# Preisach 모델을 이용한 압전액츄에이터 이력 보상

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Hysteresis Compensation in Piezoceramic Actuators Through Preisach Model Inversion

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## **ABSTRACT**

In precision positioning applications, such as scanning tunneling microscopy and diamond turning machines [1], it is often required that actuators have nanometer resolution in displacement, high stiffness, and fast frequency response. These requirements are met by the use of piezoceramic actuators. A major limitation of piezoceramic actuators, however, is their lack of accuracy due to hysteresis nonlinearity and drift. The maximum error due to hysteresis can be as much as 10-15% of the path covered if the actuators are run in an open-loop fashion. Hence, the accurate control of piezoceramic actuators requires a control strategy that incorporates some form of compensation for the hysteresis. One approach is to develop an accurate model of the hysteresis and the use the inverse as a compensator. The Preisach model has frequently been employed as a nonlinear model for representing the hysteresis, because it encompasses the basic features of the hysteresis phenomena in a conceptually simple and mathematically elegant way.

In this paper, a new numerical inversion scheme of the Preisach model is developed with an aim of compensating hysteresis in piezoceramic actuators. The inversion scheme is implemented using the first-order reversal functions and is presented in a recursive form. The inverted model is then incorporated in an open-loop control strategy that regulates the piezoceramic actuator and compensates for hysteretic effects. Experimental results demonstrate satisfactory regulation of the position of the piezoceramic actuator to the desired trajectories.

Key Words: Piezeceramic actuator, Hysteresis compensation, Preisach model inversion

# 1. Introduction

In precision positioning applications, such as scanning tunneling microscopy and diamond turning machines [1], it is often required that actuators have nanometer resolution in displacement, high stiffness, and fast frequency response. These requirements are met by the use of piezoceramic actuators. A major limitation of piezoceramic actuators, however, is their lack of accuracy due to hysteresis nonlinearity and drift. The maximum error due to hysteresis can be as much as 10-15% of the path covered if the actuators are run in an open-loop fashion.

Without modeling and incorporating hysteresis in the controller design, the hysteresis will act as an unmodeled phase lag presence and will cause instability in a close-loop control system. Reliable modeling and predictions of hysteresis would be a valuable tool when these piezoceramic actuators as part of close-loop system for purposes of motion control system such as active control and micro-positioning. If hysteresis effects of these material systems could be predicted, then actuator

controllers could be designed to correct these effects and the whole controller system could be made to appear as a device with a single-valued output function and possible even a linear device

Hence, the accurate control of piezoceramic actuators requires a control strategy that incorporates some form of compensation for the hysteresis. One approach is to develop an accurate model of the hysteresis and the use the inverse as a compensator. The Preisach model has frequently been employed as a nonlinear model for representing the hysteresis, because it encompasses the basic features of the hysteresis phenomena in a conceptually simple and mathematically elegant way. The problem of determining of the Preisach model and its inverse for a piezoceramic actuator has been addressed by several researchers [2]-[5].

In this paper, a new numerical inversion scheme of the Preisach model is developed with an aim of compensating hysteresis in piezoceramic actuators. The inversion scheme is implemented using the first-order reversal functions and is presented in a recursive form. The inverted model is then incorporated in an open-loop

control strategy that regulates the piezoceramic actuator and compensates for hysteretic effects. Experimental results demonstrate satisfactory regulation of the position of the piezoceramic actuator to the desired trajectories.

#### 2. Preisach Model and a Recursive Formula

#### 2.1 Preisach Model

A hysteresis system  $\hat{\Gamma}$  with input signal u(t) and output signal f(t) can be represented by the superposition of an infinite number of elementary hysteresis operators with

$$\hat{\gamma}_{\alpha\beta}u(t) = \begin{cases} +1 & \text{for } u(t) \ge \alpha \\ -1 & \text{for } u(t) \le \beta \\ \text{remainsunchanged} & \text{for } \alpha < u(t) < \beta \end{cases}$$
 (1)

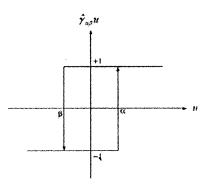


Fig. 1 Transfer characteristic of the elementary hysteresis operator

Fig. 1 displays the system behavior of this operator. With the density function  $P(\alpha, \beta)$  representing the Preisach function, the classical Preisach model for a hysteresis system is defined by

$$f(t) = \hat{\Gamma}u(t) := \iint_{\alpha \ge \beta} P(\alpha, \beta) \hat{\gamma}_{\alpha\beta} u(t) \, \mathrm{d}\alpha \mathrm{d}\beta \tag{2}$$

The investigation of model (2) is considerably facilitated by its geometric interpretation. This interpretation is based on the fact that there is a one-to-one correspondence between operators  $\hat{\gamma}_{\alpha\beta}$  and points  $(\alpha, \beta)$  of the half plane  $\alpha \geq \beta$ . At any instant of time, the support triangle T is subdivided into two sets:  $S^+(t)$  consisting of points  $(\alpha, \beta)$  for which  $\hat{\gamma}_{\alpha\beta}u(t)=1$  and  $S^-(t)$  consisting of points  $(\alpha, \beta)$  for which  $\hat{\gamma}_{\alpha\beta}u(t)=-1$ . The staircase separation curve between  $S^+(t)$  and  $S^-(t)$  is denoted L(t)

and its vertices have  $(\alpha, \beta)$  coordinates coinciding with local maxima and minima of input at previous instants of time. The final link of L(t) is attached to the line  $\alpha = \beta$  and moves when the input changes. This link is a horizontal one and moves up when the input increases, and it is a vertical one and moves from right to left when the input decreases. Using the above interpretation, the model (2) can be represented in the following equivalent form:

$$f(t) = \iint_{S^{+}(t)} P(\alpha, \beta) \, d\alpha d\beta - \iint_{S^{-}(t)} P(\alpha, \beta) \, d\alpha d\beta$$
(3)

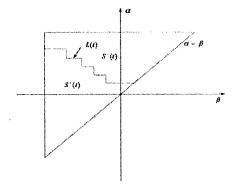


Fig. 2 A geometric interpretation of the Preisach model

To determine  $P(\alpha, \beta)$ , the set of first order reversal curves should be experimentally found. This can be done by bringing first the input to such a value that outputs of all operators  $\hat{\gamma}_{lphaeta}$  are equal to -1. If we now gradually increase the input value, then we will follow along a limiting ascending branch (see Fig. 3). This branch is called limiting because there is no branch below it. The notation  $f_{\alpha}$  will be used for the output value on this branch corresponding to the input  $u = \alpha$ . The firstreversal curves are attached to the limiting ascending branch. Each of these curves is formed when the above monotonic increase of the input is followed by a subsequent monotonic decrease. The notation  $f_{\alpha\beta}$  will be used for the output value on the reversal curve attached to the limiting ascending branch at the point  $f_{\alpha}$ . This output value corresponding to the input  $u = \beta$ . Now we can define the function:

$$F(\alpha, \beta) = (f_{\alpha} - f_{\alpha\beta})/2 \tag{4}$$

Using the geometric interpretation of the model, it is easy to prove that:

$$F(\alpha_1, \beta_1) = \iint_{T(\alpha_1, \beta_1)} P(x, y) dxdy = \iint_{\beta_1} \left( \int_{\beta_1} P(x, y) dx \right) dy$$
 (5)

where  $T(\alpha, \beta)$  is the triangle formed by the intersection

of the lines  $\alpha = \alpha_1$ ,  $\beta = \beta_1$  and  $\alpha = \beta$  (see Fig. 3). From (5), we find

$$P(\alpha, \beta) = -\frac{\partial^2 F(\alpha, \beta)}{\partial \beta \partial \alpha} \tag{6}$$

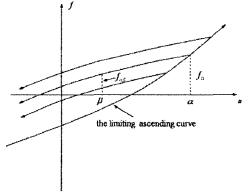


Fig. 3 First-order reversal curves.

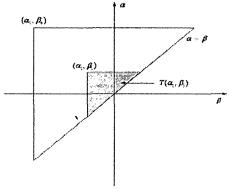


Fig. 4 Graphical representation of (5)

## 2.2 Recursive Formula

We derive a recursive formula for the model. Assume that the input has the latest local maxima  $\alpha_N$  at the time  $t_{\alpha_N}$  and the latest local minima  $\beta_{N-1}$  at the time  $t_{\beta_{N-1}}$ . And assume that the input is monotonically decreasing and not less than  $\beta_{N-1}$  i.e.  $\dot{u}(t) < 0, \ u(t) \ge \beta_{N-1}$ . As shown by Fig. 5, the region  $S^+(t)$  is the difference of  $S^+(t_{\alpha_N})$  and  $\Delta S$  i.e.

$$S^{+}(t) = S^{+}(t_{\alpha_{v}}) - \Delta S \tag{7}$$

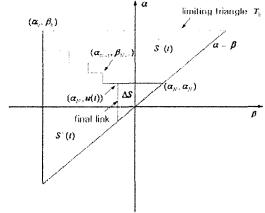


Fig. 5 Interpretation for the decreasing input

Using  $S^{-}(t) = T_0 - S^{+}(t)$ , (3) can be represented as follows:

$$f(t) := f_0 + 2 \iint_{S^*(t)} P(\alpha, \beta) \, d\alpha d\beta, \ f_0 = \iint_{T_0} P(\alpha, \beta) \, d\alpha d\beta$$

Using (7) and (4), (8) becomes

$$f(t) = f_0 + 2 \iint_{S^*(t_{\alpha_N})} P(\alpha, \beta) \, d\alpha d\beta - 2 \iint_{\Delta S} P(\alpha, \beta) \, d\alpha d\beta$$

$$= f(t_{\alpha_N}) - 2F(\alpha_N, u(t))$$

$$= f(t_{\alpha_N}) - [f_{\alpha_N} - f_{\alpha_N u(t)}]$$

To sum up, for the decreasing input the output of the model becomes

$$f(t) = f(t_{\alpha_N}) - [f_{\alpha_N} - f_{\alpha_N u(t)}] \text{ for } \dot{u}(t) < 0, u(t) \ge \beta_N.$$
(10)

Assume that the input has the latest local maxima  $\alpha_N$  at the time  $t_{\beta_N}$  and the latest local minima  $\beta_N$  at the time  $t_{\beta_N}$ . And assume that the input is monotonically increasing and not larger than  $\alpha_N$  i.e.  $\dot{u}(t)>0, \, u(t)\leq\alpha_N$ . As shown by Fig. 6, the region  $S^+(t)$  is the sum of  $S^+(t_{\alpha_N})$  and  $\Delta S$  i.e.

$$S^{+}(t) = S^{+}(t_{\beta_{N}}) + \Delta S$$
 (11)

Similarly in the decreasing case, we can get the following:

$$f(t) = f(t_{\beta_N}) + [f_{u(t)} - f_{u(t)\beta_N}] \text{ for } \dot{u}(t) > 0, u(t) \le \alpha_N$$
(12)

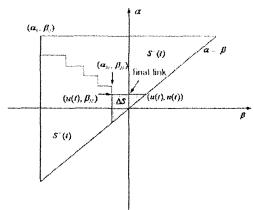


Fig. 6 Interpretation for the increasing input

In order to implement the model equations (10) and (12) to estimate the piezoceramic expansion due to an arbitrary voltage input, a series of first-order reversal function for the specific piezoelement being used are experimentally determined. These functions need to be determined for all pairs  $(\alpha, \beta)$  within the limiting triangle  $T_0$ . In this work, a mesh covering the  $\alpha - \beta$  plane is created, and a corresponding value  $F(\alpha_i, \beta_j)$  is experimentally obtained. In practice, the finite numbers of grid points and corresponding measured output values are insufficient i.e. some actual values of the input voltages are not on grid points. Thus, the mesh is divided into the finer form and the bilinear spline interpolation is employed to obtain the first-order reversal functions.

## 3. Numerical Inversion

Consider the case a desired output is decreasing. At the time t, assume that we know  $\alpha_N$ ,  $t_{\alpha_N}$  and  $f(t_{\alpha_N})$ . We make an assumption that if a desired output is decreasing  $f(t+\Delta t) < f(t)$  then the corresponding input is not increasing  $u(t+\Delta t) \le u(t)$ . Using (10), we can find the input needed to produce the desired output:

$$u(t + \Delta t) = \beta^*$$

$$= \min_{i, \beta_i \le u(t)} \left[ F(\hat{\alpha}_N, \beta_i) - \{ f(t + \Delta t) - f(t_{\alpha_N}) \} \right]$$
(13)

where 
$$\hat{\alpha}_N = \min_j \left[ \alpha_N - \alpha_j \right]$$

Consider the case a desired output is increasing. At the time t, assume that we know  $\beta_N$ ,  $t_{\beta_N}$  and  $f(t_{\beta_N})$ . We make an assumption that if a desired output is increasing  $f(t+\Delta t)>f(t)$  then the corresponding input is not decreasing  $u(t+\Delta t)\geq u(t)$ . Using (12), we can find the input needed to produce a desired output:

$$u(t + \Delta t) = \alpha^* = \min_{i, \alpha_i \ge u(t)} \left[ F(\alpha_i, \hat{\beta}_N) - \left\{ f(t + \Delta t) - f(t_{\beta_N}) \right\} \right]$$
(14)

where 
$$\hat{\beta}_N = \min \left[ \beta_N - \beta_j \right]$$

## 4. Experimental Results

The experimental setup is shown in Fig. . A fast tool servo is used in this experiment, a piezoceramic actuator is incorporated into the mechanical structure and its displacement is measured through a capacitive displacement sensor. An NI DAQ board was used to produce an input signal, record the input voltages and tip displacement of a piezoceramic actuator.

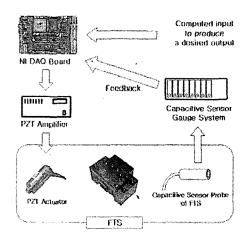


Fig. 7 Experimental setup

A desired output signal is as follows:

$$f_d(t) = 2[1 - \cos(0.2\pi t)\cos(0.02\pi t)] Volt, 0 \le t \le 200 \text{ sec}$$
 (15)

Using Eq. (13) and (14), the corresponding input was computed and input into a power amplifier. The sensor outputs one voltage per 2.5  $\mu m$  translation of the tip. Data was recorded using a NI DAQ board. Each data set consisted of the samples  $(u_k, f_k)$  where the term  $u_k$  is a sample of the output voltage from the NI board to the power amplifier, and the term  $f_k$  is a sample of the displacement sensor output voltage. Data was sampled at 1 KHz and 20,000 samples were recorded. This was 20 seconds of data or 2 cycles of the desired output voltage. The samples  $(u_k, f_k)$  were plotted in Fig. 8.

The desired output and real output displacement voltage were plotted in Fig. 9. The maximum absolute error between the desired and real output data is 0.091 V which is 1.38 % of the maximum input voltage. The error mean is 0.023 V which is 0.36 % of the maximum input

voltage. The results show satisfactory regulation of the position of the piezoceramic actuator to the desired trajectories.

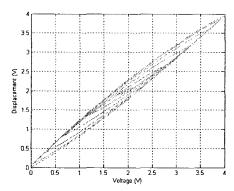


Fig. 8 Displacement hysteresis curve

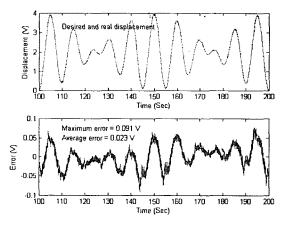


Fig. 9 Desired, real displacement and error

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