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Design Optimization of Pin-Fin Shape to Enhance Heat Transfer

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Key words: response surface method, turbulent heat transfer, pin-fin arrays

Abstract

This work presents a numerical procedure to optimize the elliptic-shaped pin fin arrays to enhance turbulent heat transfer. The response surface method is used as an optimization technique with Reynolds-averaged Navier Stokes analysis of flow and heat transfer. Shear stress transport (SST) turbulence model is used as a turbulence closure. Computational results for average heat transfer rate show a reasonable agreement with the experimental data. Four variables including major axis length, minor axis length, pitch and the pin fin length nondimensionalized by duct height are chosen as design variables. The objective function is defined as a linear combination of heat transfer and friction-loss related terms with weighting factor. D-optimal design is used to reduce the data points, and, with only 28 points, reliable response surface is obtained. Optimum shapes of the pin-fin arrays have been obtained in the range from 0.0 to 0.1 of weighting factor.

1. Introduction

Finned surfaces are frequently used in heat exchanging devices for the purpose of increasing the heat transfer between a primary surface and the surrounding fluid by increasing surface area and inducing turbulent mixing of flow. In the past two decades, heat transfer and friction characteristics of pin fin array systems have been the subject of extensive investigation because of its importance in a wide variety of engineering applications, such as compact heat exchangers, the cooling of advanced gas turbine blades and electronic devices.

Recently, with the rapid development of computer capacity and numerical algorithms, RANS (Reynolds-averaged Navier–Stokes equations) analysis has become a practical method for the analysis of complicated turbulent flows and heat transfer. Among the methods of numerical optimization, response surface method [1], as a global optimization method, has many advantages over the gradient based methods. Recently, with these advantages, the response surface methods are being applied to many single and multidisciplinary optimization problems [2-5].

There have been many experimental investigations and the theoretical analysis on the heat transfer characteristics in pin fin arrays. Van Fossen [6] conducted an experiment to examine the pin fin length effect on the overall heat transfer coefficients in rectangular ducts with staggered short pin fin arrays ($0.5 < l/d < 2.0$). Metzger and Haley [7] experimentally compared the endwall heat transfer to the overall pin array heat transfer in rectangular ducts inserted with a staggered pin array. Metzger et al. [8] further studied the effects of using flattened pins and of varying the orientation of the pin fin array with respect to the main flow direction on the heat transfer and pressure drops in pin fin ducts. A numerical investigation of the flow through diamond shaped pin fin arrays is performed by

Grannis and Sparrow [9] using a finite element based solution method. A flow visualization study along with heat transfer measurements for in-line circular and square pin fins is conducted by Minakami et al. [10]. Li et al. [11] investigated elliptical pin fin arrays for Reynolds numbers between 900 and 9000.

In these literatures mentioned above, however, few researchers tried to find the optimal geometric parameters for better thermal performance of pin fin arrays. The optimal shape of finned ducts, considering wide ranges of geometric variables, has not yet been suggested. However, for the case of rib-roughened channels, a series of investigations had been carried out by Kim and Kim [4,5] found the optimum shapes of the rib roughened-channel by using a numerical optimization technique coupled with Reynolds averaged Navier Stokes analysis of turbulent flow and heat transfer. In addition, Kim and Choi [3] also tried to optimize shape for a dimpled channel in the same way.

In this work, a numerical optimization is carried out for the design of heat transfer surfaces of three-dimensional duct inserted with elliptic-shaped pin fin arrays.

2. Method of analysis

For the analysis of convective heat transfer in three-dimensional pin fin arrays duct, commercial CFD code, CFX-5.7, was used in our work. It solves the RANS equations with high resolution scheme for advection term and Rhie-Chew interpolation for the pressure velocity coupling.

To adopt periodic boundary conditions, modifications of source terms in streamwise momentum and energy equations have been made to calibrate the gradual decrease and increase of pressure and temperature, respectively. Finally, for three-dimensional steady incompressible flows, mass, momentum, and energy conservation equations in tensor form can be written as follows:

$$\frac{\partial U_i}{\partial x_i}=0$$

$$U_j\frac{\partial U_i}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\nu\frac{\partial U_i}{\partial x_j}\right)-\frac{1}{\rho}\frac{\partial \hat{p}}{\partial x_i}+\gamma\delta_{1i}$$

$$\frac{\partial}{\partial x_j}\left(\rho c_p U_j \hat{T}\right)=\frac{\partial}{\partial x_j}\left(k\frac{\partial \hat{T}}{\partial x_j}\right)-\sigma U_j\delta_{1j}$$

where $\hat{p}(x,y,z)$ and $\hat{T}(x,y,z)$ are the pressure and temperature transformed as follows in order to use the periodic boundary condition.

$$\hat{p}(x,y,z)=p(x,y,z)+\gamma x$$

$$\hat{T}(x,y,z)=T(x,y,z)-\sigma x$$

Here, γ is the pressure gradient along the streamwise direction, and σ is the rate of bulk temperature increase due to wall heat flux.

$$\sigma=\frac{q_0 A_d}{PiU_b A_{in}}$$

SST turbulence model with scalable wall functions [12] is used as a turbulence closure. Basically, SST model combines the advantages of the k-ε and k-ω models with turbulent heat transfer [13].

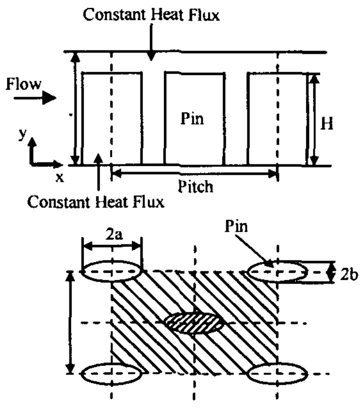

Fig. 1. Design variables and calculation domain.

The geometric parameters and computational domain are shown in Fig. 1. The computational domain is composed of one pitch of pin fin displacement with periodic boundary conditions on the surfaces normal to the streamwise direction, and also with symmetric conditions on the surfaces normal to the cross-streamwise direction. At the inlet and outlet of the domain, the periodic conditions are adopted. At the all wall boundaries, the wall function based on empirical wall law for near-wall turbulence is adopted for mean axial velocity. In Fig. 2, grid system is shown for whole computational domain and around the pin-fin.

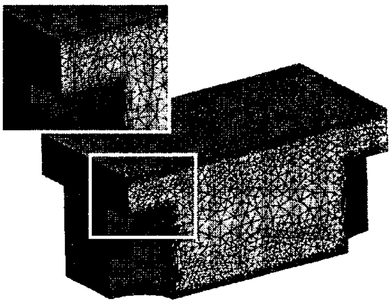

Fig. 2 Example of grid system

In the present study, unstructured tetrahedral grid system is used with the hexahedral at the wall region to resolve high velocity gradient. Bulk velocity corresponding to Reynolds number based on hydraulic diameter, 5000, and constant temperature are set at all computational nodes as initial values to help the faster convergence of the iterative calculation.

With the periodic conditions, it is difficult to assign specific flow rate to the calculated flow. Thus, an iterative procedure is inevitably employed. At the initial stage of the calculation, the pressure gradient in the streamwise direction is assumed through the source term in the streamwise momentum equation, and is continually updated until the Reynolds number reaches within 1% of the target Reynolds number.

3. Optimization techniques

In order to obtain an optimum shape of the pin fin arrays in three-dimensional duct, the response surface method (RSM) [1] as a numerical optimization is used in this work. RSM is an optimization technique which performs a series of experiments or numerical analyses, for a prescribed set of design points, and to construct a response surface of the measured quantity over the design space.

The optimization problem is defined as a minimization of an objective function F(x) with $x_i^l \le x_i \le x_i^u$, where x is a vector of design variables, and x_i^l and x_i^u are lower and upper bounds of each design variable, respectively.

The response surface is usually expressed as a second-order polynomial. The unknown polynomial coefficients are determined by least square method. Prescribed set of design points, so called training points, was selected by D-optimal design. When the second-order polynomial is used, the response surface is expressed as follows.

$$\eta = c_0 + \sum_{j=1}^{n} c_j x_j + \sum_{j=1}^{n} c_{jj} x_j^2 + \sum \sum_{i \neq j} c_{ij} x_i x_j$$

where η is the response function, x_i and x_j are the design variables, and c's are the unknown polynomial coefficients which are determined by least square method.

The efficiency of response surface method is verified by Shyy et al [14]. and Papila and Shyy [15] in their works of designing rocket engine injector, supersonic turbines, diffuser, and rocket engine component and supersonic turbines, respectively.

D-optimal design is a useful and reliable way of constructing response surface with a small number of design points which is only 1.5~2.5 times the number of polynomial coefficients.

The three-dimensional duct with pin fin arrays walls, as shown in Fig. 1 with six geometric variables; height of the duct (D), length of the pin fin (H), major axis length (2a), minor axis length (2b), pitch (Pi), and width distance of two neighbored pin fins (W). In the present optimization, W is set to be Pi/2.0, and height of the duct is kept as the 12.75mm. Thus, there are four dimensionless variables; H/D, 2a/D, 2b/D and Pi/D. In the present optimization, these four dimensionless geometric variables are selected as design variables.

To maximize the performance of the pin-fin arrays, the optimum shape should be determined by compromising between the enhancement of heat transfer and reduction of friction loss. Average Nusselt number and 1/3 power of friction factor became indexes for representing the thermal performance, as has been used in many experimental works, and also in the present optimization. These two objectives are combined with a weight factor which is frequently adopted in multi objective optimizations. On this purpose, the objective function defined as follows is minimized in the optimization process.

$$F=F_{Nu}+\beta F_f$$

where weighting factor, β is adjusted to the purpose of the designer.

The heat-transfer related term on the right-hand side is defined as an inverse of Nusselt number.

$$F_{Nu}=\frac{1}{Nu_a}$$

$$Nu_a=\frac{\int_A \frac{Nu}{Nu_s}dA}{A}, \quad Nu_s=0.023Re^{0.8}Pr^{0.4}$$

Nu is calculated on the finned surfaces, and Nu_s is the Nusselt number obtained from the Dittus-Boelter correlation, which is for the fully developed turbulent flows in a smooth pipe, and the integration is performed over the heated surface (A).

The friction-loss related term in equation (2) is defined as follows.

$$F_f=\left(\frac{f}{f_0}\right)^{1/3}$$

$$f=\frac{\Delta p D_h}{2\rho \bar{U}^2 L}, \quad f_0=2(2.236\ln \mathrm{Re}-4.639)^{-2}$$

where f_0 is a friction factor for fully developed flow in a smooth pipe, and is obtained from Petukhov empirical correlation [16] which is modified from the Karman-Nikuradse correlation for the best fit in the range, $10^4<\mathrm{Re}<10^6$.

4. Results and discussion

For the validation of present numerical solution, the results for the distribution of average Nusselt number are validated based on the experimental data of Li et al. [11] at Reynolds number, 5,000, where heights of the duct (D), major axis length (2a), minor axis length (2b) and pitch (Pi) are 12.75, 16, 9 and 60 mm, respectively, with W/Pi = 0.5 and H/D =1.

In order to determine the optimum number of grids, the numerical solution of the grid dependency test is shown in Fig. 3. Four different numbers of grids are tested in the case of staggered arrays of pin fin for the three-dimensional duct with the Re=5,000.

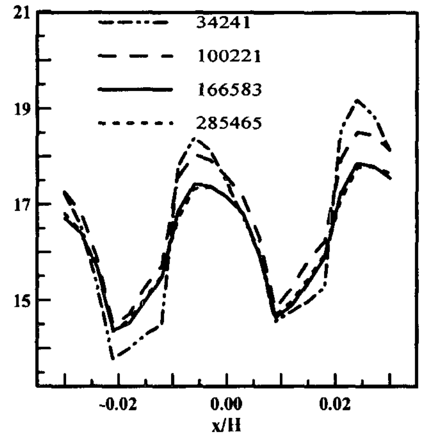

Fig. 3. Grid dependency test (2a/D=1.255, 2b/D=0.71, Pi/D=4.706, H/D=1 and Re = 5,000).

From the results, Grid 3 with 1.67×10^5 grids is selected as an optimum one.

Fig. 4 shows the results of the validation of numerical solutions. For the wide range of Reynolds numbers, pin fin averaged Nusselt numbers normalized by Nusselt number obtained from empirical relation for the fully developed turbulent flow in a smooth pipe are in good agreements with the experimental data.

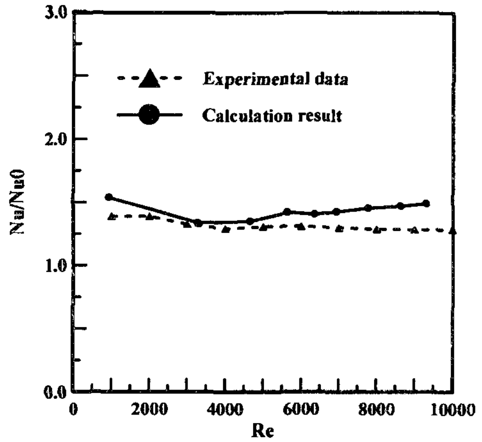

Fig. 4. Comparison of calculation and experiment averaged Nusselt number (2a/D=1.255, 2b/D=0.71, Pi/D=4.706, H/D=1 and Re = 5,000).

As the flow conditions for the present optimization, Reynolds number based on hydraulic diameter and 25□ air is 5,000. Uniform heat flux is imposed on both walls is 600W/m. For the optimization, response surface based optimization is used and the 28 training points are selected by D-optimal design to construct the response surface. Ranges of each design variable are listed in Table 1.

Table 1 Design variables and design spaces

Design variable	Lower limit	Upper limit
2a/D	0.7	1.25
2b/D	0.4	0.7
Pi/D	2.2	4.7
H/D	0.69	1

To measure uncertainty in the set of coefficients in a polynomial, analysis of variance (ANOVA) and regression analysis provided by t-statistic [1] is used and the results are shown in Table 2.

Table 2 Results of ANOVA and regression analysis

weight	R	R Square	Adjusted R Square	Std. Error of the Estimate
0.02	0.985	0.971	0.940	0.019293962

Results of optimization for β= 0.02 are shown in Table 3 to compare with the reference case. The optimum shape shows the good performances where average Nusselt number is improved by 43%, and friction-loss related term decreases by 28%. Finally, the objective function is reduced by 16% of the value of reference shape.

Table 3 Results of optimization for $\beta = 0.02$

	Design variable				Nu_a	F_f	F
	2a/H	2b/H	Pitch/H	Height/H			
Reference	1.255	0.71	4.706	1	23.641	3.7775	0.1178487
optimum	0.94	0.43	4.314	0.69	33.8535	2.71361	0.0838112

Fig. 5 shows the comparison of optimized Nusselt number distribution with that of reference case. With the considerable enhancement observed, it means that the overall average Nusselt number is increased.

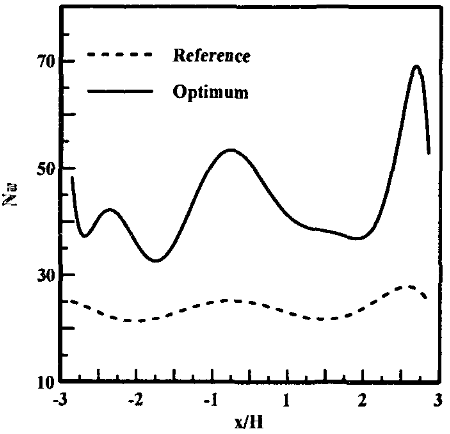

Fig. 5. Optimized Nusselt number distribution for $\beta = 0.02$

5. Conclusions

Shape optimization of pin-fin arrays inserted in three-dimensional duct is optimized to enhance heat transfer using the response surface-based optimization method coupled with Reynolds-averaged Navier Stokes analyses. Average Nusselt number calculated shows reasonable agreement with experimental data for several different Reynolds numbers, enough to be used in the optimization process. Twenty-eight training points selected by D-optimal design for four design variables construct a reliable response surface. In short, the Numerical optimization using response surface method combined with RANS analysis provides quite reliable and economic way of designing heat-transfer duct with finned surfaces.

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