Low-Complexity Maximum-Likelihood Decoder for V-BLAST Architecture

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ABSTRACT

In this paper, a low-complexity maximum-likelihood (ML) decoder based on QR decomposition, called real-valued LCMLDec decoder or RVLCMLDec for short, is proposed for the Vertical Bell Labs Layered Space-Time (V-BLAST) architecture, a promising candidate for providing high data rates in future fixed wireless communication systems [1]. Computer simulations, in comparison with other detection techniques, show that the proposed decoder is capable of providingthe V-BLAST schemes with ML performance at low detection complexity.

Keywords

Space-time coding, multiple-input multiple-output, maximum likelihood detection, sphere decoding, and wireless communication

Ⅰ. Introduction

The use of multiple antennas at both transmitter and receiver sides, resulting in the so-called multiple input multiple output (MIMO) systems, is theoretically proved to have capability of remarkably increasing spectral efficiencies [2].In order to experimentally verify the fact, a MIMO architecture, called V-BLAST [1], has been implemented in real time and demonstrated its performance in an indoor slow-fading environment. The results of the V-BLAST showed that very high spectral efficiencies, from 20 to 40bits/s/Hz, can be obtained, whereby making it very promising for high-data-rate applications in wireless communication systems.

In order to detect transmitted symbols of the V-BLAST, different detection algorithms can be utilized. Of cause, brute-force maximum likelihood (ML) decoder is the optimal one for the V-BLAST. However, its complexity, which grows exponentiallywith the number of transmit antennas, is a big disadvantage, preventing it to be a preferable decoder. To avoid the complexity problem associated with brute-force ML detection, linear and nonlinear suboptimal detection schemes have been proposed.Some possible suboptimal methods are zero forcing (ZF), minimum mean square error (MMSE) [3], and decoders using interference nulling and successive interference cancellation such as ZF-BLAST [1], QR-decomposition (QRD) [3], sorted QR decomposition (SQRD) [4], or MMSE-SQRD [5]-[6].Nonetheless, for the V-BLAST with equal numbers of transmit and receive antennas, the use of interference suppression, either by ZF, by MMSE, or by QRD, causes the diversity order of the first detected symbol to reduce to one, leading to high bit error rate (BER).

Recently, it has been reported in the literature that ML performance can be obtained for the V-BLAST at low detection complexity by means of sphere decoding [7]-[8].Sphere decoding is actually a joint ML detection technique where the ML decision metric is computed over all constellation points enclosed in a sphere of a given radius $\sqrt{C}$, whereby causing an enormous reduction in the number of signal points to be tested. However, one of the disadvantages of sphere decoding is that the performance and complexity of the algorithm greatly depend on the initial choice of the sphere radius. Specifically, a small value of the sphere radius may lead to an empty sphere, whereas a large value of the sphere radius may lead to a large number of signal points to be tested, and hence high detection complexity.

In this paper, we propose a low complexity ML decoder based on QR decomposition, called RVLCMLDec, for the V-BLAST. Similar to the real-valued sphere decoders (SDs), our proposed algorithm is also constructed using QR decomposition. Nonetheless, it is distinguished from the real-valued SDs in the following points:

1. It does not require determining any sphere radius. Thus ML solution can always be found.
2. The use of a sorting rule enables the proposed decoder to find ML solution with low complexity.

With the aid of our proposed decoder, the V-BLAST is now able to attain ML performance at low detection complexity. Simulation results are provided to demonstrate the performance and complexity of the RVLCMLDec decoder.

II. System model

We consider an uncoded V-BLAST configuration with n_T transmit and $n_R \geq n_T$ receive antennas as shown in Fig. 1.

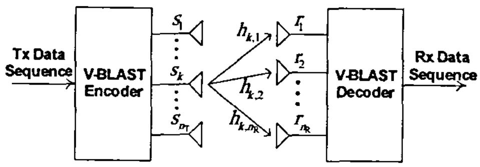

Fig.1: Model of a V-BLAST scheme with n_T transmit and n_R receive antennas.

At the transmitter, the input data sequence is divided into n_T sub-streams (or layers), which are then modulated by *M*-Quadrature Amplitude Modulation (*M*-QAM) schemes and transmitted from different transmit antennas. For convenience, we assume one-time-slot complex baseband signal model, where at each symbol period a $n_T \times 1$ transmit signal vector $\mathbf{s}$ composed of n_T symbols, s_i, $i = 1, 2, \ldots, n_T$, is sent through n_T transmit antenna.Under the assumption that the signals are narrowband and the channel remains constant over some block of arbitrary length and changes from one block to the next (*i.e.*, block fading channel), the $n_R \times 1$ received signal vector, $\mathbf{x}$, is given by:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{1}$$

where $\mathbf{w} = \left[w_1, w_2, \ldots, w_{n_R}\right]^T$ represent the noise samples at n_R receive antennas, which are modeled as independent samples of a zero-mean complex Gaussian random variable with noise variance σ^2, T denotes the transpose of a matrix, $\mathbf{H}$ is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas modeled as the samples of a zero-mean complex Gaussian random variable with equal variances of 0.5 per complex dimension. In the paper, we also assume that the signals transmitted from individual antenna have equal powers of P/n_T and the channel gains are perfectly known at the receiver.

Similar to the sphere decoders [7]-[8], our proposeddecoder deals with the real and imaginary parts of (1) separately. Therefore, the system model can be equivalently rewritten as:

$$\mathcal{X} = \mathcal{H}\mathcal{s} + \mathcal{w} \tag{2}$$

with the real-valued channel matrix, signal and noise vectors respectively given by:

$$\mathcal{H} = \begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix} \tag{3}$$

$$\mathcal{X} = \begin{bmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{bmatrix}, \mathcal{s} = \begin{bmatrix} \Re(\mathbf{s}) \\ \Im(\mathbf{s}) \end{bmatrix}, \mathcal{w} = \begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix} \tag{4}$$

Note here that the corresponding dimensions of $\mathcal{H}$, $\mathcal{X}$, $\mathcal{s}$, and $\mathcal{w}$ are $N_R \times N_T$, N_R, N_T, and N_R, where $N_R = 2n_R$ and $N_T = 2n_T$.

As perfect channel state information is available, the optimal ML decoder searches $\mathcal{s} = \left(s_1, s_2, \ldots, s_{N_T}\right)$ over the finite set of integer values, Z_{QAM}, which is used to generate a QAM constellation, and decides the ML solution $\hat{s}$ that minimizes the following decision metric:

$$\hat{s} = \arg \min_{s \in Z_{QAM}} \| x - \mathcal{H}s \|^2 \quad (5)$$

Using brute-force ML detection to solve (5) is infeasible for large values of N_T and/or high-level modulation schemes, *i.e.*, large M since the complexity is of order M^{N_T}. In the sequel, we develop an optimal detection scheme for solving (5) with moderate complexity.

Ⅲ. Proposed RVLCMLDec Decoder

Using the Modified Gram-Schmidt (MGS) algorithm [9], the channel matrix $\mathcal{H}$ can be factorized as:

$$\mathcal{H} = \mathrm{QR} \quad (6)$$

where Q is a $N_R \times N_T$ unitary matrix, *i.e.*, $\mathrm{Q}^H \mathrm{Q} = \mathrm{I}_{N_T}$, and R is a $N_T \times N_T$ upper triangular matrix.

Pre-multiplying both sides of (2) with Q^H yields:

$$\mathbf{v} = \mathbf{R}s + \mathbf{n} \quad (7)$$

where $\mathbf{v} = [v_1, v_2, \ldots, v_{N_T}] = \mathrm{Q}^H x$ and $\mathbf{n} = [n_1, n_2, \ldots, n_{N_T}] = \mathrm{Q}^H w$ are respectively the $N_T \times 1$ received signal vector and the $N_T \times 1$ noise vector after QR decomposition.

Since Q is unitary, the statistical properties of the noise term $\mathbf{n}$ remain unchanged. The proposed RVLCMLDec decoder chooses the ML solution, $\hat{s}$, from Z_{QAM} that satisfies:

$$\hat{s} = \arg \min_{s \in Z_{QAM}} \| \mathbf{v} - \mathbf{R}s \|^2 = \arg \min_{s \in Z_{QAM}} \sum_{k=1}^{N_T} d_k^2 \quad (8)$$

where,

$$d_k^2 = \left(v_k - \sum_{i=k}^{N_T} R_{k,i} s_i \right)^2 = \left(v_k - \xi_k - R_{k,k} s_k \right)^2 \quad (9)$$

with $\xi_k = \sum_{i=k+1}^{N_T} R_{k,i} s_i$

Thanks to the special form of d_k, exhaustive search over the set Z_{QAM} for decoding $s = (s_1, s_2, \ldots, s_{N_T})$ can be avoided, thereby considerably speeding up the decoding process. Let $\mathbf{r}$ be the array of length L containing all the integers in the set Z_{QAM}, for example, $\mathbf{r} = (-3, -1, 1, 3)$ and $L = 4$ for 16-QAM, and **AbsEuclDist()** be the function that utilizes $\mathbf{r}$ and (9) to compute the absolute values of d_k, and rearrange (sort) those values of $|d_k|$ and the corresponding values of $\mathbf{r}$ in an ascending order of $|d_k|$. The inputs of **AbsEuclDist()** are v_k, ξ_k, $R_{k,k}$, $\mathbf{r}$, while the outputs are $\mathbf{d}_k$ containing the sorted values of $|d_k|$ and $\mathbf{y}_k$ containing the sorted values of $\mathbf{r}$. The proposed RVLCMLDec decoder can be summarized as follows.

RVLCMLDec (Input $\mathbf{v}$, $\mathbf{R}$, $\mathbf{r}$, L, Output $\hat{s}$)

- *Initialization*

Set $k := N_T$, $T_k := 0$, $\xi_k := 0$.
<Loop>
  $I_k := 1$
  if ($k = N_T$), then
  [$\mathbf{d}_k$, $\mathbf{y}_k$]=AbsEuclDist(v_k, ξ_k, $R_{k,k}$, $\mathbf{r}$)
  else
    $T_k := \mathbf{d}_{kk,1}^2 + T_{kk}$
    $\xi_k := \sum_{i=kk}^{N_T} R_{k,i} \mathbf{y}_{i,1}$
    [$\mathbf{d}_k$, $\mathbf{y}_k$]=AbsEuclDist(v_k, ξ_k, $R_{k,k}$, $\mathbf{r}$)
  endif
  $\hat{s}_k := \mathbf{y}_{k,1}$
  $kk := k$
  $k := k - 1$
go to <Loop>
Set $D_c := (\mathbf{d}_0^2 + T_0)$

- *Searching*

Set $k := 2$, $I_k := I_k + 1$

2 go to Step 2

3 Step 1: $[\mathbf{d}_k, \mathbf{y}_k]$=AbsEuclDist(v_k, ξ_k, $R_{k,k}$, $\mathbf{r}$) and $l_k := 1$

4 Step 2: $t := T_k + \mathbf{d}_{k,l_k}^2$

5 Step 3: if $(t \geq D_c)$ or $(l_k > L)$, then if $(k = N_\mathrm{T})$, terminate, else set $k := k+1$, $l_k := l_k + 1$, and go to Step 2

6 Step 4: if $(k > 1)$, then $kk := k-1$, $T_{kk} := t$, $\xi_{kk} := \sum_{i=k}^{N_\mathrm{T}} R_{kk,i} \mathbf{y}_{i,l_i}$, $k := kk$, and go to Step 1

7 Step 5: New solution found, let $D_c := t$, save $\hat{s}_k := \mathbf{y}_{k,l_k}$, $k = 1, \ldots, N_\mathrm{T}$, then set $k := k+1$, $l_k := l_k + 1$, and go to Step 2.

IV Simulation results

In order to evaluate performance and complexity of the proposed RVLCMLDec decoder, we consider its application in different V-BLAST configurations. For simplicity, each V-BLAST scheme with n_T transmit, n_R receive antennas as the $(n_\mathrm{T}, n_\mathrm{R})$ system. In the simulation, the burst length is set equal to 100 symbol durations. In addition, the channel matrix is assumed to stay constant within one burst and changes randomly from one burst to the next.

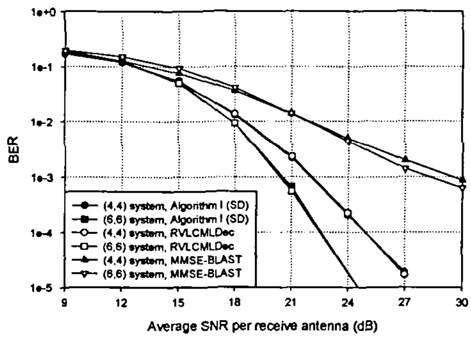

Fig. 2: BER performances of RVLCMLDec decoder, sphere decoder, and MMSE-BLAST decoder for (4,4) and (6,6) systems; 16-QAM modulation.

In Fig. 2, BER performances versus average signal-to-noise power ratio (SNR) per receive antenna of the proposed RVLCMLDec, the sphere decoder (Algorithm I) [8], and the MMSE-BLAST decoder [10] for (4,4) and (6,6) systems using 16-QAM modulation are provided. The sphere radius of Algorithm I is a function of noise variance and determined in such a way that the probability of finding a valid point insider the sphere is 0.99. If no valid point is found, the sphere radius will be multiplied by a factor of 1.2.

From Fig. 2, we can see that both RVLCMLDec decoder and sphere decoder significantly outperform MMSE-BLAST decoder. In addition, RVLCMLDec decoder and the sphere decoder have almost the same BER performances. Clearly, RVLCMLDec decoder is capable of providing V-BLAST schemes with ML performances.

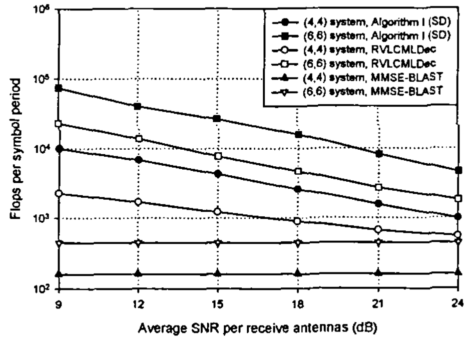

Fig. 3: Average complexities of RVLCMLDec decoder, sphere decoder, and MMSE-BLAST decoder in (4,4) and (6,6) systems with a 16-QAM constellation.

The complexities as functions of SNR for the three decoders are shown in Fig. 3 in terms of numbers of floating point operations (flops).To obtain the complexities of RVLCMLDec decoder and sphere decoder, 20000 channel realizations are generated. The complexities of RVLCMLDec decoder and sphere decoder include those of the preprocessing stage and of the searching stage. It can be observed from Fig. 3 that MMSE-BLAST decoder has lowest complexity, and yet poorest BER performance. For the same system configuration, the proposed decoder has remarkably lower complexity than does the sphere decoder, especially in the low and medium SNR regions. For example, in (4,4) system, at SNR of 9dB, the complexity of the sphere decoder is roughly 4 times higher than that of the proposed decoder.

V Conclusion

In this paper, we propose a low-complexity ML decoder based on QR decomposition, namely, RVLCMLDec, for the V-BLAST. Computer simulation shows that the proposed decoder has ML performance, while it offers a noticeably reduction in detection complexityas compared to the sphere decoder. Furthermore, the propose decoder does not require the determination of any sphere radius, thereby making it simple to be applied. The complexity of the proposed decoder can be further reduced by employing appropriate preprocessing techniques such as sorted QR decomposition (SQRD) or MMSE-SQRD.

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