Analysis of the depth limitation for curved lens array system based on integral imaging

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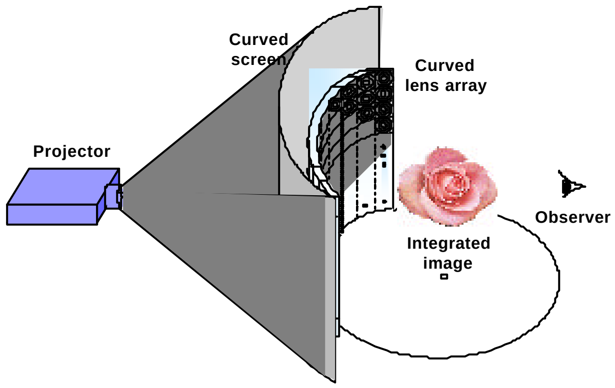
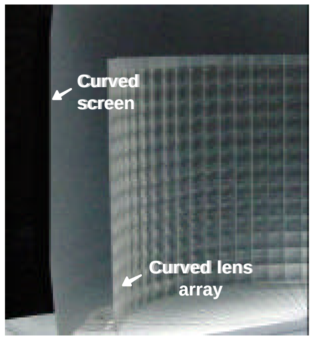
Abstract
Integral imaging attracts much attention as an autostereoscopic three-dimensional (3D) display technique for its many advantages. Recently the method that uses a curved lens array with a curved screen has been reported to overcome the limitation of viewing angle in integral imaging. This method widens the viewing angle remarkably. However, to understand the proposed system we need to know how the depth is limited in the proposed method also. We analyze the depth limitation and show the simulation results.

1. Introduction
Integral imaging (integral photography) is a three-dimensional display technique, first proposed by Lippmann in 1908. Recently the integral imaging became to attracts much attention as an autostereoscopic three-dimensional display technique for its many advantages. It has continuous viewpoints within the viewing angle and does not require any special glasses. It also provides full parallax and can display real time 3D animated images owing to the development of the display devices. However, in integral imaging the primary disadvantages are the limitation of viewing angle and the limitation of expressible depth.

To overcome the limitation of viewing angle some methods have been proposed. Among them the method that uses a curved lens array has been reported recently [1, 2]. This method widens the viewing angle considerably compared with the conventional method. Generally each elemental lens has its corresponding area, elemental image region on the display panel. To prevent the image flipping, the elemental image that exceeds the corresponding area is discarded. Therefore the number of the elemental image is limited. However, in the curved lens array system each elemental image does not exceed the corresponding area. It owes the curved structure and this characteristic widens the viewing angle. We reported previously the enhancement of the viewing angle. Figure 1 shows the configuration of the proposed method.



(a)



(b)

Figure 1. Curved lens array method with curved screen (a) configuration of the system (b) the experimental setup

A wide viewing angle is accomplished using the proposed method. However, for displaying 3D images with good quality, we also need to know how the depth is limited in the proposed method.

2. Limitation of depth
Generally integral imaging system uses a lens array composed of many elemental lenses. Thus there exists the central depth plane. The central depth plane is the focused plane determined by the focal length of elemental lenses and the gap between the lens array

and the display panel by using the lens law. Integral imaging reconstructs the original 3D object image exactly only at the central depth plane, and the image quality degrades as it goes away from the central depth plane. Therefore the thickness of the 3D image to be displayed cannot be large due to severe image quality degradation.

2.1 Image distortion

Previously, the causes of the depth limitation have been known as the diffraction of the waves. However, in the case that the elemental lens size is larger than 1mm, the effect of the diffraction is small and we cannot explain the image degradation using diffraction. Recently another method was proposed to analyze the depth limitation [3]. Figure 2 shows the concept of the analysis of the image distortion.

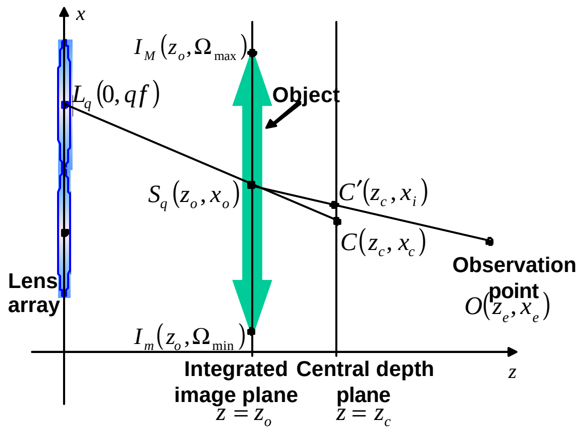


Figure 2. Analysis of the image distortion

Figure 2 shows the situation when the object is located out of central depth plane. The plane where the object is located is defined as the integrated image plane. In this case the observer recognizes the integrated image by observing focused image at the central depth plane. When the observation point is O, the observer see S_q through the point C that is on the intersection between the line $\overline{L_q S_q}$ and the central depth plane. However, the observer would see S_q through the point C' if the image were reconstructed as it is. The difference between C and C' is the error and this causes the distortion of image displayed out of central depth plane. In this analysis d is defined as the square of the difference between C and C', which is the representative parameter of the distortion.

2.2 Image distortion in curved case

In curved lens array case it is more advantageous to use large size elemental lenses for relaxing the limitation of lateral image size [4]. We can apply the above analysis to the curved lens array method. When it comes to the curved lens array case, the coordinate of each elemental lens is changed and each elemental lens has different central depth plane as shown in Fig. 3. The coordinate of *q-th* elemental lens, L_q, can be expressed as

$$L_q = (-R\cos(2qq), R\sin(2qq)), \qquad (1)$$

$$q = \arctan(\frac{j}{2R}), \qquad (2)$$

where R is the radius of curvature of the lens array and j is the pitch of elemental lens.

The central depth plane of each elemental lens is the focused plane. Since *q-th* elemental lens and the corresponding elemental image are inclined with the angle of $2qq$, the central depth plane of the *q-th* elemental lens is also on the line of

$$x = \cot(2qq)z. \qquad (3)$$

Figure 3 shows this situation in detail. The central depth planes are not planar but curved toward the observation point.

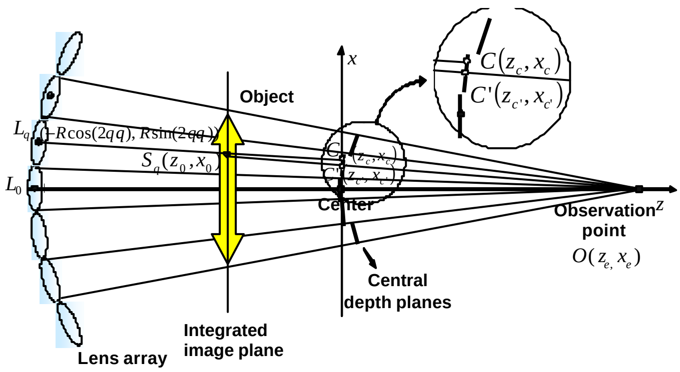


Figure 3. Analysis of the image distortion in curved lens array case

On the same principle of conventional depth analysis, when the object is located out of central depth plane, the observer recognizes the integrated image by observing focused image at the central depth plane. The observer looks at S_q by the focused point, C. However, if the image were reconstructed as it is, the observer would look at S_q by the point, C'. The difference between C and C' is the error and d is defined as the square of the difference. Thus we can calculate the numerical value of d according to each x point in central depth plane. As a result, d can be expressed as a function of x_c.

3. Simulation results

In simulation, square elemental lenses are used as the lens system. It has a width of 10 mm and focal length of 22 mm. The radius of curvature of curved lens array is 100 mm. The pixel pitch of the elemental image is 0.25 mm.

Figure 4 shows the simulation result in the curved lens array system. The distortion, d, is plotted versus x in central depth planes for different integrated image planes. The image size is 60 mm.

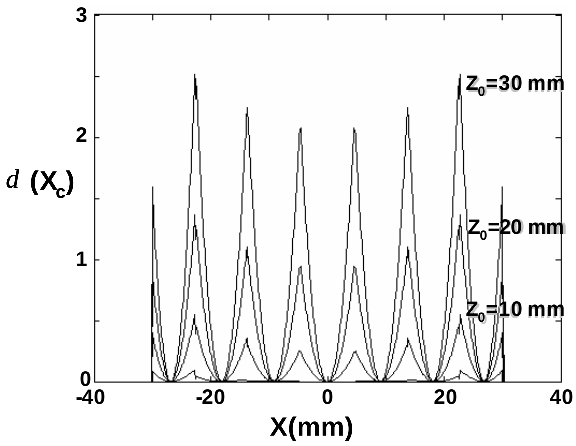


Figure 4. d **versus** x **for different integrated image planes** Z_0**(mm)**

As expected, the distortion gets larger when the integrated image plane becomes farther from the central depth region, the center. The point with no error appears periodically. In the conventional case the peak of the distortion $d(x_c)$ was constant. However, in the curved lens array case the peak is not constant. It is small around the center and increases as the image gets farther from center in the curved case. The peak of the distortion exists even when the image located at Z_0=0 mm.

It is because the central depth plane is curved and the image is planar as shown in Fig. 3. Thus the distortion occurs as the image is distant from the center point.

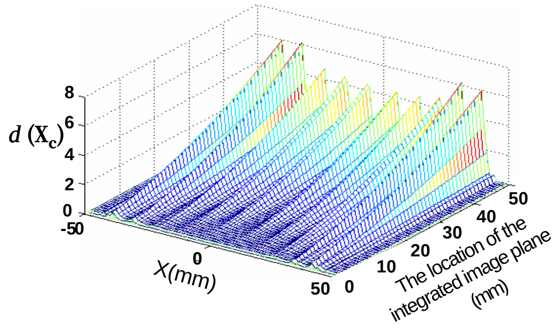
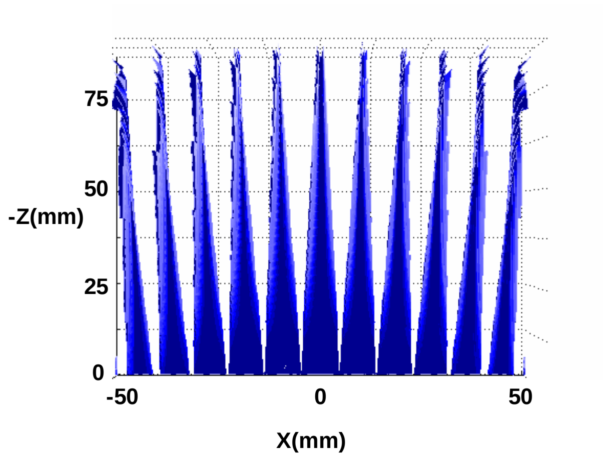


Figure 5. d **versus** x **and the location of the integrated image plane**

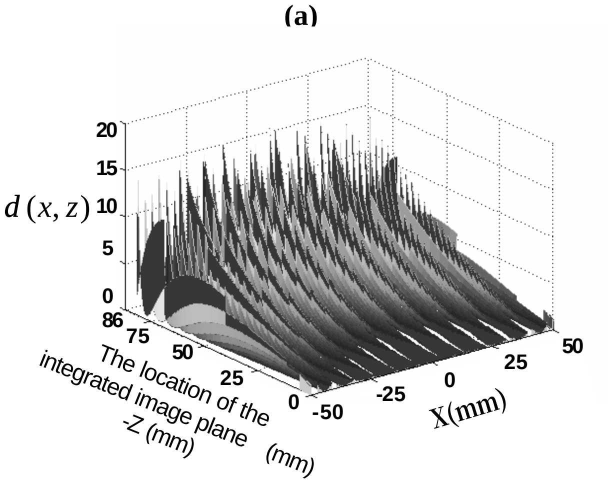
Figure 5 shows the increase of $d(x_c)$ continuously as the integrated image plane gets far from the plane, Z_0=0.

The image size is 100 mm. The peak is on the increase and the distortion is small around the center. However, the image that we want to display is not a planar 2D image, but a volumetric 3D image which has a depth. Thus the calculation of $d(x,z)$ on planar image is not enough to understand the distortion of 3D image and an expressible depth. We should expand the simulation of $d(x_c)$ on a space to understand how distortion occurs at each point in space. Figure 6 shows d on the space. We can see d according to the location of image point. Figure 6(a) is the top view of the d on the space. The regions of dark color indicate the little distortion and the color gets brighter as the distortion increase. There are regions of no color. Any image located in the region cannot be observed.

Figure 6(b) shows the 3D view of the $d(x,z)$ on the space. The distortion is small around the center and equi-distortion region is elliptical as shown in Fig. 6(b). If we set the durable limit of distortion as d_{limit} and decide the expressible depth region as the region of smaller d than the d_{limit}, the expressible depth region will have an elliptical shape. It owes to the curved structure of lens array and screen. The shape of depth region is the difference between the conventional flat lens array case and the curved lens array case. The discrete peaks are because of the finite pixel size.



(a)



(b)

Figure 6. Distortion d according to the location of the integrated image point (a) 2D view (b) 3D view

4. Conclusion

Utilizing the analysis on the depth limitation in conventional case, we explained the depth limitation in the curved lens array case and analyzed the image distortion numerically. As the simulation results show, the curved lens array system has the curved central depth planes and the distortion is minimum around the center. Thus equi-distortion region has an elliptical shape. Knowledge of the image distortion according to the location of the integrated image plane, and also to the image point on the space will help us to display wide-viewing 3D images in curved lens array system with minimized error.

5. Acknowledgement

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