Analysis on the viewing parameters of floating display system based on integral imaging

Joohwan Kim[1], Sung-Wook Min[2], and Byoungho Lee[1]
[1]School of Electrical Engineering, Seoul National University, Seoul 151-744, Korea
byoungho@snu.ac.kr
[2]Digital Media Laboratory, Information and Communications University, Seoul, Korea

Abstract

An analysis on the viewing parameters of floating display system based on integral imaging is introduced. A floating display system based on integral imaging is a three-dimensional display system which can display three-dimensional moving pictures with impressive feeling of depth. The analysis given in this paper will optimize the design of the floating display system for the specific application.

1. Introduction

Integral imaging (InIm), also called integral photography, is a three-dimensional image display system which was first proposed by Lippmann in 1908. InIm provides full parallax and continuous viewpoints and can be easily implemented using the lens array and two-dimensional image display devices such as LCD monitors. Elemental images, which have periodicity and contain three-dimensional information, are represented on the two-dimensional display device and integrated through the lens array. A number of researches are performed recently to analyze and enhance InIm scheme.

Floating display is a well-known three-dimensional display scheme which can be observed often in museums or magic shows. It uses one large convex lens or concave mirror to produce a three-dimensional image of an object in a position near to the observer. Three-dimensional image produced by floating display gives great feeling of depth to the observer.

Floating display system based on InIm[1] is proposed to display three-dimensional moving pictures with the aid of InIm and give great feeling of depth to the observer by the virtue of floating display system. However, there is no theoretical analysis which is necessary for the design of the floating display system based on InIm. The objective of this paper is to give an analysis on the viewing parameters of floating display system based on InIm.

2. Analysis

Analyzed viewing parameters of the floating display system based on InIm are viewing angle, expressible depth range and the window. Among these attributes, viewing angle and expressible depth range are mainly determined by InIm, which itself has limitation on viewing angle and expressible depth range[2,3]. Therefore, viewing angle and expressible depth range are analyzed focusing on the relationship between viewing parameters of InIm and floating display system.

In InIm system, the observer out of the viewing angle can see repeated images. Repeated images are produced where the elemental images are integrated through the wrong elemental lenses. In floating display system based on InIm, there exists a window through which repeated images are not observable. Designers can prevent the observer from seeing repeated images by concealing the whole system but the window. The location and the size of the window are analyzed in terms of the viewing angle of the InIm and the focal length of the floating lens.

In this paper, the floating display method using a convex lens is analyzed. It can be applied to the concave mirror case as well.

2.1 Viewing angle

Before finding out the relationship between the viewing angles of InIm and floating display system based on InIm, it is necessary to analyze the relationship between the gradient of a ray incident on a convex lens and the gradient of the ray after it passes the lens. The angle of a ray can be easily calculated by taking the inverse of tangent to the gradient of the ray. Then the viewing angle of a floating display system can be calculated by using the viewing angle of an InIm system which is experimentally obtained.

Consider Fig. 1, where a ray is incident on a convex lens after passing $P_{o1}(y_{o1},z_{o1})$ and $P_{o2}(y_{o2},z_{o2})$. $P_{i1}(y_{i1},z_{i1})$ and $P_{i2}(y_{i2},z_{i2})$ are the respective images of $P_{o1}(y_{o1},z_{o1})$ and $P_{o2}(y_{o2},z_{o2})$ by the convex lens.

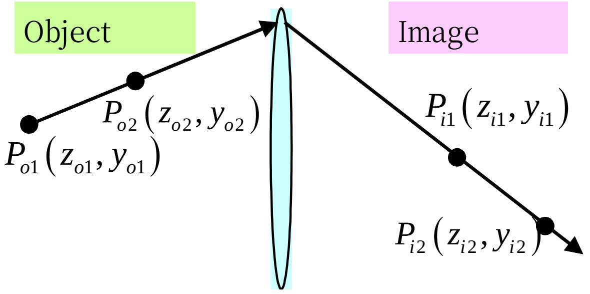


Figure 1. Geometry for viewing angle calculation

Assume Cartesian coordinates with its origin at the center of the floating lens. Coordinates in the object space which is left to the floating lens have negative *z* values, while coordinates in the image space which is right to the floating lens have positive *z* values.

According to the Gauss's lens law and considering the sign of *z*-coordinate value, P_{o1} and P_{i1} are related as in Eqs. (1) and (2),

$$\frac{1}{z_{i1}} = \frac{1}{z_{o1}} + \frac{1}{f}, \qquad (1)$$

$$y_{i1} = y_{o1}\frac{z_{i1}}{z_{o1}}, \qquad (2)$$

where f is the focal length of the convex lens. Similarly, P_{o2} and P_{i2} are also related in the same way. Now we define a_o as the gradient of the ray in object space, and it is related to P_{o1} and P_{o2} as in Eq. (3).

$$a_o = \frac{y_{o1} - y_{o2}}{z_{o1} - z_{o2}} \qquad (3)$$

To calculate a_i, the gradient of the ray in image space, we use Eqs. (1)-(3) and Eq. (4) is obtained.

$$a_i = \frac{y_{i1} - y_{i2}}{z_{i1} - z_{i2}} = a_o\left(1 + \frac{z_{o1}}{f}\right) - \frac{y_{o1}}{f} \qquad (4)$$

For simplicity, Eq. (4) is expressed in terms of y_{o1} and z_{o1}. It can also be expressed in terms of y_{o2} and z_{o2}.

In floating display system based on InIm, Eq. (4) describes the relationship between the gradients of a ray before and after it passes the floating lens. Thus, the viewing angle of the floating display system based on InIm can be calculated by applying Eq. (4) to a ray with viewing angle of InIm system in image space to find out the gradient of the ray after it passes the floating lens. Detailed calculations will be given in experimental results.

2.2 Expressible depth range

In InIm, there exists a central depth plane and only objects apart from the central depth plane by less than certain distance are displayable. This certain distance is called expressible depth range. If we let Δ_o be the expressible depth range of InIm and z_c be the central depth plane, we get expressible depth as in Eq. (5) and it is from z_{o1} to z_{o2}.

$$z_{o1} = z_c - \Delta_o, z_{o2} = z_c + \Delta_o \qquad (5)$$

By substituting Eq. (1) into Eq. (5), we obtain expressible depth range of floating display Δ_i as in Eq. (6).

$$\Delta_i = \frac{z_{i2} - z_{i1}}{2} = \frac{f^2\Delta_o}{2(z - \Delta_o - f)(z + \Delta_o + f)} \qquad (6)$$

Opposing to the case of the viewing angle, the expressible depth range is larger when InIm is close to the focal plane of the convex lens, and gets smaller when InIm moves farther from the focal plane. The designer should consider this trade-off relation when designing the floating display system based on InIm.

2.3 Window

Window is the two-dimensional rectangle where all valid rays of floating display system based on InIm passes through. In floating display system based on InIm, repeated images can be observed to the observer even if he or she is in the viewing zone of the right image. However, repeated images can be removed by positioning a mask which only reveals the window of the system.

Figure 2 shows the concept of the window.

Following the geometrical optics, every ray

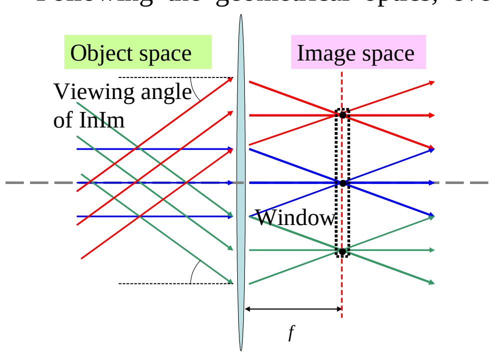


Figure 2. Geometry of the window of floating display system based on InIm

with same gradient in object space will converge to a certain point at the focal plane, and the point in image space will be farther from the axis if the gradient of the ray in object space is steeper. Therefore, window is the area which is composed of points where parallel rays in object space with angles smaller than the viewing angle of the InIm are converging to. Rays out of this area possess possibility of displaying repeated images because repeated images are composed of rays which are out of the viewing angle of InIm system. This area is the window of the floating display system based on InIm.

The size of window can be determined by calculating where the rays with the viewing angle of InIm system converge. Figure 3 shows this procedure geometrically.

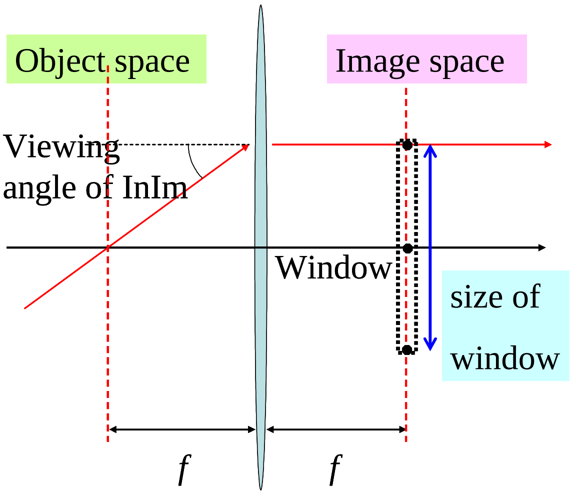


Figure 3. Geometry for the calculation of the size of the window

Let a_{ov} be the gradient of the ray with the viewing angle of the InIm system in the object space. By geometrical optics, all rays departing from the focal point of a lens should be parallel after they pass the lens. Thus the size of the window h can be found as

$$h = 2a_{ov} f \qquad (7)$$

by considering a ray with the gradient of a_{ov} passing the focal point $(-f,0)$ in the object space of the floating lens.

3. Experimental results

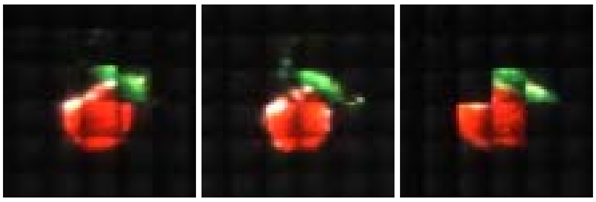
In this section, the analyses given are experimentally verified. Parameters of experimental setup are represented in Table 1.

Table 1. Parameters of experimental setup

Setup	Specifications	Characteristics
Image displayed by LCD projector	Resolution	1024 X 768
	Pixel pitch	150μm
	Distance to the lens array	18mm
Object image	Distance to the floating lens	350mm
	Size	30mm(H) X 35mm(V)
Lens array	Lens pitch	10mm
	Number of elemental Lenses	13(H) X 13(V)
	Focal length	22mm
Floating lens	Focal length	175mm
	Pitch	300mm

First, the viewing angle of the InIm system is experimentally measured. By using the measured viewing angle of the InIm system, the viewing angle of the floating display system based on InIm system and the size of window are calculated and experimental results to prove the calculations are given.

The viewing angle of InIm system used in experiment is measured to be 14°. Figure 4 shows left, center and right views of the reconstructed image of InIm system.



(a) left 14° (b) center (c) right 14°

Figure 4. Left, center and right views of the reconstructed image of InIm

By comparing the center image to the side images, it can be noticed that the leaf of the left image and the cherry of the right image are clipped.

The horizontal viewing angle can be calculated by substituting a_o to the tangent of 14°, y_{o1} to -15mm, z_{o1} to -350mm and f to 175mm in Eq. (4). y_{o1} is substituted to -15mm because the smallest value of y_{o1} gives the smallest absolute value of a_i, and the smallest value of y_{o1} is -15mm since the width of the object is 30mm. The viewing angle of the floating display system can be obtained by taking the inverse of tangent to a_i, which is 9°. Figure 5 verifies this calculation.

(a) left 9° (b) center (c) right 9°

Figure 5. Left, center and right views of the reconstructed image of floating display system based on InIm

In Fig.5, the leaf in the left image and the left side of the cherry in right image are clipped.

The size of the window can be calculated by using Eq. (7) and substituting tan14° to a_{ov} and 175mm to f, and it is 87mm in each side. Figure 6 shows the left, center and right views taken from the same position as in Fig. 5 and the only difference is the existence of the mask which conceals the system but the area of the window.

(a) left 9° (b) center (c) right 9°

Figure 6. Left, center and right views of the reconstructed image of floating display system based on InIm with mask

The comparison between Fig. 5 (b) and Fig. 6 (b) shows that the repeated images are effectively removed by locating mask. By carefully observing Fig. 6 (b), one can observe a small right end of the leaf of the left repeated image is just out of the mask. The cherry of the right repeated image is completely hiden by the mask. Also, the left and right views of Figs. 5 and 6 show that the original image is not interfered by the existence of the mask. Moreover, it was the border of the mask where right image started to disappear. Therefore, placing a mask is essential for the design of the floating display system based on InIm.

4. Conclusion

The analyses on the viewing parameters of floating display system based on InIm are proposed and experimentally verified.

Acknowledgment

This work was supported by the ITRC program of the Ministry of Commerce, Industry and Energy of Korea.

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