

카오스 로봇에서의 상호 연동 제어

배영철, 김천석
여수대학교 전자통신전기공학부

A study on Mutual Cooperative Control in the Chaos Mobile Robot

Youngchul Bae, Chunsuk Kim
*Nat'l Yosu University
E-mail : ycbae@yosu.ac.kr

Abstract

In this paper, we propose that the mutual cooperative control in the chaotic mobile robot. In order to achieve mutual cooperative control in the mobile robot, we apply coupled synchronization technique and driven synchronization technique in the mobile robot with obstacle.

1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods with mutual cooperative control.

In this paper, we propose a chaotic mobile robots that have unstable limit cycles in a chaos trajectory surface with Arnold equation, Chua's equation. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaos robots meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos

circuit equations such as the Arnold equation or Chua's equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided by using mutual cooperative control with an Arnold equation or Chua's equation.

2. Chaotic Mobile Robot embedding Chaos Equation

2.1. Mobile Robot

As the mathematical model of mobile robots, we assume a two-wheeled mobile robot as shown in Fig. 1.

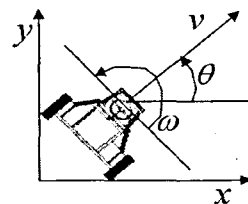


Fig. 1 Two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity [rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{1}$$

where (x,y) is the position of the robot and θ is the angle of the robot.

2.2 Some Chaos Equations

In order to generate chaotic motions for the mobile robot, we apply some chaos equations such as an Arnold equation or Chua’s equation.

1) Arnold equation [10]

We define the Arnold equation as follows:

$$\begin{aligned} \dot{x}_1 &= A \sin x_3 + C \cos x_2 \\ \dot{x}_2 &= B \sin x_1 + A \cos x_3 \\ \dot{x}_3 &= C \sin x_2 + B \cos x_1 \end{aligned} \tag{2}$$

where A, B, C are constants .

2) Chua’s Circuit Equation

Chua’s circuit is one of the simplest physical models that has been widely investigated by mathematical, numerical and experimental methods. We can derive the state equation of Chua’s circuit.

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g (x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= - \beta x_2 \end{aligned} \tag{3}$$

where .

$$g(x) = m_{2n-1}x + \frac{1}{2} \sum_{k=1}^{2n-1} (m_{k-1} - m_k) (|x + c_k| - |x - c_k|)$$

2.3 Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use the Arnold equation and Chua’s circuit equation as follows.

1) Arnold equation

We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= D v + C \cos x_2 \\ \dot{x}_2 &= D v + B \sin x_1 \\ \dot{x}_3 &= \omega \end{aligned} \tag{4}$$

where B, C, and D are constant.

Substituting (1) into (2), we obtain a state equation on \dot{x}_1 , \dot{x}_2 , and \dot{x}_3 as follows:

$$\begin{aligned} \dot{x}_1 &= Dv + C \cos x_2 \\ \dot{x}_2 &= Dv + B \sin x_1 \\ \dot{x}_3 &= \omega \end{aligned} \tag{5}$$

We now design the inputs as follows [10]:

$$\begin{aligned} v &= A / D \\ \omega &= C \sin x_2 + B \cos x_1 \end{aligned} \tag{6}$$

Finally, we can get the state equation of the mobile robot as follows:

$$\begin{aligned} \dot{x}_1 &= A \sin x_3 + C \cos x_2 \\ \dot{x}_2 &= B \sin x_1 + A \cos x_3 \\ \dot{x}_3 &= C \sin x_2 + B \cos x_1 \\ \dot{x} &= V \cos x_3 \\ \dot{y} &= V \sin x_3 \end{aligned} \tag{7}$$

Equation (7) includes the Arnold equation. Fig. 5 shows the trajectory of mobile robot of Arnold equation, when there is no boundary.

2) Chua’s Equation

Using the methods explained in equations (4)-(7), we can obtain equation (8) with Chua’s equation embedded in the mobile robot.

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g (x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= - \beta x_2 \\ \dot{x} &= V \cos x_3 \\ \dot{y} &= V \sin x_3 \end{aligned} \tag{8}$$

Using equation (8), we obtain the embedding chaos robot trajectories with Chua’s equation.

3. Mutual cooperative control by using synchronization methods

To achieve mutual cooperative control in the mobile robot, we applied the chaotic synchronization technique from the several mobile robot trajectories. Firstly, we applied coupled synchronization method and then we also applied driven synchronization method for mutual cooperative control between the several robots.

3.1 Coupled synchronization method

In order to accomplish mutual cooperative control in the several chaos mobile robots, we applied a coupled synchronization method proposed by Cuomo [11] in the Chua's chaos mobile robots.

To applied coupled synchronization method in the Chua's circuit, transmitter-receive state equations are following:

Transmitter state equation

$$\begin{aligned} C_1 \frac{dv_{c_1}}{dt} &= \frac{1}{R}(v_{c_2} - v_{c_1}) - g(v_{c_1}) + \frac{1}{R_x}(v_{c_1}' - v_{c_1}) \\ C_2 \frac{dv_{c_2}}{dt} &= \frac{1}{R}(v_{c_1} - v_{c_2}) + i_L \\ L \frac{di_L}{dt} &= -v_{c_2} \end{aligned} \tag{9}$$

Receiver state equation

$$\begin{aligned} C_1' \frac{dv_{c_1}'}{dt} &= \frac{1}{R}(v_{c_2}' - v_{c_1}') - g(v_{c_1}') + \frac{1}{R_x}(v_{c_1} - v_{c_1}') \\ C_2' \frac{dv_{c_2}'}{dt} &= \frac{1}{R'}(v_{c_1}' - v_{c_2}') + i_L' \\ L' \frac{di_L'}{dt} &= -v_{c_2}' \end{aligned} \tag{10}$$

In order to accomplish synchronization of the Eq. (9), (10), we need to find stable coupled-register R_x value between the transmitter and the receiver.

3.2 Coupled mutual cooperative control in the Chua's chaos robot by using coupled synchronization

To accomplish synchronization of the two chaos robot embedding Chua's circuit, first we formed each state equation for Eq. (11), (12). Then

found coupled coefficient k and k' by using stability criteria. After that, we applied k and k' within stable area to perform computer simulation.

Main chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a(x_2 - g(x_1)) + k \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{11}$$

Sub chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a(x_2 - g(x_1)) + k' \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{12}$$

3.2 Driven mutual cooperative control in the Chua's chaos robot by using driven synchronization

To accomplish synchronization of the two chaos robot embedding Chua's circuit, first we formed each state equation for Eq. (13), (14). Then found driven coefficient k and k' by using stability criteria. After that, we applied k and k' within stable area to perform computer simulation.

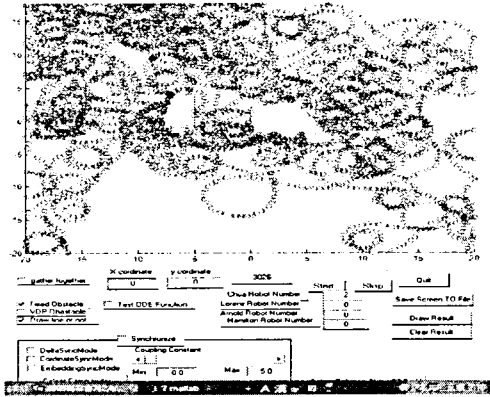
Main chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a(x_2 - g(x_1)) + k \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{13}$$

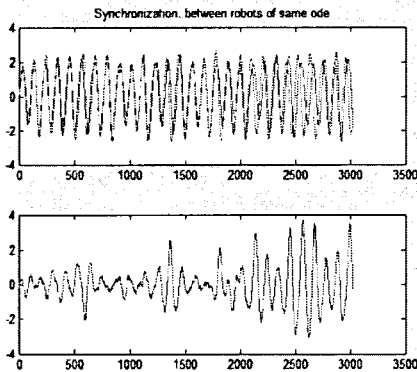
Sub chaos robot's state equation

$$\begin{aligned} \dot{x}_1 &= a(x_2 - g(x_1)) + k' \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x} &= v \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{14}$$

The Fig. 2 and 3 showing synchronization of two Chua's chaos robot after using Eq.(11) and (12). Fig. 2 is showing the result of synchronization at fixed obstacle.



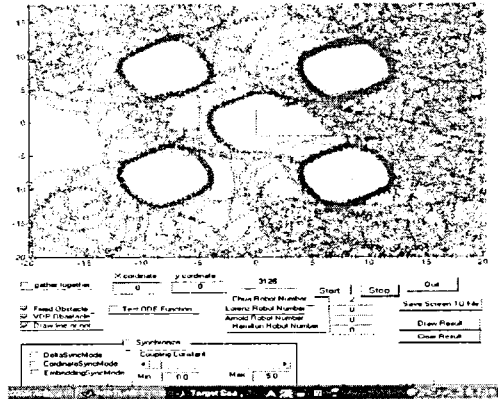
(a) Robot trajectory



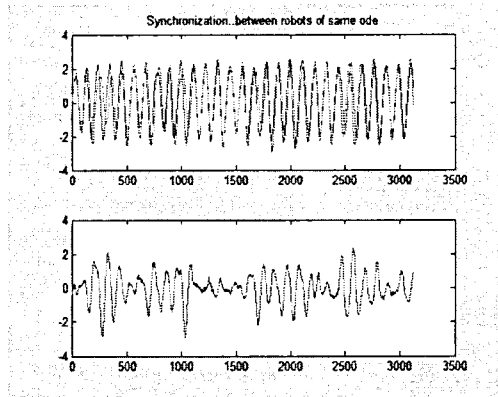
(b) The result of synchronization

Fig.2 The result of synchronization in the Chua's robot with fixed obstacles by using coupled mutual cooperative control

The Fig. 3 showing synchronization of two Chua's chaos robot after using Eq.(13) and (14). Fig. 3 is showing the result of the synchronization after applying hidden obstacle, VDP.



(a) Robot trajectory



(b) The result of synchronization

Fig. 3 The result of synchronization in the Chua's robot with hidden obstacles using driven mutual cooperative control

4. Conclusion

In this paper, we proposed a chaotic robots, which employs a robots with Chua's equation trajectories, and also proposed a robot synchronization methods in which coupled-synchronization and driven synchronization.

We designed chaotic robot trajectories such that the total dynamics of the robots was characterized by a Chua's equation and we also designed the chaotic robot trajectories to include an obstacle avoidance method. As a result, we realized that the

result of synchronization is generalized synchronization.

REFERENCE

- [1] E. Ott, C.Grebogi, and J.A York, "Controlling Chaos", *Phys. Rev.Lett.*, vol. 64, no.1196-1199, 1990.
- [2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, "Using small perturbations to control chaos", *Nature*, vol. 363, pp. 411-417, 1993.
- [3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" *IEICE. Trans. Fundamentals.* vol.E77-A, no. , pp.1000-1005, 1994.
- [4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" *J. Circuit. Systems and computers*, vol. 3, no. 1, pp. 93-108, 1993.
- [5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, "Chaos Synchronization in Coupled Chua Circuits", *IEICE. NLP.* 92-51. pp.33-40. 1992.
- [6] K. M. Short, "Unmasking a modulated chaotic communications scheme", *Int. J. Bifurcation and Chaos*, vol. 6, no. 2, pp. 367-375, 1996.
- [7] L. Kocarev, "Chaos-based cryptography: A brief overview," *IEEE*, Vol. pp. 7-21. 2001.
- [8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", *Phys. Rev. E* 67, pp. 036208, 2003.
- [9] K. Krantz, f. H. Yousse, R.W , Newcomb, "Medical usage of an expert system for recognizing chaos", *Engineering in Medicine and Biology Society*, 1988. *Proceedings of the Annual International Conference of the IEEE* , 4-7, pp. 1303 -1304,1988 .
- [10] Nakamura, A. , Sekiguchi. "The chaotic mobile robot", *IEEE Transactions on Robotics and Automation*, Volume: 17 Issue: 6 , pp. 898 -904, 2001.
- [11] K. Cuomo, A.V.Oppenheim, and S.H.Strogatz, "Synchronization of lorenz-based chaotic circuits with application to communications," *IEEE Trans. Circuits Syst. II* vol.40, pp.626-633, Oct. 1993.
- [12] J.P. Eckmann, S.O.Kamphorst, D.Ruelle, S.Ciliberto, "Liapunov exponents from time series", *Phys. Rev. A*34(6), pp. 4971-4979, 1986.
- [13] Okamoto and H. Fujii, *Nonlinear Dynamics*, Iwanami Lectures of Applied Mathematics, Iwanami, Tokyo, 1995, vol. 14.
- [14] Y. Bae, J. Kim, Y.Kim, "The obstacle collision avoidance methods in the chaotic mobile robot", *2003 ISIS*, pp.591-594, 2003.
- [15] Y. Bae, J. Kim, Y.Kim, "Chaotic behavior analysis in the mobile robot: the case of Chua's equation" , *Proceeding of KFIS Fall Conference 2003*, vol. 13, no. 2, pp.5-8, 2003.
- [16] Y. Bae, J. Kim, Y.Kim, "Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , *Proceeding of KFIS Fall Conference 2003*, vol. 13, no. 2, pp.110-113, 2003.