Denoising and Deblurring Images Using Backward Solution of Nonlinear Wave Equation

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ABSTACT

In this paper, we introduce the backward solution of nonlinear wave equation for denoising. The PDE method is approved about 4 PSNR value compare with any convolution method. In neuro images, denoising process using proposed PDE is good about 0.2% increased Voxel Region

Keyword: Nonlinear Wave Equation, Backward Solution

I. Introduction

Denosing is the process with which we reconstruct a signal from a noisy one. We know kernel estimators or spline estimators do not resolve local structures well enough. This is necessary when dealing with signals that contain structures of different scales and amplitudes such as neurophysiological signals. When Fourier based signal processing, we arrange our signals such that the signals and any noise overlap as little as possible in the frequency domain. But this linear filtering approach cannot separate noise from signal where their Fourier spectra overlap. The noise overlap as little as possible in the frequency domain and linear time-invariant filtering will approximately separate them. As a non-linear method, this idea is to have the amplitude, rather than the location of the spectra is as different as possible for that of the noise. This allows shrinking of the amplitude of the transform to separate signals or remove noise. One such traditional problem in image restoration is that of reconstructing a noisy image, in which the resultant image should faithfully represent the original. Some nonlinear past efforts can be found in [1-4]. Here the motivation is to apply a fast and efficient method, based on nonlinear variational formulation and PDEs, to perform denoising in a singe large time step. Let us assume that during an image acquisition stage, the original image is noised by a known point-spread function (PSF). The image degradation model is of the form

f(x; y) = (d * u)(x; y) + n(x; y)

where u(x; y) is the desired original image, d is the known PSF, denotes the two-dimensional convolution, f is the observed degraded image, and n denotes the additive noise that is present in that image. By advancing the following PDE to its steady-state solution

$$\frac{\partial u}{\partial t} = \nabla \cdot (\psi t(|\nabla u|^2)\nabla u + \alpha d * (d * u - f)$$

[5].

In this paper, we introduce the backward solution of nonlinear partial differential equation for denosing as the nonlinear Klein Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + V_u(u) = f \qquad (1)$$

where Δ is the Laplacian operator in R^2, $V_u(u)$ is the derivative of the "Newtonian potential function" V, and f is a source term independent of the solution u, in various areas of optical physics refer to [6]

II. Backward Solution

A beam propagates and takes an image through the lens. In this case, we consider the image is confused by any other rays. That is, this image is not original objects. We can use backward solution of nonlinear wave equation as (1). As a backward solution, for any time t we compute a given equation then we get a solution t=0 as in Figure.1.

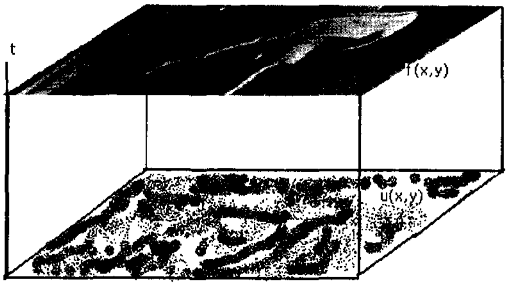


Figure.1. At t=0 as bottom is backward solution.

We reset the equation (1) as

$$V_u(u) = du * nu * u = dnu^3 \qquad (2)$$

where d is blurring factor and n is noising factor. Then we get from (1)

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + dnu^3 = f \qquad (3)$$

By using Lagrange polynomial interpolation such that let $I(x)$ be an interpolation function with n-node x_i of an arbitrary function $f(x)$, if we need an interpolation for $f(x)^3$, then we shall use $f(x_i)^3 I_i(x)$ instead of $I(x)^3$ where $I(x) = \sum_i f(x_i) I_i(x)$, we can get the numerical solutions where $l_i(x)$ is a N- degree Lagrange polynomial with $N+1$ nodes as

$-1 = x_0 < x_1 < x_2 < \ldots < x_N = 1$ We can get

$$u^N(x,t) = \sum_{i=0}^{N} a_i(t) l_i(x)$$

into

$$\frac{d^2 a_i(t)}{dt^2} - (l_0''(x_i)a_0(t) + \ldots + l_N''(x_i)a_N(t)) + dna_i(t)^3 = f(x_i, t)$$

$i=0,1,2,\ldots,N.$

As for stability, we would like to approximate the solution of the following problem

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}+|u|^{\alpha}u=f$$

The solution $u^N(x,t)$ of the Lagrange approximation of this problem is for all t > 0 a polynomial of degree N in x. , which is zero at $x=\pm 1$ and satisfies the equations, we obtain

$$(1/2)\frac{d}{dt}\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)}+(1/2)\frac{d}{dt}\|u_x^N(t)\|^2+(1/p)\beta\frac{d}{dt}\|u^N(t)\|^N_{L^p(-1,1)}$$
$$\leq (1/2)\|f(t)\|^2_{L_w(-1,1)}+(1/2)\|P_{N-2}u_t^N(x)\|^2_{L_w(-1,1)}$$

$$\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)}+\|u_x^N(t)\|^2+(2/p\beta)\|u^N(t)\|^N_{L^p(-1,1)}$$

$$\leq\|P_{N-2}u_t^N(0)\|^2_{L_w(-1,1)}+\|u_x^N(0)\|^2+(2/p\beta)\|u^N(0)\|^N_{L^p(-1,1)}$$
$$+\int_0^t\|f(s)\|^2_{L_w(-1,1)}\,ds+\int_0^t\|P_{N-2}u_t^N(s)\|^2\,ds$$

Applying Gronwall's inequality we complete the proof.
This theorem shows the stability of the approximate solution of u^N for

$$0=\int_{-1}^{1}(u^N(x,0)-u_0(x))u^N_{0_{xx}}dx=-\int_{-1}^{1}(u_x^N(x,0)-u_{0_x}(x))u^N_{0_x}dx$$
$$\int_{-1}^{1}u_x^N(x,0)u^N_{0_x}dx=\int_{-1}^{1}u_{0_x}(x)u_{0_x}(x)u^N_{0_x}dx$$

$$\leq c\int_{-1}^{1}u_{0_x}(x)u_{0_x}(x)dx\leq c\|u_0\|^2_{H_0^1(\Omega)}$$

As for convergence, we may assume that $m>2$.
Let

$$M=CN^{1-m}\|u_{tt}\|_{H^{m-2}(-1,1)}+CN^{1-m}\|u\|_{H^m(-1,1)}+CN^{-\frac{1}{2}}\||u|^{\alpha}u\|_{H^{m-2}(-1,1)}$$

clearly $M\to 0$ as $N\to\infty$.

$$\frac{1}{2}\frac{d}{dt}\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)}+\frac{1}{2}\frac{d}{dt}\|e_x(t)\|^2$$
$$\leq\frac{1}{2}M^2+\frac{1}{2}\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)}$$

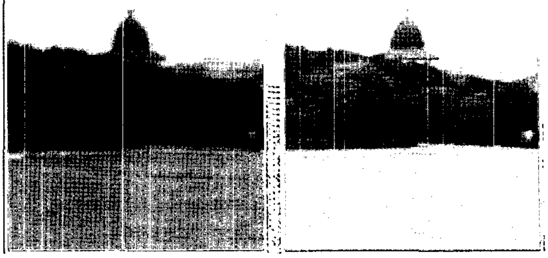
where P is a projection and e is an error term.

$$\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)}+\|e_x(t)\|^2$$
$$\leq\|P_{N-2}e_t(0)\|^2_{L_w(-1,1)}+\|e_x(0)\|^2+\int_0^t M^2ds+\int_0^t\|P_{N-2}e_t(x,s)\|^2_{L_w(-1,1)}ds$$

We know that $\|e_x(0)\|^2\leq c\|e_0\|^2_{H_0^2(\Omega)}$ and applying Gronwall's inequality we conclude the process.

III. Experimental Results

We can control the factor d or n, then the deblurring image and the denoising image are get out. In Fig.2, we show deblurred image when t=0.01 and 100th iterations, in this right image (a) we can see some trees and in (b) some hairs.



(a) We see some trees in church image

(b) We see some hairs.
Figure.2 Left is blurred image and right is deblurred image

If n is emphasized, we get denoised image as in Figure.3. The right image is denoised t=0.01 and 100th iterations..

Figure.3 Left is noised Image and right is denoised image

The PSNR is computed by using

$$PSNR=20\log_{10}^{(\frac{255}{\sigma})}$$

where σ is the root mean squared error. We show the PSNR of the denoised image from t=0.01 to t=0.001 with step 0.1 and 200th terations in Figure.4 and t=0.001 in Figure.5.

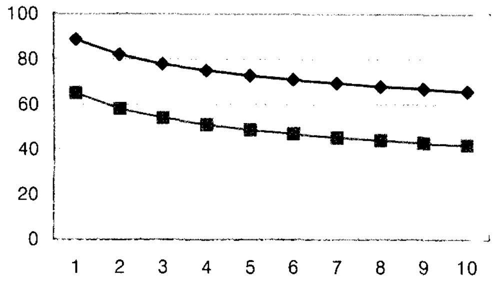


Figure.4. Upper line is convolution type denoised and lower line is PDE type denoised PSNR when t=0.01.

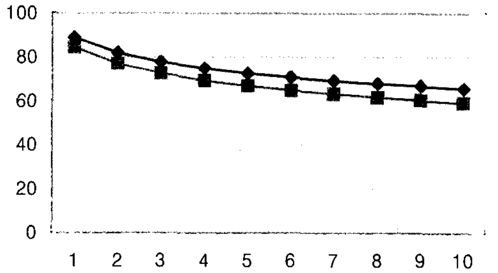
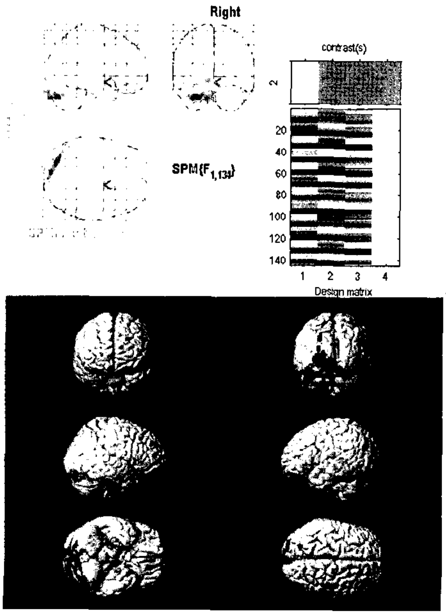
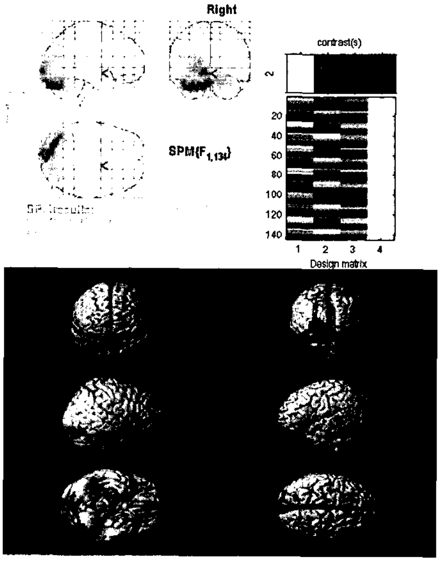


Figure.5. . Upper line is convolution type denoised and lower line is PDE type denoised PSNR when t=0.001.

We can apply this method to SPM for fMRI data, in Figure. 6 we show denoised neroimage, (a) is not denoised, (b) is denoised.



(a) Not denoised ICA-SPM image



(b) Denoised ICA-SPM image

Figure.6. Denoised image is good about 0.2% Voxel Region increased

IV. Conclusion

. In this research, the PDE method is approved about 4 PSNR value compare with any convolution method. In neuro image Denoising Process using proposed PDE is good about 0.2% increased Voxel Region

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