To assess model feasibility, an example project from El-Rayes [4] is used to illustrate model capability. Based on the optimized results from the example project, related case is analyzed and discussed. Through cost study, the importance of outsourcing resources is then recognized.

2. CONSTRAINT PROGRAMMING

Constraint programming (CP) incorporates the techniques from mathematics, artificial intelligence and operations research [12]. The main core idea of CP is to solve the constraint satisfaction problems (CSPs), and CP utilizes suitable heuristic algorithms to rapidly determine the solutions. By declaring clean variables and constraints, a simple framework of problems is presented and relevant solution fields are identified. Compared with mathematical programming, CP can search for solutions more simply depending on the algorithm chosen by users, and CP is not restricted by any particular type of model formulation, such as linear equations. In other words, CP programs can be simply adapted to changing requirements. Furthermore, program efficiency and flexibility strengthens the competitive ability of CP since computer techniques are rapidly developing nowadays.

CP has been widely and successfully applied to handle complex combinatorial problems in different fields, including rostering, and scheduling problems [13], but has not been widely adopted by civil engineering researchers [11]. This study employs CP techniques to allocate resources required by a repetitive project for optimization purposes. By exploiting CP programs, relevant objectives and constraints can be simply modified to meet different requirements. Moreover, some logical and sequential constraints existing in the model for linear scheduling problems are more convenient to declare compared with other optimization methods such as mathematical programming. Consequently, the proposed model provides the capability to allocate resources, and finds the optimum solution in relation to the minimization of total cost.

3. OUTSOURCING RESOURCES

It is essential to prepare feasible resource plans for construction projects, depending on schedule plans and budgetary limitations. In practice, it is common to purchase the extra quantity of temporary resources to refine resource plans, to shorten project duration or possibly decrease total cost. In other words, the decision mentioned above is sometimes referred to as "project duration compression". Therefore, outsourcing actions such as temporary subcontracting are generally considered and executed to make up for deficiencies in the original resource plan. However, it is sometimes difficult to determine the exact resource quantity purchased, due to several complicated reasons such as the corresponding influences on project duration and cost. This study presents an optimization-based view for providing the information for making outsourcing decisions. Thus, outsourcing resources implemented in this study are defined as temporary additional resources required to fulfill managerial goals through shortening the duration of specific activities, and undoubtedly purchased at a higher than normal unit price. Moreover, once those resources are assigned to specific activities, they cannot be released until those activities finish.

The importance of outsourcing resource usage lies in enhancing the flexibility of project scheduling during the planning phases. Outsourcing actions can be executed by adding crew formations, construction equipment, or working hours per day, and the concept of using outsourcing resources is similar to project duration compression.

Outsourcing resources are usually used for critical activities to improve project performance. Generally, the utilization of more outsourcing resources means higher production rate. However, the main issues for planners making outsourcing decisions are the following questions: (1) which activities need outsourcing resources? (2) how many of outsourcing resources are needed? (3) when should outsourcing resources be needed and dismissed? Therefore, the concept of outsourcing resources is implemented in the proposed model via a set of decision variables for each activity to determine the timing of outsourcing resource assignation and the quantity of such resources to be assigned.

Essentially, quantities of outsourcing resources are limited by construction project environment. For instance, quantities of construction equipment are constrained by working areas of construction sites, and project budget limitation. Thus, setting a reasonable range for different outsourcing resources to comply with such constraints is an important part of successfully resource planning. This study defines the range of outsourcing resources as a parameter. For real practices, it should be assigned based on planner experience.

4. MODEL FORMULATION

The key goal of this study is to identify the optimum solution with respect to the objective of minimizing project total cost for a linear project. To fit the characteristics of linear scheduling problems, the constraints in constraint programming formulation are described in two parts, including activity and resource constraints, and total cost calculation are also illustrated.

However, several assumptions must be made before deriving the formulation, as follows:

1. The influence of learning behavior on crew formations for repetitive sections is ignored.
2. The production rate for outsourcing resources which is defined in this study is assumed to be identical for the same type of activities.
3. Given the upper limit of quantities of outsourcing resources, outsourcing resources remain available until the project ends.

Two types of constraints comprising activity precedence relationships and outsourcing resources are described , and the components of cost are listed as follows.

4.1 Activity Constraints

Four typical scheduling relationships and job continuity logic of repetitive activities are shown as the following equations. Additionally, the production rate associated with outsourcing resources is introduced in this section. The constraints of each type of activity relationship are, respectfully, as follows:

$$S_j^i \geqq F_j^{i-1} \quad (1a)$$

$$S_j^i \geqq S_j^{i-1} \quad (1b)$$

$$F_j^i \geqq F_j^{i-1} \quad (1c)$$

$$F_j^i \geqq S_j^{i-1} \quad (1d)$$

Where:
S_j^i = Start date of repetitive activity type i in section j;
F_j^i = Finish date of repetitive activity type i in section j.

For the same type of repetitive activities, in order to maintain job continuity, a successor activity can start only once its predecessor finishes. It is a basic constraint for repetitive project scheduling problems as follows:

$$S_j^i \geqq F_{j-1}^i \quad (2)$$

For each activity, the following precedence logic is used：

$$F_j^i \geqq S_j^i + D_j^i \quad (3)$$

Where:
D_j^i = Duration of repetitive activity type i in section j.

Depending on work quantity and production rate associated with outsourcing resource utilization, the duration for each activity is calculated:

$$D_j^i = Q_j^i \;/\; [\; P^i + (\; OR_j^i \times EP^i \;)\;] \quad (4)$$

Where:
Q_j^i = Quantity of work for repetitive activity type i in section j; P^i = Production rate of crew formation for repetitive activity type i; OR_j^i = Quantity of outsourcing resources per day added to crew formation for repetitive activity type i in section j, which is determined through the optimization process; EP^i = Unit production rate for outsourcing resource added to crew formation for repetitive activity type i.

Unit production rate for outsourcing resource (EP^i) is determined by the following equation:

$$EP^i = (\; P^i \;/\; CS^i \;) \times IFP^i \quad (5)$$

Where:
CS^i = Crew size of crew formation for repetitive activity type i;
IFP^i = Influence factor of unit production rate for outsourcing resource added to repetitive activity type i, which may be higher or lower than 1, depending on the evaluation for the productivity of outsourcing resource.

4.2 Resource Constraints

One topic investigated in this study is maintaining or accelerating project progress by allocating outsourcing resources to the crew formation for specific activities in order to adjust the production rate. However, the upper limit of the outsourcing resource should be reasonable and practical based on considerations of budget and outsourcing resource availability. Eqn. 6 shows the outsourcing resource constraints:

$$0 \leqq OR_j^i \leqq ORA^i \quad (6)$$

Where:
ORA^i = Upper limit per day of the quantity of outsourcing resource utilized for repetitive activity type i.

4.3 Project Total Cost

In this study, project total cost equals the sum of the direct and indirect costs, as illustrated in Eqn. 7. Direct cost comprises material, equipment, labor, and outsourcing resource costs, as shown in Eqn. 8, and indirect cost which is calculated on a daily basis is defined as the expression of Eqn. 9.

$$TC = DC + IC \quad (7)$$

$$DC = (\; MC + EC + LC + ORC \;) \quad (8a)$$

$$MC = \sum_1^i \sum_1^j Q_j^i \times MC^i \quad (8b)$$

$$EC = \sum_1^i \sum_1^j D_j^i \times EC^i \quad (8c)$$

$$LC = \sum_1^i \sum_1^j D_j^i \times LC^i \quad (8d)$$

$$IC = ICP \times T \quad (9)$$

$$ORC = \sum_1^i \sum_1^j OR_j^i \times ORC^i \times D_j^i \quad (10)$$

$$ORC^i = [\; \sum_1^j (\; LC^i + EC^i \;) \;/\; CS^i \;] \times IFC^i \quad (11)$$

Where:
TC = Total cost; IC = Indirect cost; DC = Total direct cost; MC = Total material cost; EC = Total equipment cost; LC = Total labor cost; ORC = The sum of outsourcing resource cost utilized through a repetitive project; MC^i = Unit material cost per cubic meter for repetitive activity type i ; EC^i = Unit labor cost per day with crew formation for repetitive activity type i ; LC^i = Unit equipment cost per day with crew formation for repetitive activity type i ; ICP = Indirect cost per day; T = Project duration; ORC^i = Unit cost for outsourcing resource utilized for repetitive activity type i, which may be higher than the normal contract price, since it is a temporary supplementary resource; IFC^i = Influence factor of unit cost for outsourcing resource added to repetitive activity type i, which may be higher or lower than 1, depending on the evaluation on the expense of outsourcing resource.

5. CASE STUDY

This study investigates one case to illustrate the utilization of the proposed model. The bridge example employed in previous studies is used in the following case. The original input data is shown in Table 1 and moreover, Table 2 displays other modified input data, including five types of repetitive activities, two non-repetitive activities, and the relevant crew information. Besides the repetitive activities, the example adds two activities, the ground improvements, which are considered non-repetitive activities. The ground improvement activities can start only after excavation activities are completed. Furthermore, both the cost and duration of non-repetitive activities are assigned, and no outsourcing resources are involved. The project schedule based on the modified example is shown in Table 3, and is compared with the results of the following optimized schedule.

Table 1. Basic input data

Repetitive Activity	Quantities in Cubic Meter			
	Section 1	Section 2	Section 3	Section 4
Excavation	1,147	1,434	994	1,529
Foundation	1,032	1,077	943	898
Columns	104	86	129	100
Beams	85	92	104	80
Slabs	0	138	114	145

5.1 Variable Definition

Several variables defined in this case study are designated, and related parameters are listed in Table 2. For each type of repetitive activity, several parameters based on planners' assessment are illustrated as follows:

1. Production rate regarding outsourcing resource (EP^{i}) should be reasonable, meaning that the values must be derived from planner evaluation. Referring to Eqn.5, EP^{i} is calculated based on influence factor of production rate (IFP^{i}), which is set to 0.6 in this study.
2. The limit of the outsourcing recourses for (ORA^{i}) is assigned depending on availability, and regarded as the resource available for utilization per day. Referring to Eqn. 6, ORA^{i} is shown in Table 2.
3. The unit cost of outsourcing resource for repetitive activity type i (ORC^{i}) is usually higher than normal price since the procurement behavior is temporary-based. The calculation of ORC^{i} is shown as Eqn.11, and influence factor of unit cost for outsourcing resource (IFC^{i}) is set to 1.2.

5.2 Model Objective

This case study attempts to optimize project total cost (Minimize TC) by using outsourcing resources, given an assigned duration. Similar to the concept of project duration compression in construction projects, the proposed model allows planners to refine a schedule plan for ensuring that a project finishes no latter than a desirable duration, given the information of the least total cost optimized in the model. Due to the need for crashing plans, the concept of outsourcing resources is introduced in the proposed model for solving such problems. Then the influence of outsourcing resources on project duration will be clearly demonstrated.

The desirable duration is set to 145 days in this scenario, and the objective and related constraint are shown as follows:

Objective: Minimize TC

Project Duration Constraint: T $\leqq$ 145 (days)

Finally, the model formulation generates 145 variables and 95 constraints to achieve its objective.

5.3 Optimized Results

After engaging outsourcing resources, the optimum solution and related information is shown in Table 4. Based on the result listed in Table 4, the optimum total cost is \$1,486,709, which is less than the total cost of original project schedule (\$1,487,370) in Table 3, and the total duration in this scenario is 143.0 days which is better than the desirable duration (145 days).

Table 2. Example data

Repetitive activity	Base Crew Data					Outsourcing Recourse Information		
	Crew size	Output (m^3/day)	Material cost (\$ / m^3)	Labor cost (\$ / day)	Equipment cost (\$ / day)	Limit	Unit Cost (\$)	Unit production rate per day (m^3)
Excavation	6	91.75	0	340	566	3	181	9.18
Foundation	6	53.86	92	1,902	436	3	468	5.39
Columns	10	5.73	479	1,875	285	5	259	0.34
Beams	4	5.66	195	1,850	148	2	599	0.85
Slabs	8	7.76	186	1,878	149	4	304	0.58

Non-Repetitive Activity

Activity	Total cost (\$)	Duration (day)	
Ground Improvement_1	12,000	10.5	No outsourcing resources involved
Ground Improvement_2	10,000	8.5	Project indirect cost: \$1,000 per day

Table 3. Original project schedule

Section	Activity											
	Excavation		Ground Improvement		Foundation		Columns		Beams		Slabs	
	Start	Finish	Start	Finish	Start	Finish	Start	Finish	Start	Finish	Start	Finish
1	0	12.5	–	–	12.5	31.6	31.6	49.7	49.7	64.7	–	–
2	12.5	28.1	28.1	38.6	38.6	58.5	58.5	73.5	73.5	89.7	89.7	107.4
3	28.1	38.9	38.9	47.4	58.5	76.0	76.0	98.5	98.5	116.3	116.3	130.9
4	38.9	55.5	–	–	76.0	92.6	98.5	115.9	116.3	130.4	130.9	**149.5**
Project Duration	**149.5 days**											
Total Cost	**$1,487,370**											

Table 4. Optimum project duration with outsourcing resources.

Section	Activity											
	Excavation		Ground Improvement		Foundation		Columns		Beams		Slabs	
	Start	Finish	Start	Finish	Start	Finish	Start	Finish	Start	Finish	Start	Finish
1	0	9.6	–	–	9.6	28.7	28.7	46.8	46.8	61.8	–	–
2	9.6	21.6	21.6	32.1	32.1	52.0	52.0	67.0	67.0	83.2	83.2	100.9
3	21.6	32.4	32.4	40.9	52.0	69.5	69.5	92.0	92.0	109.8	109.8	124.4
4	32.4	49.0	–	–	69.5	86.1	92.0	109.4	109.8	123.9	124.4	**143.0**
Project Duration	**143.0 days**											
Total Cost	**$1,486,709**											

Compared with the total cost of original schedule in Table 3, the optimum total cost of this scenario ($1,486,709) is less than that of original schedule ($1,487,370). Therefore, the benefits of outsourcing resources in this scenario lie not only in compressing the project duration, but also in simultaneously reducing the total cost. Even adding the expense of outsourcing resources ($11,728) shown in Table 5, the total cost is reduced by around $661 ($1,487,370 to $1,486,709) owing to project duration compression and reduced indirect costs. However, the cost reduction cannot be guaranteed since it depends on the difference between the outsourcing resource cost and the indirect cost.

The resource assignment schemes shown as Table 5 can help planners identify the critical actives for reducing project duration. Moreover, LSM Diagram (Figure 1) visualize the influence of outsourcing resources for project schedule. For example, the duration of the excavation activities in section 1 and 2 is needed to be shortened by utilizing outsourcing resources, in order to satisfy the objective of total cost minimization. Planners should pay attention to the influence on project duration due to project duration compression for those activities when rearranging project schedules.

8. CONCLUSION

This paper presents an optimization model for handling the linear scheduling problems involving resource assignment. The proposed model adopts constraint programming techniques and implements the concept of outsourcing resources for optimizing the total cost or duration of repetitive projects, even when several non-repetitive activities are involved.

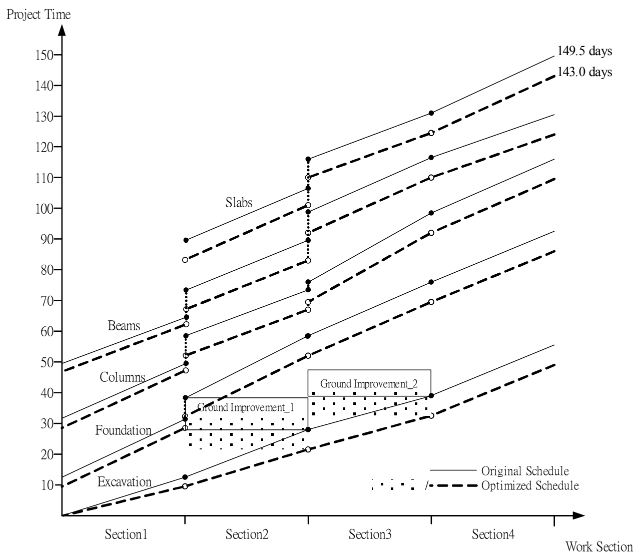

Figure 1. LSM diagram

This study uses an example bridge project to illustrate the capabilities of the proposed model. In case study, given the assigned project duration, the minimization of project total cost is conducted while considering outsourcing resources. The outcomes demonstrate that the model provides an efficient tool for solving these problems, such as project duration compression.

Table 5. Outsourcing resource assignment

Activity	Amount of Outsourcing Resource Usage				Outsourcing Resource Cost	Direct Cost
	Section 1	Section 2	Section 3	Section 4		
Excavation	**3**	**3**	0	0	**$11,728**	$56,122
Ground Improvement_1	–	–	–	–	–	$12,000
Ground Improvement_2	–	–	–	–	–	$10,000
Foundation	0	0	0	0	0	$534,307
Columns	0	0	0	0	0	$358,381
Beams	0	0	0	0	0	$195,883
Slabs	–	0	0	0	0	$177,016
Cost Information						
Total Direct Cost	$1,343,709					
Total Indirect Cost	$143,000 ($1,000/per day)					
Project Total Cost	**$1,486,709**					

Additionally, the concept of outsourcing resources provides users with an opportunity to review work plans and evaluate better options for schedule and resource plans, and also to make appropriate decisions regarding the optimization of either project total cost or duration.

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