한국전산구조공학회
2005년 봄 학술발표회 논문집

상시 진동을 사용한 교량 상부 구조계의 휨강성 추정기법

Substructural Identification of Bending Stiffness in Bridge Deck

구 기 영* 윤 정 방** 이 진 학***
Koo, Ki Young Yun, Chung Bang Yi, Jin-Hak

ABSTRACT

This paper proposed a new substructural identification method to estimate the bending stiffness of a bridge deck, a fundamental structural health index of a super-structure. The proposed method can estimate the bending stiffness without considering actual supporting conditions by using substructural identification method while most of conventional methods need reasonable assumptions on supporting conditions which are hard to be assessed in a real bridge in operation. The mathematical formulation is derived and the results of laboratory tests are summarized. It was verified that the proposed method gives consistent estimation results regardless of actual supporting conditions.

1. INTRODUCTION

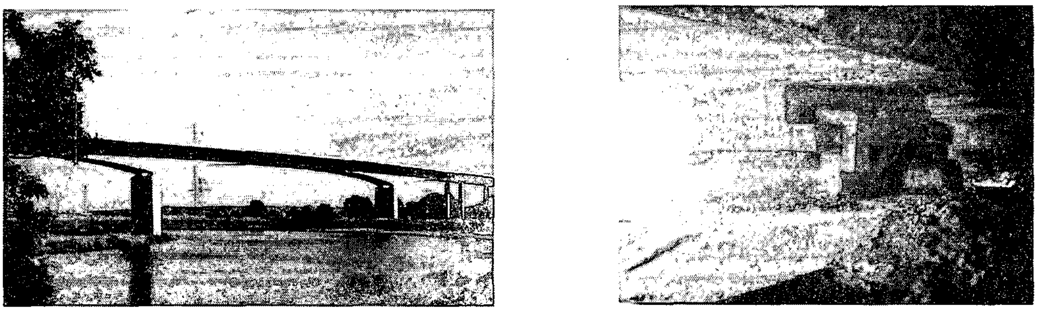
Structural health monitoring system is one of the most important topics for optimal bridge managements. In the case of bridge monitoring, the super-structure is considered as major member of structural system and acceleration, displacement and strain of a bridge deck are usually measured and used to assessment for structural health of a super-structure of a bridge in operations. However measured responses are affected by not only structural integrity of a bridge deck but also supporting conditions. Although these supporting conditions are originally constructed as a hinge, roller or friction supporting by design specification, it may not remain the same conditions due to various environment effects (example shown in Fig. 1) and traveling vehicles. In practice, it is very hard to assess the support conditions exactly in real bridges in operation. Therefore, a method to estimate the bending stiffness, as a major structural integrity indexes without information on supporting conditions is needed.

To solve this problem, the substructural identification method can be utilized. Substructural identification

* Ph.D. Candidate, Dept. of Civil & Environ. Engrg., KAIST
** Professor, Dept. of Civil & Environ. Engrg., KAIST
*** Research Professor, Smart Infra-Structure Technology Research Center, KAIST

methods were first introduced to reduce DOF's of complex structural system and make an identification task simple and easy. Substructural identification of a plan frame structure was carried out in time-domain by numerical simulation (Koh et al. 1991 and Oreta et al. 1994) and substructural identification for a pier-deck substructure of the New-Lian River Bridge was carried out in frequency domain based on earthquake time histories measured (Loh et al. 1998).

In this study, a simple and easy method is proposed to identify the bending stiffness of a bridge deck incorporating ambient vibration measurements regardless of considering actual supporting conditions. The proposed method is developed by using the analytical solution of the Bernoulli-Euler beam equation and an amplitude relationship between power spectral density values to make procedure simple and easy.

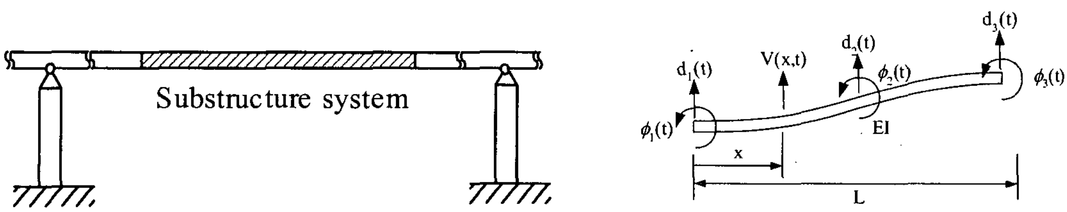


a) A typical bridge structure with bearing supporting b) A degraded support condition caused by corrosion

Fig. 1. A bridge with unknown supporting conditions

2. SUBSTRUCTURAL IDENTIFICATIONS FOR A BRIDGE DECK

The substructural identification can be stated as follows "when we measure interfacial and interior responses of the substructure, we can construct the substructural system having interfacial response as inputs and interior responses as outputs. By identifying these inputs/outputs relations, substructural properties can be estimated regardless of the properties of total structure."



a) A substructural system b) Inputs and outputs of a substructure system

Fig. 2. A substructural system

Fig. 2 (a) shows a substructural system for a bridge. As shown in Fig. 2 (b), the measurements $d_1(t)$, $\phi_1(t)$, $d_3(t)$ and $\phi_3(t)$ are used as inputs and $d_2(t)$ and $\phi_2(t)$ are used as outputs of the substructural system, when $d_i(t)$ are vertical displacements and $\phi_i(t)$ are rotational displacements. In next section, inputs/outputs relationship, i.e. transfer function $H(\omega)$ will be derived analytically by solving the differential equation of the Bernoulli-Euler beam and a new substructural identification method using the amplitude relations between the power spectral densities will be described subsequently.

2.1 Transfer function of the substructural system

The governing equation of Bernoulli-Euler beam is

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 v}{\partial x^2}\right]+\rho A\frac{\partial^2 v}{\partial t^2}+\eta A\frac{\partial v}{\partial t}=0$$

where EI is the bending stiffness, ρ is the mass density, A is the cross sectional area and η is the damping coefficient per unit volume.

The complete solution can be obtained as

$$v(x,t)=\sum_{\omega}\left[c_1e^{-i\beta x}+c_2e^{-\beta x}+c_3e^{+i\beta x}+c_4e^{+\beta x}\right]e^{i\omega t} \tag{1}$$

where $\beta^2=\sqrt{\dfrac{\omega^2\rho A-i\omega\eta A}{EI}}$

The substructural beam is subjected to the time varying boundary conditions as follows

$$v(0,t)=d_1(t),\quad v(L,t)=d_3(t),\quad \left.\frac{\partial v(x,t)}{\partial x}\right|_{x=0}=\phi_1(t),\quad \left.\frac{\partial v(x,t)}{\partial x}\right|_{x=L}=\phi_3(t)$$

From this, the set of linear algebraic equations can be obtained.

$$\sum_{\omega}\begin{bmatrix}1 & 1 & 1 & 1\\ -i\beta & -\beta L & i\beta & \beta L\\ e^{-i\beta L} & e^{-\beta L} & e^{i\beta L} & e^{-\beta L}\\ -i\beta e^{-i\beta L} & -\beta Le^{-\beta L} & i\beta e^{i\beta L} & \beta Le^{\beta L}\end{bmatrix}\begin{Bmatrix}c_1\\ c_2\\ c_3\\ c_4\end{Bmatrix}e^{i\omega t}=\begin{Bmatrix}d_1(t)\\ \phi_1(t)\\ d_3(t)\\ \phi_3(t)\end{Bmatrix}$$

Let the boundary inputs be written as following form

$$\{d_1(t)\ \ \phi_1(t)\ \ d_3(t)\ \ \phi_3(t)\}^T=\sum_{\omega}\mathbf{B}(\omega)e^{i\omega t}$$

The general solution can be obtained

$$v(x,t)=\sum_{\omega}\left[e^{-i\beta x}\ \ e^{-\beta x}\ \ e^{+i\beta x}\ \ e^{+\beta x}\right]\mathbf{A}(\omega)^{-1}\mathbf{B}(\omega)e^{i\omega t} \tag{2}$$

where

$$\mathbf{B}(\omega) = \{b_1(\omega) \quad b_2(\omega) \quad b_3(\omega) \quad b_4(\omega)\}^T$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i\beta & -\beta L & i\beta & \beta L \\ e^{-i\beta L} & e^{-\beta L} & e^{i\beta L} & e^{-\beta L} \\ -i\beta e^{-i\beta L} & -\beta L e^{-\beta L} & i\beta e^{i\beta L} & \beta L e^{\beta L} \end{bmatrix}$$

By using Eqn (2), the transfer function between substructural response and boundary inputs can be obtained

$$H_s(x,\omega) = \left[e^{-i\beta x} \quad e^{-\beta x} \quad e^{+i\beta x} \quad e^{+\beta x}\right]\mathbf{A}(\omega)^{-1} \tag{3}$$

where $\beta^2 = \sqrt{\dfrac{\omega^2 \rho A - i\omega\eta A}{EI}}$

Finally, the transfer function between interior responses at $x = L/2$ and boundary inputs can be obtained in the form of dimensionless variable ξ using Eqn (3)

$$\begin{Bmatrix} d_2(\omega) \\ \phi_2(\omega) \end{Bmatrix} = \begin{bmatrix} h_{11}(\xi) & Lh_{12}(\xi) & h_{13}(\xi) & Lh_{14}(\xi) \\ \frac{1}{L}h_{21}(\xi) & h_{22}(\xi) & \frac{1}{L}h_{23}(\xi) & h_{24}(\xi) \end{bmatrix} \begin{Bmatrix} d_1(\omega) \\ \phi_1(\omega) \\ d_3(\omega) \\ \phi_3(\omega) \end{Bmatrix}$$

where $\xi = bL = \sqrt[4]{\dfrac{\omega^2 \rho A - \eta A}{EI}}L$

$$\mathrm{h}_{11}(\xi) = \frac{\frac{1}{2}(1+i)\left[\sin(\frac{\xi}{2}) + \sinh(\frac{\xi}{2})\right]}{\sin(\frac{1}{2}(1+i)\xi) + \sinh(\frac{1}{2}(1+i)\xi)}, \mathrm{h}_{12}(\xi) = \frac{\frac{1}{2}(1+i)\left[\sin(\frac{\xi}{2}) + \sinh(\frac{\xi}{2})\right]}{\xi\left[\sin(\frac{1}{2}(1+i)\xi) + \sinh(\frac{1}{2}(1+i)\xi)\right]}, \mathrm{h}_{13}(\xi) = \mathrm{h}_{11}(\xi), \mathrm{h}_{14}(\xi) = -\mathrm{h}_{12}(\xi)$$

$$\mathrm{h}_{21}(\xi) = \frac{\frac{1}{2}(1-i)\left[\cos(\frac{\xi}{2}) - \cosh(\frac{\xi}{2})\right]\xi}{\sin(\frac{1}{2}(1+i)\xi) - \sinh(\frac{1}{2}(1+i)\xi)}, \mathrm{h}_{22}(\xi) = \frac{\frac{1}{2}(1-i)\left[\sin(\frac{\xi}{2}) - \sinh(\frac{\xi}{2})\right]}{\sin(\frac{1}{2}(1+i)\xi) - \sinh(\frac{1}{2}(1+i)\xi)}, \quad \mathrm{h}_{23}(\xi) = -\mathrm{h}_{21}(\xi), \mathrm{h}_{24}(\xi) = \mathrm{h}_{22}(\xi)$$

2.2 Bending Stiffness (EI) estimation algorithm

The basic input-output relationship derived previously is given below

$$\{d_2(\omega) \quad \phi_2(\omega)\}^T = \mathrm{H}(\xi)\{d_1(\omega) \quad \phi_1(\omega) \quad d_3(\omega) \quad \phi_3(\omega)\}'$$

Using relationships between $h_{ij}(\xi)$

$$d_2(\omega) = h_{11}(\xi)(d_1(\omega) + d_3(\omega)) + Lh_{12}(\phi_1(\omega) - \phi_3(\omega))$$

$$\phi_2(\omega) = \frac{1}{L}h_{21}(\xi)(d_1(\omega) - d_3(\omega)) + h_{22}(\phi_1(\omega) + \phi_3(\omega))$$

The power-spectral densities relationship can be obtained by using equations above.

$$\begin{aligned} S_{d_2 d_2}(\omega) &= h_{11}(\xi)\left(S_{d_1 d_2}(\omega) + S_{d_3 d_2}(\omega)\right) + Lh_{12}(\xi)\left(S_{\phi_1 d_2}(\omega) - S_{\phi_3 d_2}(\omega)\right) \\ S_{\phi_2 \phi_2}(\omega) &= \frac{1}{L}h_{21}(\xi)\left(S_{d_1 \phi_2}(\omega) - S_{d_3 \phi_2}(\omega)\right) + h_{22}(\xi)\left(S_{\phi_1 \phi_2}(\omega) + S_{\phi_3 \phi_2}(\omega)\right) \end{aligned} \tag{4}$$

The power spectral densities can be estimated based on measurements of 4 boundary inputs and two interior outputs. Then, we can obtain the non-dimensional variable ξ by minimize the error satisfying Eqn (4). The bending stiffness of the substructure can be estimated by using definition of ξ.

$$EI = \rho A \omega_i^2 L^4 / \mathrm{Re}\left(\hat{\xi}^4\right)$$

The procedure to estimate bending stiffness of the substructural beam from measurement is summarized below.

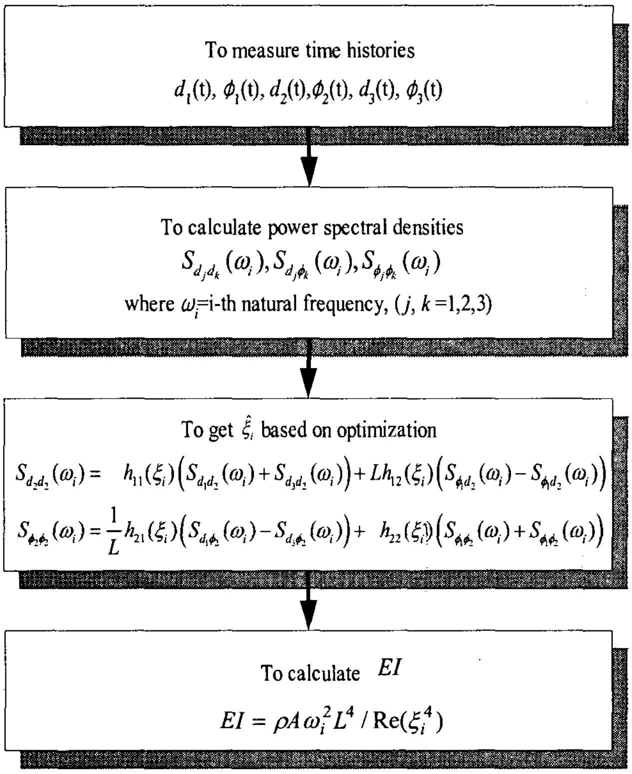


Fig. 3. Flow chart to estimate bending stiffness using substructural identification

3. LABORATORY TEST FOR BOUNDARY INDEPENT IDENTIFICATIONS

In the previous section, a new method is proposed to identify the bending stiffness of a bridge deck regardless of supporting conditions. To verify the independency of supporting condition, the laboratory tests were carried out using the test bridge with variable support conditions as shown in Fig. 4. The substructure was selected as a middle part of the total span as shown in Fig. 5 and accelerometers were installed to measure vertical and rotational accelerations. The mean value of the adjacent two accelerations, i.e. A1 and A2 or A4 and A5, is used for the vertical acceleration response, and the difference of the two accelerations was used for the rotational acceleration response as depicted in Fig. 5. Four cases of experiments with different supporting conditions (Table 1) were carried out by combining four types of support conditions as shown in Fig. 6. The free vibration responses were measured by the miniature accelerometers (B&K model 4370)

Typical raw acceleration measurements are shown in Fig. 7 and the input and output accelerations for substructural beam evaluated by raw acceleration measurement is shown in Fig. 8. The identification results are shown in Table 2. For the experiment cases, the first natural frequency is quite different due to the different supporting conditions, however, the bending stiffness were estimated very consistently regardless of the actual support conditions.

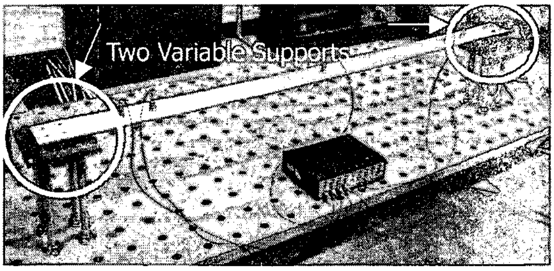


Fig. 4. The laboratory test bridge with two variable boundary supporting.

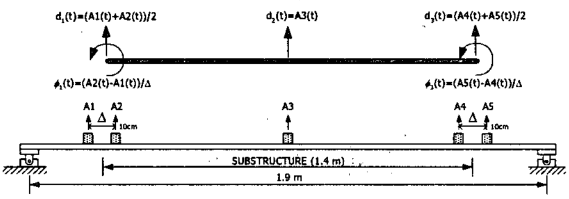
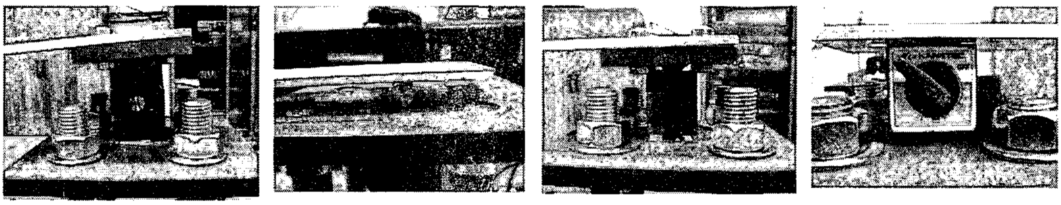


Fig. 5. The substructure and accelerometer configuration to measure $d_1(t)$, $\phi_1(t)$, $d_2(t)$, $d_3(t)$ and $\phi_3(t)$



a) Hinged support b) Roller support c) Fixed support d) Friction support

Fig. 6. Four types of boundary conditions used in the experiment.

Table 1. The support conditions used for each experiment cases

Case number	#1	#2	#3	#4
Left end support condition	Fixed	Roller	Roller	Friction
Right end support condition	Fixed	Fixed	Hinged	Hinged

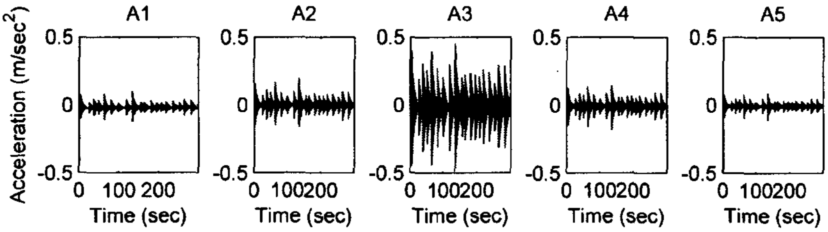


Fig. 7 Typical raw acceleration measurements for each sensor (case number 1: the sensor positions are described in Fig. 5)

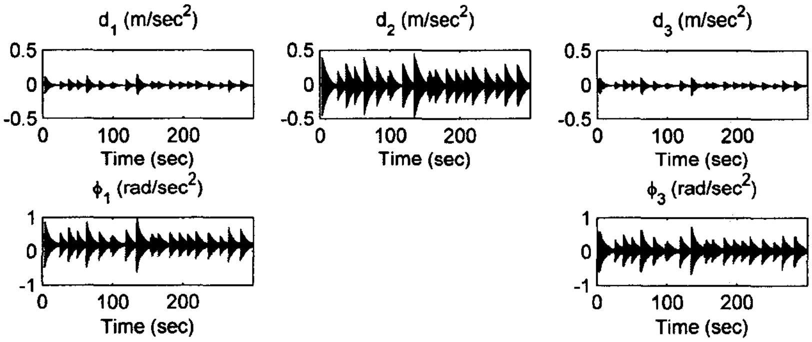


Fig. 8 Typical input/output accelerations of substructural beam evaluated by five accelerometers

Table 2. The Identification results (True EI = 128.8 Nm2)

CASE No.	First natural frequency (Hz)	ξ	EI (Nm^2)	Relative error (%)
#1	8.179	3.436	122.0	-4.8
#2	5.981	2.886	131.1	2.2
#3	3.845	2.346	124.0	-3.2
#4	4.883	2.615	129.6	1.0

3. CONCLUSION

A new method is proposed to identify the bending stiffness of a super-structure of bridge systems without information on support condition. Its applicability is verified by the laboratory test using the simple bridge model with the variable support conditions. The bending stiffness was estimated for the four different support conditions and the results show that the newly proposed method gives very reasonable and consistent estimations regardless of the actual boundary conditions. In addition, the proposed method has several merits over other conventional methods as following

- The proposed method can be easily applied to a complex bridge by divide and conquer strategy while the identification procedure may be difficult to consider all structural elements at once for a continuous and complex bridge.
- The proposed method can give us quantitative results, i.e. bending stiffness (EI), to indicate structural health of a super-structure. Hence, it is not necessary to know the initial information of structural health of a bridge.
- The proposed method is relative simple and easy to use. It only needs calculation for the power spectral densities and single variable optimization using amplitudes relations of the power spectral densities.
- The proposed method can be used for a damage detection algorithm by using small length of a substructure.

REFERENCES

1. Koh, Chang Ghee, See, Lin Ming and Balendra, Thambirajah, "Estimation of Structural parameters in time domain: a substructure approach", Earthquake Engineering and Structural dynamics, Vol. 20, 1991, pp. 787-801
2. Oreta, Andres W. and Tanabe, Tada-aki "Element Identification of Member Properties of Framed Structures", Journal of Structural Engineering, Vol. 120, No. 7, 1994, pp. 1961-1976
3. Lee, Zheng-Kuan and Loh, Chin-Hsiung, "Substructural Identification of Bridge - A FFT-Based Spectral Analysis", Journal of Structural Mechanics and Earthquake Engineering, JSCE, Vol. 15, No.1, 1998, pp. 41-51
4. Lee, HyungJin and Yun, ChungBang, "Efficient Structural Identifications Using Substructural Technique", Proceedings of annual conference of KSCE, 1995, pp. 145-148
5. Craig, Jr., Roy R., "Structural Dynamics: An Introduction to Computer Methods", John Wiley & Sons, Inc., 1981