Digital Holography for 3D Color Display of Real Objects incoherently illuminated

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Abstract

The proposed method is based on extracting information from 3-D Fourier spectra calculated from some projection incoherent images. Three colored computer-generated holograms (CGHs) are synthesized from 3-D Fourier spectra. Optically reconstructed full-color images are presented.

1. Introduction

In many hologram applications, CGH is one of the most important among them because it can reconstruct virtual objects and modulate wavefronts arbitrarily. Various kinds of CGHs have been reported in order to create virtual three-dimensional objects. There are some methods that make use of projection processes for wide viewing-angles. These methods require calculations of the projections for virtual objects. Li *et al*[1] and Sando *et al*[2] proposed new methods for creating real existing objects, as opposed to virtual objects, using projection processes and computational processing. In theses methods, CGHs are synthesized from several projection images recorded with white light. The absence of a coherent light source in these methods makes them suitable for CGHs of real existing objects. The superposition of three color CGHs corresponding to Red, Green and Blue components can reconstruct full-color 3-D objects[3-5].

In this paper, we describe a new method for the 3-D reconstruction of full-color real existing objects based on the previous report. In addition, methods for deciding certain parameters such as angular ranges, angular increments and projection numbers are presented for the purposes of making the magnifications constant for each color. Verification of this method by both numerical and optical reconstruction is also presented.

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1. Theory of Method

The method proposed here mainly consists of three processes. In the first process, projection images are recorded (in this report, synthesized using a computer for ease) with a color CCD scanning two dimensionally and parts of the 3-D Fourier spectrum of objects are calculated in accordance with the principle of CT[2]. The Fourier components required to generate a CGH are extracted from the 3-D Fourier spectrum in the second process. Finally, a Fresnel hologram is synthesized from it directly.

2.1 Calculation of 3-D Fourier Spectrum

The proposed method is based on the relationship between one projection image recorded by a CCD camera and its Fourier plane, similar to the principle of computer tomography. According to the principle of 3-D CT, calculating the 2-D Fourier transform of the projection image $O_2(x_{ij}, y_{ij})$ of a 3-D object $O_1(x_{ij}, y_{ij}, z_{ij})$, as shown in Fig. 1, in which the coordinates are defined in the projection plane at an angle (θ_i, ϕ_j) gives a section of the 3-D Fourier spectra;
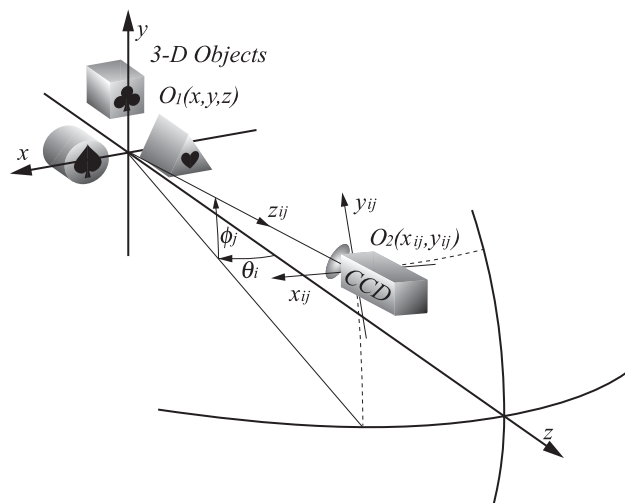

Figure 1 Recording system for projection images by 2-D color CCD camera.

$$O_3(u_{ij}, v_{ij}) = \iint O_2(x_{ij}, y_{ij})$$
$$\times \exp[-i2\pi(u_{ij}x_{ij} + v_{ij}y_{ij})]dx_{ij}dy_{ij}$$

(1)

where (u_{ij}, v_{ij}) is the coordinates for the 2-D sectional plane in the Fourier domain.

2.2 Extraction of Required Components

It should be noted that the coordinates (x_{ij}, y_{ij}) have the following relation with the coordinates $(x, y.z)$ in the 3-D object space.

$$x_{ij} = x\cos\theta_i - z\sin\theta i$$
$$y_{ij} = -x\sin\theta_i \sin\phi_j + y\cos\phi_j - z\cos\theta_i \sin\phi_j$$

(2)

Using some approximations, we have the Fourier spectra corresponding to the projection

$$G_{\theta_i, \phi_j} = O_3(u_{ij} = 2\theta_i / \alpha\lambda, v_{ij} = 2\phi_j / \alpha\lambda)$$

$$= \iiint dxdydzO_1(x, y, z)$$

$$\times \exp\left\{-i\frac{4\pi}{\alpha\lambda}[\theta_i(1-\phi_j^2)x + \phi_j y - (\theta_i^2 + \phi_j^2)z]\right\}$$

(3)

where λ is the same wavelength, and α is the magnification of the z-axis.

2.3 Fresnel CGH Synthesis

In the final process, a Fresnel CGH is synthesized from the Fourier spectrum $G(u_0, v_0)$. Calculating a Fresnel diffraction directly from the Fourier spectrum is given by

$$h(x_0, y_0) = FourierTransform$$
$$\times \left\{G(u_0, v_0) \cdot \exp[-i\pi\lambda R(u_0^2 + v_0^2)]\right\}$$

(4)

2. Analysis of the Method

3.1 Extraction of components

Here, we illustrate the extraction represented by Eq.(3) in the 3-D Fourier space (u, v, w). The diagram is shown in Fig. 2.

(a)

(b)
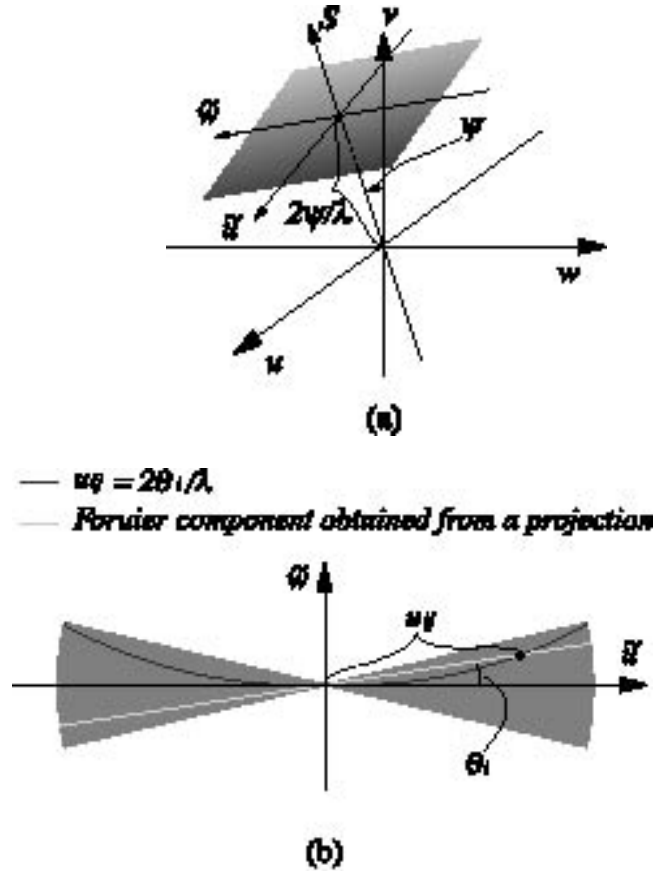
Figure 2 Diagram of the component extraction: (a) in 3-D Fourier space and (b) on the ψ plane.

However, it is very difficult to illustrate a 3-D diagram. This time, by considering the case in which the angle ϕ_j is fixed to $\phi_j = \psi$ and only the angle θ_i is changed, the extraction only by $u_{ij} = 2\theta_i\lambda$ on the sectional plane (ψ plane) is illustrated two-dimensionally in Fig. 2(b). The ψ plane is defined in Fig. 2(a). In Fig. 2(a), the vector S is inclined by ψ to v axis on the $v - w$ plane. The ψ plane is the one orthogonal to the vector S and positioned at $2\psi\lambda$ from the origin. According to the 3-D CT, the 2-D Fourier transform of a projection image gives one of the sectional planes in the 3-D Fourier space and the line crossed between the ψ plane and the sectional Fourier plane is represented by light gray line in

Fig.2(b). The gray area shows the locus of the crossed line by the change of θ_i while Eq.(3) is effective. The dark gray curve represents the essential components for synthesizing a CGH and only intersections between the dark and light gray lines are necessary and extracted. Iterating this procedure for each ϕ_j two-dimensionally makes it possible to obtain the full distribution given by Eq.(3). This means only components at the curved surface of the 3-D Fourier space are required to make a CGH.

3.2 Magnification

We discuss the case in which the value of α is not 1 in Eq.(3). Performing variable conversion by $z' = \alpha z$ gives another representation of Eq.(3) as follows:

$$G_{\theta_i,\phi_j} = \frac{1}{\alpha} \iiint dxdydz' O_1(x,y,\frac{z'}{\alpha})$$
$$\times \exp\left\{ -i\frac{i2\pi}{\lambda}[\frac{u'x+v'y}{f} - \frac{(u'^2+v'^2)z'}{2f^2}]\right\}$$

(5)

This equation means the complex amplitude at the Fourier plane in Fig. 1 for the objects magnified at α times in the z direction. So, the magnification in the z direction is α. It goes without saying the magnifications in the x and y directions are 1.

It is desired that the magnification in each direction is 1 or the same, but in this case it is difficult because of the resolution of projection images. Here, we explain how to determine the value of α in consideration of the descretization. It is assumed that the size of a projection image, the pixel number of it and the maximum projecion angles are $W \times W$, $N \times N$ and $\pm \theta_0$, $\pm \phi_0$, respectively. It is enough to consider about olny x axis. The maximum spatial frequecy in this condition is $N/2W$, which should correspond to the maximum of u_{ij}. Therefore, the following relation is formed and the value of α is determined.

$$\frac{N}{2W} = u_{ij\,max} = \frac{2\theta_0}{\alpha\lambda}$$
$$\alpha = \frac{4W\theta_0}{N\lambda}$$

(6)

3.3 Color Reconstruction

It is easy to extend our method to the full-color reconstruction if projection images are recorded with a color CCD. As can be seen from Eq.(1), a wavelength λ has no connection with a recording process and the principles of CT. Wavelengths are introduced by the extension of components in Eq.(3) for the first time. λ is a simple proportional factor. Therefore, if R, G and B components of a projection image are obtained separetely with a color CCD, it is easily able to synthesize three CGHs for R, G and B components by changing the wavelength used in the extraction process. It should be paid attention to that the magnification in the z direction depends on a wavelength.

4. Experiments
4.1 Computer Simulation

To simplify the recording process of projection images, they are generated in the computer. The size of each projected object is about 2.7mm x 2.7mm x 2.7mm and the marks Heart, Club and Spade are located at about z=-2.7mm, 0mm and 2.7mm, respectively. The size of each projection image is 1cm x 1cm and the pixel size of it is 256 x 256. Ar-ion (457.9nm), ND:YAG (532nm) and He-Ne (632.8nm) are used as a blue, green and red light source. Though the angular increment between two successive projection images is 1 degree for each wavelength, the angular ranges are different from each other in order to accord the magnification in the z direction (see Section 3.2). Therefore, the angular ranges of both θ_i and ϕ_j are ± 11.6, ± 13.5 and ± 16 for Ar-ion, ND:YAG and He-Ne lasers , respectively. Consequently, 33 x33 = 1089 projection images with color are generated in the computer and CGHs for R, G and B components are synthesized from them. The pixel size of each CGH is 1024x1024.
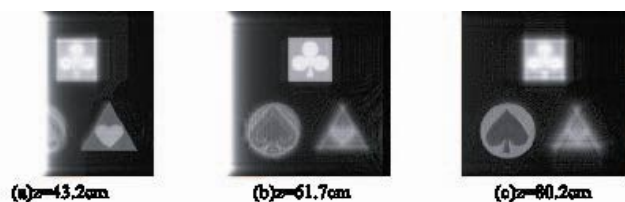
Figure 3 Computer simulation:reconstructed images from three CGHs.

Figure 3 shows the simulation results with three CGHs for reconstruction of the objects at each position. These images also seem gray-toned, but they are reconstructed with color actually. The diffraction length R and the inclined angle ω of the reference beam are 61.7cm and 0.928 degrees. The magnification in the z direction calculated from Eq.(6) is 69.0. So, it is assumed that three marks are reconstructed at z= 43.2cm, 61.7cm and 80.2cm. The marks Heart in Fig. 3 (a), Club in (b) and Spade in (c) are clearly in focus without blurs in any direction. And the same objects are focused at the same position for each wavelength. Because different marks are focused at different positions and there is no difference in the positions for reconstruction by the difference of the wavelengths, this method can reconstruct 3-D objects colorfully.

4.2   Optical Reconstruction

The reconstructed images from it are shown in Fig. 4, respectively. The optical system is set up to match the distances from each CGH to the color-CCD. In the same way as Fig. 3, the success of our method is verified from Fig. 4.

Figure 4 Experimental results of optical reconstructtion.

3.   Conclusion

In conclusion, we have proposed the method for synthesizing Fresnel CGHs from a series of projection images recorded along both horizontally and vertically. This method does not need a coherent light source and lenses in reconstruction and is impervious to vibration. This method essentially requires parts of 3-D Fourier spectra. Therefore, it can be easily combined with methods which have connection with 3-D Fourier spectra such as X-ray CT, ultrasonic CT, MRI imaging and so on. The diagrams for the extraction of essential components and the method for color reconstruction are also described. To verify our method, we have demonstrated the numerical and optical reconstruction including color process. The reconstructed images are clearly focused and have no blur in any direction and the 3-D reconstruction of objects is confirmed. This means that the method proposed here has the high potentiality for application to the wide fields of holographic cameras, printers, security and CG displays.

5.   References

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