

카오스 수중 로봇

김천석, 배영철

여수대학교 전자통신전기공학부

A Chaotic Underwater Robot

Chunsuk Kim, Youngchul Bae

*Nat'l Yosu University

E-mail : ycbae@yosu.ac.kr

Abstract

In this paper, we propose a chaotic underwater robots that have unstable limit cycles in a chaos trajectory surface with Arnold equation, Chua's equation. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. We also show computer simulation results of Arnold equation and Chua's equation chaos trajectories with one or more Van der Pol obstacles

1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a chaotic underwater robots that have unstable limit cycles in a chaos trajectory surface with Arnold equation, Chua's equation. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaos robots meet

obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Arnold equation or Chua's equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided with an Arnold equation or Chua's equation.

2. Chaotic Underwater Robot embedding Chaos Equation

2.1. Underwater Robot

As the mathematical model of underwater robots, we assume a underwater robot as shown in Fig. 1.

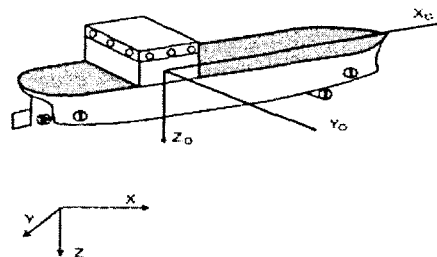


Fig. 1 Underwater robot

Let the position and yaw angle of vehicle, $\eta = [X, Y, \psi]^T \in R^3$ can be obtained by

$$\dot{\eta} = J(\eta)v \tag{1}$$

where $J(\eta)$ is a transform matrix between the earth-fixed coordinate and the body-fixed coordinate. The transformation matrix $J(\eta)$ has the following form:

$$J(\eta) = J(\Psi) = \begin{vmatrix} \cos(\Psi) & -\sin(\Psi) & 0 \\ \sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{2}$$

where $J(\Psi)$ is nonsingular for all Ψ and notice that $J^{-1}(\Psi) = J^T(\Psi)$. The state equation of the underwater robot is written as follows:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Psi} \end{bmatrix} = \begin{vmatrix} \cos(\Psi) & -\sin(\Psi) & 0 \\ \sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{vmatrix} v \tag{3}$$

where (X, Y) is the position of the underwater robot and Ψ is the yaw angle of an underwater robot.

2.2 Some Chaos Equations

In order to generate chaotic motions for the chaos underwater robot, we apply some chaos equations such as an Arnold equation or Chua's equation.

1) Arnold equation [10]

We define the Arnold equation as follows:

$$\begin{aligned} \dot{x}_1 &= A \sin x_3 + C \cos x_2 \\ \dot{x}_2 &= B \sin x_1 + A \cos x_3 \\ \dot{x}_3 &= C \sin x_2 + B \cos x_1 \end{aligned} \tag{4}$$

where A, B, C are constants.

2) Chua's Circuit Equation

Chua's circuit is one of the simplest physical models that has been widely investigated by

mathematical, numerical and experimental methods. We can derive the state equation of Chua's circuit.

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{5}$$

where .

$$g(x) = m_{2n-1}x + \frac{1}{2} \sum_{k=1}^{2n-1} (m_{k-1} - m_k)(|x + c_k| - |x - c_k|)$$

2.3 Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the underwater robot, we define and use the Arnold equation and Chua's circuit equation as follows.

1) Arnold equation

We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= D \dot{y} + C \cos x_2 \\ \dot{x}_2 &= D \dot{x} + B \sin x_1 \\ \dot{x}_3 &= \theta \end{aligned} \tag{6}$$

where B, C, and D are constant.

Substituting (3) into (4), we obtain a state equation on \dot{x}_1 , \dot{x}_2 , and \dot{x}_3 as follows:

$$\begin{aligned} \dot{x}_1 &= Dv + C \cos x_2 \\ \dot{x}_2 &= Dv + B \sin x_1 \\ \dot{x}_3 &= \omega \end{aligned} \tag{7}$$

We now design the inputs as follows [10]:

$$\begin{aligned} v &= A / D \\ \omega &= C \sin x_2 + B \cos x_1 \end{aligned} \tag{8}$$

Finally, we can get the state equation of the underwater robot as follows:

$$\begin{aligned}
 \dot{x}_1 &= A \sin x_3 + C \cos x_2 \\
 \dot{x}_2 &= B \sin x_1 + A \cos x_3 \\
 \dot{x}_3 &= C \sin x_2 + B \cos x_1 \\
 \dot{x} &= v (\cos x_3 - \sin x_3) \\
 \dot{y} &= v (\sin x_3 + \cos x_3)
 \end{aligned}
 \tag{9}$$

Equation (9) includes the Arnold equation.

2) Chua's Equation

Using the methods explained in equations (6)-(9), we can obtain equation (10) with Chua's equation embedded in the underwater robot.

$$\begin{aligned}
 \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\
 \dot{x}_2 &= x_1 - x_2 + x_3 \\
 \dot{x}_3 &= -\beta x_2 \\
 \dot{x} &= v \cos x_3 - v \sin x_3 \\
 \dot{y} &= v \sin x_3 + v \cos x_3
 \end{aligned}
 \tag{10}$$

Using equation (10), we obtain the embedding chaos underwater robot trajectories with Chua's equation.

3. Numerical Analysis of the Behavior of the Chaos Underwater Robot

We investigated by numerical analysis whether the underwater robot with the proposed controller actually behaves in a chaotic manner. In order to computer simulation, we applied mirror mapping and have shown it in Fig. 2. The parameters and initial conditions are used as follows:

3.1 Arnold equation case

Coefficients:

$$v=1[m/s], A=0.5[1/s], B=0.25[1/s], C=0.25[1/s]$$

Initial conditions:

$$x_1 = 4, x_2 = 3.5, x_3 = 0, x = 0, y = 0$$

3.2 Chua' equation scase

Coefficients:

$$\alpha = 9, \beta = 14.286$$

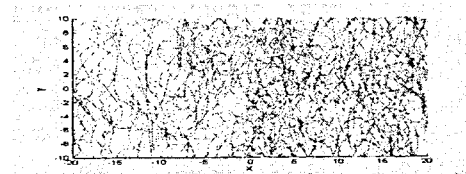
$$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, m_3 = m_1$$

$$c_1 = 1, c_2 = 2.15, c_3 = 3.6$$

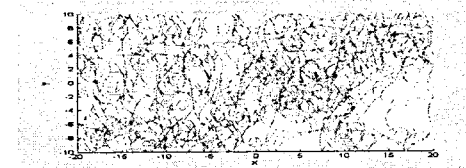
Initial conditions:

$$x_1 = 4, x_2 = 3.5, x_3 = 0, x = 0, y = 0$$

Fig. 2 shows the trajectories in which mirror mapping was applied only on the outer wall. In this case, the chaos underwater robot has no obstacles, and we can confirm that the robot is adequately meandering along the trajectories of Arnold and Chua's equation and are covering the whole space in their chaotic manner.



(a)



(b)

Fig. 2 Trajectory of the underwater robot, when boundary exists. (a)Arnold equation, (b) Chua's equation

4. The Underwater Robot with Van der Pol equation obstacle.

In this section, we will discuss the underwater robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the underwater robot can not move close to the obstacle and the obstacle is avoided.

4.1 VDP equation as an obstacle

In order to represent an obstacle of the underwater, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2)y - x \end{aligned} \tag{9}$$

From equation (9), we can get the following limit cycle as shown in Fig. 3.

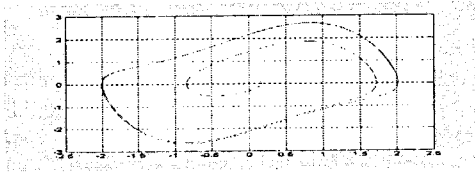


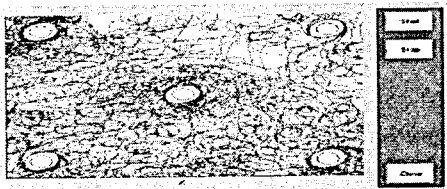
Fig. 3 Limit cycle of VDP

5. Dangerous degree obstacle avoid method

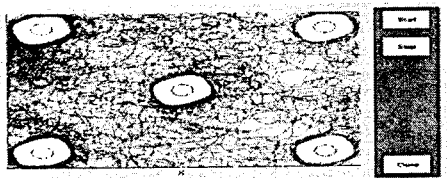
In this section, we proposed a new obstacle avoidance method which is according to dangerous degree with Arnold equation, Chua's equation. We have to ensure against underwater robot risk with distance limit, if there is a dangerous situation when robots are avoid to obstacle. To do this, we constrained approach obstacle distance for the degree of robots.

5.1. Arnold equation

In Fig. 4 we can see the robot trajectories of obstacle avoidance results, dangerous degree are low situation (a), high situation (b) in the Arnold chaos underwater robot respectively.



(a)

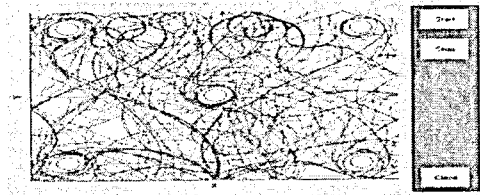


(b)

Fig. 5 An obstacle avoidance result for dangerous degree in the Arnold chaos underwater robot, dangerous degree low (a), high (b)

5.2. Chua's equation

In Fig. 5, we can see the underwater robot trajectories of obstacle avoidance results, dangerous degree are low situation (a), high situation (b) in the Chua's chaos underwater robot respectively.



(a)



(b)

Fig. 6 An obstacle avoidance result for dangerous degree in the Chua's equation, dangerous degree low (a), high (b)

6. Conclusion

In this paper, we proposed a chaotic underwater, which employs a underwater robot with Arnold equation, Chua's equation trajectories, and also proposed an obstacle avoidance method in which we assume that the obstacle has a Van der Pol equation with an unstable limit cycle.

In order to make an obstacle avoidance result in the underwater robot system, we applied the Arnold equation and Chua's equation with dangerous degree. As a result, we realized that there are satisfy to obstacle avoidance method with dangerous degree.

REFERENCE

- [1] E. Ott, C.Grebogi, and J.A York," Controlling Chaos", Phys. Rev.Lett., vol. 64, no.1196-1199, 1990.
- [2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, " Using small perturbations to control chaos", Nature, vol. 363, pp. 411-417, 1993.

- [3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundamentals. vol.E77-A, no. , pp.1000-1005, 1994.
- [4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.
- [5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, "Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.
- [6] K. M. Short, "Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.
- [7] L. Kocarev, "Chaos-based cryptography: A brief overview," IEEE, Vol. pp. 7-21. 2001.
- [8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", Phys. Rev. E 67, pp. 036208, 2003.
- [9] K. Krantz, F. H. Yousse, R.W, Newcomb, "Medical usage of an expert system for recognizing chaos", Engineering in Medicine and Biology Society, 1988. Proceedings of the Annual International Conference of the IEEE, 4-7, pp. 1303 -1304,1988 .
- [10] Nakamura, A. , Sekiguchi. "The chaotic mobile robot", , IEEE Transactions on Robotics and Automation , Volume: 17 Issue: 6 , pp. 898 -904, 2001.
- [11] J.A.K.Suykens, "N-Double Scroll Hypercubes in 1-D CNNs" Int. J. Bifurcation and Chaos, vol. 7, no. 8, pp. 1873-1885, 1997.
- [12] Y. Bae, J. Kim, Y.Kim, " The obstacle collision avoidance methods in the chaotic mobile robot", 2003 ISIS, pp.591-594, 2003.
- [13] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Chua's equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp.5-8, 2003.
- [14] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003.
- [15] Y. C. Bae, J.W. Kim, Y.G. Kim , " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003
- [16] Y. C. Bae, J.W. Kim, N.S. Choi ,The Collision Avoidance Method in the Chaotic Robot with Hyperchaos Path", KIMICS Conference 2003 Fall , vol. 7, no. 2, pp.584-588, 2003.
- [17] Y. C. Bae, J.W. Kim, N.S. Choi , " The Analysis of Chaotic Behaviour in the Chaotic Robot with Hyperchaos Path ov Van der Pol(VDP) Obstacle", KIMICS Conference 2003 Fall , vol. 7, no. 2, pp.589-593, 2003.
- [18] Y. C. Bae, J.W. Kim, Y.I, Kim, Chaotic Behaviour Analysis in the Mobile of Embedding some Chaotic Equation with Obstacle", J.ournal of Fuzzy Logic and Intelligent Systems, vol. 13, no.6호, pp.729-736, 2003.
- [19] Y. C. Bae, J. W. Kim, Y.I, Kim, Obstacle Avoidance Methods in the Chaotic Mobile Robot with Integrated some Chaotic Equation", International Journal of Fuzzy Logic and Intelligent System, vol. 3, no. 2. pp. 206-214, 2003.
- [20] Y. C. Bae, J.W. Kim, Y.I, Kim, "The Obstacle Collision Avoidance Methods in the Chaotic Mobile Robots", ISIS 2003 Proceeding of the 4th International symposium on Advanced Intelligent System, pp. 591-594, 2003.