Integer Frequency Offset Estimation of OFDM Systems

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Abstract *- A blind-mode integer frequency offset estimation algorithm is proposed for an OFDM system. Imperfect integer frequency offset estimation causes ambiguity in the data sub-carrier position. Morelli's blind integer frequency offset estimation algorithm exploits the likelihood function by comparing the power in sub-carriers. It, however, shows performance degradation when there is the fractional frequency offset. The proposed algorithm solves this by using interpolation in the frequency domain. In this algorithm, it is exploited that the effect of the frequency offset is shown as a shift of power spectrum. By calculating the covariance of over-sampled samples, most approximate samples to integer point are obtained. It enables integer frequency offset estimation less affected by the fractional frequency offset.*

Keywords: OFDM, frequency offset estimation, blind estimation.

1 Introduction

Orthogonal frequency-division multiplexing (OFDM) systems have a crucial weakness in frequency offset conditions [1]. The carrier frequency offset can be divided into two parts; integer offset and fractional offset parts. For a very high data rate system, the distance between two adjacent sub-carriers is very narrow. Thereby, a small offset of the oscillator for demodulation can generate a large integer frequency offset. The ambiguity of data position in the frequency domain incapacitates next procedures such as channel estimation and error correction.

Most parameter estimators proposed for OFDM systems are based on the periodically transmitted pilot data [2, 3, 4]. Therefore, the data rate could be reduced. Without pilot information, the system requires longer time than that with pilots to estimate exact communication parameters such as timing synchronization, carrier synchronization, channel estimation and other signal processing procedures.

Morelli [5] proposed a blind estimation algorithm that is the most commonly used. It exploits the power differences between subcarriers. However, it has severe performance degradation when fractional frequency offset estimation is imperfect. This paper proposes an integer frequency offset estimation that is independent to the fractional frequency offset and overcomes the disadvantage of Morelli's algorithm.

2 Signal Modeling

A data vector C can be described with a vector of size $N \times 1$ with indices $[-N/2, N/2)$ such as

$$C = \begin{cases} c_k, & -N_u \le k \le N_u, k \ne 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where c_k is a constellation in QAM or M-PSK, and k is the frequency index. $2N_u$ data sub-carriers are assigned in an OFDM symbol. The time domain samples after the fast furrier transform (FFT) process can be expressed as

$$x(n) = \sum_{k=-N/2}^{N/2-1} c_k e^{\frac{j(2\pi k)n}{N}}, \quad 0 \le n < N. \tag{2}$$

In order to reduce multi-path fading, a guard interval with N_G-sample is added in ahead of the OFDM symbol. The last N_G samples of the inverse FFT (IFFT) are used as the guard interval samples. The total length of the OFDM symbol becomes $N_T = N + N_G$ samples. Considering integer and fractional frequency offset, the frequency offset ν is expressed as

$$\nu = m + \varepsilon \tag{3}$$

where m is an integer within the estimation range of $[-M, M)$, ε is a fraction within $[-0.5, 0.5)$. The frequency is normalized to the distance between adjacent subcarriers. The OFDM symbols are transmitted through a channel function of $h(n)$. Then, the received signal can be expressed as [6].

$$y_l(n) = \{x_l(n) \otimes h(n)\} \cdot e^{j\left[\frac{2\pi\nu(n+lN_T)}{N}+\theta\right]} + w_l(n) \tag{4}$$

where $\otimes$ means convolution, $w(n)$ is white noise and l is the symbol number.

The received symbol affected by the channel H_n, a frequency offset ν and noise is given by

$$Y_l(k) = \frac{e^{j\theta'}}{N} \sum_{n=-N/2}^{N/2} c_n H_n \sum_{i=-N/2}^{N/2} e^{j\left[\frac{2\pi(k-n-v)i}{N}\right]} + W_l(k)$$

$$= \frac{e^{j\theta'}}{N} \sum_{n=-N/2}^{N/2} c_n H_n IF\left(k-n-v\right) + W_l(k), \quad -N/2 < k \le N/2 \tag{5}$$

in the frequency domain, where $\theta' = \theta + 2\pi l N_T / N$, and the exponential term is denoted as an interference function (*IF*) such as

$$IF\left(k-n-v\right) = e^{-j\pi(k-n-v)\left(2-\frac{1}{N}\right)} \frac{\sin\{\pi(k-n-v)\}}{N\sin\left\{\frac{\pi}{N}(k-n-v)\right\}} \tag{6}$$

From (6), $Y_l(k)$ has the spectrum shifted by a frequency offset v from the original sub-carrier position. If v is zero, it indicates a sinc function that has peak value at each point of $k = n$. If the fine frequency offset exists, its effect is similar to that of the inter-symbol interference (ISI). In the case of the integer frequency offset, the data position is moved to a different sub-carrier index.

3 Conventional Integer Frequency Offset Estimation Algorithm

Blind algorithms for integer frequency offset estimation exploit the sub-carrier power level. Morelli's method [5] is the most representative method in blind estimation. It utilizes the likelihood method. When fractional frequency offset estimation is imperfect, Morelli's algorithm has severe performance degradation.

The Morelli's blind mode likelihood function for the unknown parameters m, θ and $\tilde{c}_{l,n}$, is given by

$$\Lambda\left(\tilde{m}, \tilde{c}_{l,n}, \tilde{\theta}\right) = \prod_{l=0}^{1} \exp\left\{-\frac{1}{\sigma_n^2}\left|\sum_{k=0}^{N-1} y_l(k) - e^{j[2\pi\tilde{m}(k+lN_T)/N+\tilde{\theta}]} \cdot x_l(k)\right|^2\right\} \tag{7}$$

In (7), two consecutive symbols are used for estimation. Because the probability functions of $\tilde{m}$, $\tilde{\theta}$ and $\tilde{c}_{n,l}$ are independent to each parameter in (7), $\tilde{\theta}$ can be dropped by taking an average with respect to $\tilde{\theta}$ [3,4], therefore it can be expressed as

$$\Lambda(\tilde{m}) = \sum_{l=0}^{1} \sum_{n=-N_u}^{N_u} \left|Y_l(n+\tilde{m})\right|^2 \tag{8}$$

In (8), $\Lambda(\tilde{m})$ is the subcarrier power corresponding to $\tilde{m}$.

Assuming the received signal has a fractional frequency offset, the likelihood function must consider the fractional frequency offset as an independent parameter. Hence, (7) can be expressed after applying the Taylor series as

$$\Lambda(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{l,n}) \approx 1 + \frac{2}{\sigma_n^2} X(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{l,n}) + \frac{2}{\sigma_n^4} X^2(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{l,n}) \tag{9}$$

with

$$X(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{l,n}) \equiv \sum_{l=0}^{1} \sum_{n=-N_u}^{N_u} \tilde{c}_{l,n}^{*} Y_l(k+\tilde{m}+\tilde{\varepsilon}) e^{-j\frac{2\pi\tilde{m}lN_T}{N}} \tag{10}$$

In order to acquire blind estimation from $\Lambda\left(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{l,n}\right)$, the data parameter $\tilde{c}_{l,n}$ must be eliminated by averaging (9) with respect to $\tilde{c}_{l,n}$. When $l = 0$, (10) can be simplified as an inner product between data with assuming $\varepsilon = 0$ and a high signal-to-noise ratio (SNR) such as

$$X(\tilde{m}, \tilde{c}_{0,n}) = \sum_{n=-N_u}^{N_u} \tilde{c}_{0,n}^{*} \cdot c_{0,(n+\tilde{m})} \cdot H_n \tag{11}$$

If data have a quasi-random distribution, (11) has the maximum value when the exact $\tilde{m}$ is estimated. When the inter-carrier interference (ICI) is caused due to the frequency offset, data become dependent to other subcarriers' data from (6), and are not assumed to be quasi-random. In this case, (10) can be expressed as

$$X(\tilde{m}, \tilde{\varepsilon}, \tilde{c}_{0,n}) \equiv \sum_{n=-N_u}^{N_u} \tilde{c}_{0,n}^{*} Y_0(n+\tilde{m}+\tilde{\varepsilon}) \tag{12}$$

Substituting (12) for (5) with $\varepsilon \neq 0$ and a high SNR, data in Y_0 are interfered by other sub-carriers. Therefore, the orthogonality is damaged severely as it has a large frequency offset ε. Therefore, $\tilde{c}_{l,n}$ cannot be eliminated from (12) by taking an average with respect of $\tilde{c}_{l,n}$.

This approach is identically applied to the third term in (9). When considering that the fractional frequency offset ε impairs subcarrier orthogonality, it cannot be used before the fraction offset compensation in a blind mode.

4 Proposed Integer Frequency Offset Estimation Algorithm

The proposed algorithm uses oversampled samples in the frequency domain for estimation. The FFT result after zero padding can be expressed as

$$Y_l'(k) = \frac{1}{N} \sum_{n=-QN/2}^{QN/2-1} y'(n) e^{-j[2\pi nk/N]} + W_l(k)$$
$$= e^{j\theta'} \cdot \frac{1}{N} \sum_{n=-N/2}^{N/2-1} c_n H_n IF(k/Q - n - v) + W_l(k), \quad (13)$$
$$\theta' = \theta + 2\pi l N_T / N, -QN/2 \le k < QN/2$$

where $Y_l'(k)$ is Q times oversampled power spectrum of $Y_l(k)$. Therefore, data subcarriers are distributed every Q samples. When the channel is ideal and there is only a frequency offset of v, the spectrum will be shifted by the frequency offset v.

In order to solve the disadvantage of Morelli's algorithm, the proposed algorithm estimates the integer frequency offset by searching subcarrier positions at which the data are the closest to the original data. The criterion for sample selection is given by

$$\hat{i} = \arg \min_{0 \le i < Q} \{ \mathrm{cov}(A_i) \} \quad (14)$$

where

$$A_i = \{ a_r \mid a_r = Y_l'(-Q(N_u - M - r) + i) \} \quad (15)$$

with i and r are integers in $0 \le i < Q$ and $0 < r \le 2(N_u - M)$, respectively. Samples of A_i with the least covariance are used for estimation. After deciding A_i, (9) can be modified as

$$\Lambda(\hat{m}, \hat{\varepsilon}) = \sum_{l=0}^{1} \sum_{n=-N_u}^{N_u} \left| \sum_{k=-QN/2}^{QN/2} c_{l,n'} IF_l\left(\frac{k}{Q} - n - \hat{m} - \hat{\varepsilon} + \hat{i}\right) + W_l\left(\frac{k}{Q} - n - \tilde{m} - \tilde{\varepsilon} + \hat{i}\right) \right|^2 \quad (16)$$

By using closest samples to the original data subcarriers, the likelihood function of (16) can estimate the integer frequency offset which is independent to the fractional frequency offset.

The final estimation on integer frequency offset is given by

$$\hat{m} = \arg \max_{|\hat{m}| \le M} \{ \Lambda(\hat{m}, \hat{\varepsilon}) \}. \quad (17)$$

This estimation compares summation power of subcarriers after adjusting the sample position. Referring to (9), it can be rewritten as

$$\tilde{m} = \arg \max_{\tilde{m} \le \tilde{v}} \left\{ \sum_{l=0}^{1} \sum_{n=-N_u}^{N_u} \left| Y_l'\left(n + \tilde{m} + \hat{i}\right) \right|^2 \right\} \quad (18)$$

If we use a higher interpolation rate Q, it finally converges to the original data subcarrier which is independent to the fine frequency offset. By using a large Q, the estimation equation can be expressed as

$$\Lambda(\tilde{m}, \tilde{\varepsilon}) \approx \Lambda(\tilde{m})$$
$$= \sum_{l=0}^{1} \sum_{k=-N_u}^{N_u} \left| \sum_{n=-N/2}^{N/2} c_{l,n} IF_l\left(k - n - \tilde{m}\right) + W_l(k - n - \tilde{m}) \right|^2 \quad (19)$$

5 Simulation

The test system is targeted for the fully blind-mode operation. Therefore, only unknown data are transmitted. The considered system is shown in Table I.

Table 1. System parameters

FFT size (N)	256
Number of data subcarrier ($2N_u$)	212
Null subcarrier in guard band	44
Modulation	QPSK
Interpolation rate (Q)	4 or 8
Length of cyclic prefix	20
Frequency offset estimation range	$[-5, 5)$
Symbol number for estimation (l)	2

Additive white Gaussian noise (AWGN) and an exponential-decayed multi-path channel are considered. The multi-path channel has 15 paths with amplitudes P_i that have mean power of each path as

$$E\{P_i^2\} = \exp\{-i/3\}, i = \{0, 1, 2, \ldots 14\} \quad (20)$$

The s-curve of the proposed integer frequency offset estimation algorithm is shown in Fig. 2 under an ideal channel condition. The estimation value must be converged to an integer point, and the fractional part needs to be estimated by a separate method.

Figs. 3 and 4 illustrate the failure probability of the estimators; $\Pr\{\tilde{m} \ne m\}$. The Integer frequency offset estimation has only two cases; correct or wrong because data can be correctly detected only if data keep their subcarrier positions.

Fig. 3 indicates the failure probability tested under AWGN in terms of the frequency offset including the fractional part. Comparing with Morelli's algorithm, the proposed algorithm can estimate the frequency offset independent to the fractional part. The performance of Morelli's algorithm is severely degraded in the region of the fractional part, so it is inadequate for estimation when fractional frequency offset estimation is imperfect.

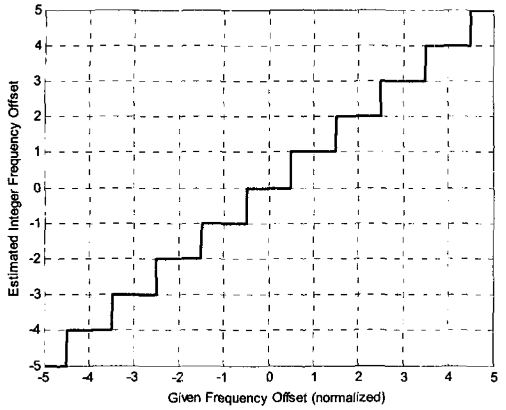

Figure 1. S-curve of the proposed algorithm.

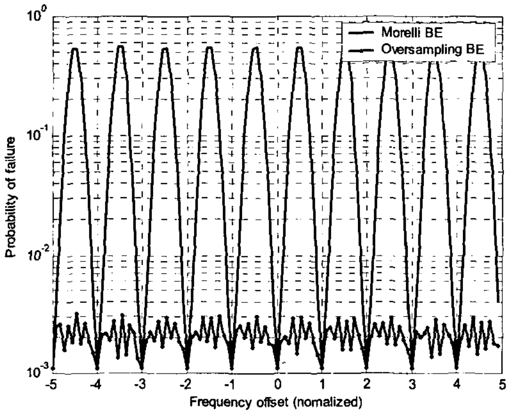

Figure 3. Estimation performance in estimation range in AWGN channel.

Fig. 4 shows the system performance in terms of the SNR with a frequency offset of 0.2. The proposed algorithm has about 3~5dB gain at a probability of failure of 0.09.

The performance improvement is obvious as the fractional offset increases. The Morelli's method does not work for large fractional frequency offsets. The proposed algorithm has a stable performance over its estimation range because the algorithm exploits samples closest to integer points. With the simulation results, we can get conclusion that the proposed algorithm is comparatively independent to fractional frequency offset estimation.

6 Conclusion

A blind-mode integer frequency offset estimation algorithm has been proposed for an OFDM system. An OFDM system has a weakness that transmitted data are easily damaged by the frequency offset. If it is failed to estimate the exact integer frequency offset, the whole data loaded in subcarriers become useless.

The proposed algorithm estimates the integer part frequency offset under the fractional part offset by interpolation in the frequency domain. It enables integer frequency offset estimation to be independent to the fractional frequency offset.

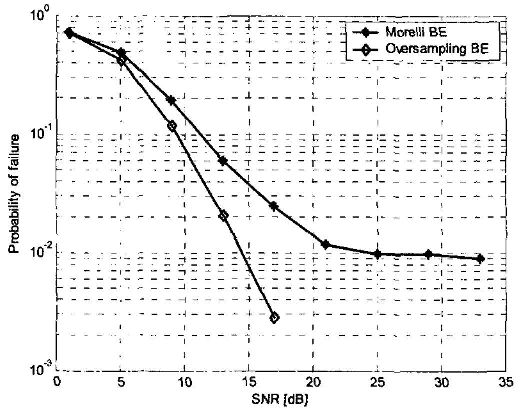

Figure 4. Probability of failure (exponential decayed channel, frequency offset = 0.2).

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