Cavitating Flow Simulation Using Two-Fluid Two-Phase Flow Model and HLL Scheme

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이유체 이상유동 모델과 HLL 스킴을 이용한 캐비테이션 유동 해석

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A compressible two-fluid two-phase flow computation model using the stiffened-gas equation of state is formulated. Since the conservation equation system is of mixed type, it gives complex eigenvalues. The sonic speeds obtained from the individual single phase have been simply used in the literature for the fastest wave speeds necessary in the HLL scheme. This method has worked fine but proved to be quite diffusive according to our test. To improve the accuracy, we here propose to utilize the analytic eigenvalues evaluated from an approximate Jacobian matrix for the fastest wave speeds. The interfacial transfer terms were dropped in constituting the Jacobian matrix for this purpose. The present scheme proved efficient, robust and accurate in comparison with other existing methods. We solved the cavitating flow problem using the present scheme. The result shows more detailed wave structure in the cavitating process caused by the strong expansion waves.

Key Words: 캐비테이션유동(Cavitating Flow) 이상유동(Two-Phase Flow), 이유체모델(Two-Fluid Model), 근사Jacobian행렬(Approximate Jacobian Matrix), HLL스킴(HLL Scheme)

1. Introduction

The mathematical formulations of two-fluid two-phase flow have been known to be non-conservative and ill-posed due to the complex eigenvalues. They make the numerical method unstable. In order to make the equations hyperbolic, various remedies have been proposed so far. Adding the virtual mass, the interfacial pressure term, or the surface tension force term to the governing equations has proved efficient in improving the stability.

Various numerical methods are available to solve the two-fluid governing equations. A method based on the staggered grid and donor-cell differencing has been much employed in the commercial two-phase codes such as RELAP, TRAC, and CATHARE [1]. However, this method introduces a large amount of numerical diffusion. Upwind methods are very efficient and less diffusive if the equation system is hyperbolic. Various upwind methods have been applied; among them, the HLL Riemann solver is very efficient and robust, but quite diffusive. Saurel and Lemetayer [2] applied this method to the two-pressure two-fluid model based on the seven

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conservation equations. The key point of the scheme lies on how well the fastest wave speeds can be estimated. Because it is difficult to obtain the wave speed information to the present model generally, a novel approach is hereby proposed.

2. Governing equations

We consider the compressible two-fluid two-phase flow model. The six conservation equations can be written as

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+H\frac{\partial \alpha_g}{\partial x}=S \tag{1}$$

where

$$U=\begin{pmatrix} \alpha_g\rho_g \\ \alpha_l\rho_l \\ \alpha_g\rho_g u_g \\ \alpha_l\rho_l u_l \\ \alpha_g\rho_g E_g \\ \alpha_l\rho_l E_l \end{pmatrix},\quad F=\begin{pmatrix} \alpha_g\rho_g u_g \\ \alpha_l\rho_l u_l \\ \alpha_g\rho_g u_g^2+\alpha_g p \\ \alpha_l\rho_l u_l^2+\alpha_l p \\ \alpha_g\rho_g u_g E_g+\alpha_g u_g p \\ \alpha_l\rho_l u_l E_l+\alpha_l u_l p \end{pmatrix} \tag{2}$$

and

$$H=\begin{pmatrix} 0 \\ 0 \\ -p^i \\ p^i \\ -p^i u^i \\ p^i u^i \end{pmatrix},\quad S=\begin{pmatrix} 0 \\ 0 \\ \alpha_g\rho_g g \\ \alpha_l\rho_l g \\ \alpha_g\rho_g u_g g \\ \alpha_l\rho_l u_l g \end{pmatrix} \tag{3}$$

The variables α, ρ, u, E and p represent the void fraction, the density, the velocity, the total energy, and the common pressure, respectively; p^i and u^i are respectively the interfacial pressure and the interfacial velocity. The constant g is the acceleration of gravity. The subscript g and l denote the gas and the liquid phase, respectively. In addition, the void fraction obeys $\alpha_g+\alpha_l=1$.

2.1 Stiffened-gas equation of state

In order to treat each phase as a compressible fluid, we employ the stiffened-gas equation of state in the form

$$p=(\gamma_k-1)\rho_k e_k-\gamma_k p_{\infty,k} \tag{4}$$

where γ_k and e_k is the specific heat ratio and the internal energy of each phase, respectively.

For the gas phase, the constants for air are

$$\gamma_g=1.4,\ p_{\infty,g}=0 \text{ Pa} \tag{5}$$

For the liquid phase, the constants for water are given by [3]

$$\gamma_l=2.8,\ p_{\infty,l}=8.5\times10^8 \text{ Pa} \tag{6}$$

The speed of sound is defined by

$$a_k^2=\frac{\gamma_k(p+p_{\infty,k})}{\rho_k} \tag{7}$$

2.2 Interfacial transfer terms

To make the two-fluid governing equation system stable, several adding terms have been used. We here adopt the following interfacial pressure model used in CATHARE code [1]:

$$p^i=p-\delta\frac{\alpha_g\alpha_l\rho_g\rho_l}{\alpha_g\rho_l+\alpha_l\rho_g}(u_g-u_l)^2 \tag{8}$$

where the damping coefficient δ is a positive constant. The system becomes hyperbolic for $\delta\geq1$.

The interfacial velocity u^i can be modeled as the velocity at the center of mass in the form [2]

$$u^i=\frac{\alpha_g\rho_g u_g+\alpha_l\rho_l u_l}{\alpha_g\rho_g+\alpha_l\rho_l} \tag{9}$$

3. Approximate Jacobian matrix

The two-phase governing equation system is of non-conservative form. The system, as it is, is not unconditionally hyperbolic. Its eigenvalues also depend on the type of added stability terms, i.e., the virtual mass and the interfacial pressure terms. We here propose a new method to obtain the system eigenvalues using an approximate Jacobian matrix.

The equation system, Eq. (1), becomes the following form when the interfacial transfer terms are dropped:

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=0 \qquad (10)$$

Then, the equations can be transformed using the primitive variables $W=(\alpha_g, p, u_g, u_l, \rho_g, \rho_l)^T$ into

$$\frac{\partial W}{\partial t}+(A^{-1}B^c)\frac{\partial W}{\partial x}=\frac{\partial W}{\partial t}+\Phi^c\frac{\partial W}{\partial x}=0 \qquad (11)$$

where A and B^c is the coefficient matrix, and Φ^c is the approximate Jacobian matrix. The six eigenvalues of this system can be found, in an analytical form, as

$$\lambda_1^c=u_g,$$
$$\lambda_2^c=u_l,$$
$$\lambda_{3,4}^c=u_g\pm a_g,$$
$$\lambda_{5,6}^c=u_l\pm a_l\sqrt{1-\frac{\alpha_g\gamma_g\gamma_l^2 p_{\infty,l}}{\alpha_g\gamma_g\gamma_l(\gamma_l-1)p_{\infty,l}+\alpha_l\rho_g a_g^2\gamma_l+\alpha_g\rho_l a_l^2\gamma_g}}$$

4. Numerical results and discussion

We consider a cavitation generation problem proposed by Saurel and Lemetayer [2]. The cavitation phenomenon is well represented by the two-fluid two-phase flow formulations. Fig. 1 shows the schematic diagram of the cavitation problem. A cavitation tube, 1 m long, is initially filled with water (with only 1% dispersed gas) at atmospheric pressure and separated in the middle by a diaphragm. Then, the fluid left to the diaphragm is set to the initial speed, -100 m/s, and that right to the diaphragm at the same but opposite speed, 100 m/s. The computation is done with 1000 grid cells and CFL=0.9. A simple

Fig.1 Schematic of the cavitation tube

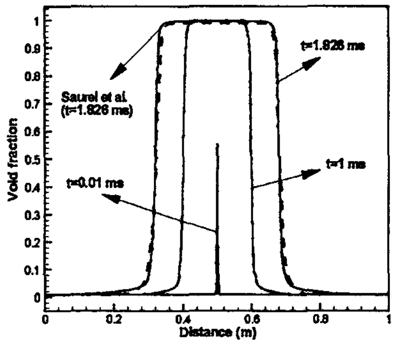

Fig. 2 Void fraction

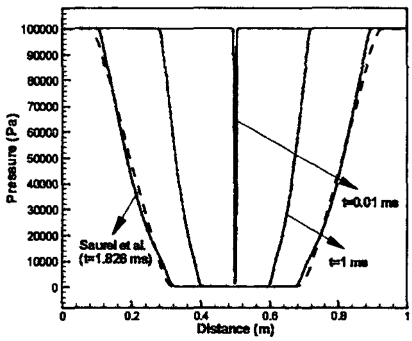

Fig. 3 Pressure

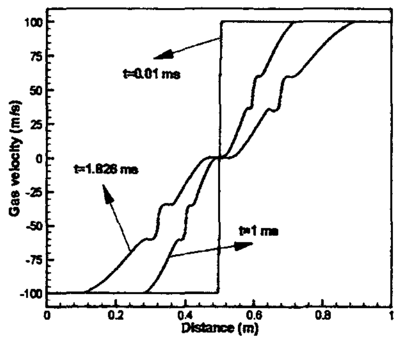

Fig. 4 Gas velocity

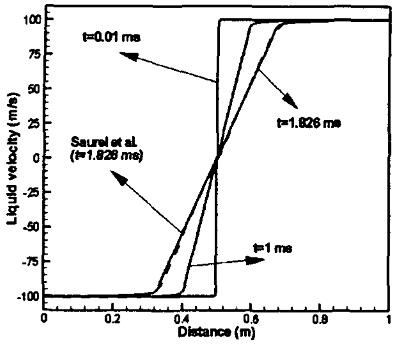

Fig. 5 Liquid velocity

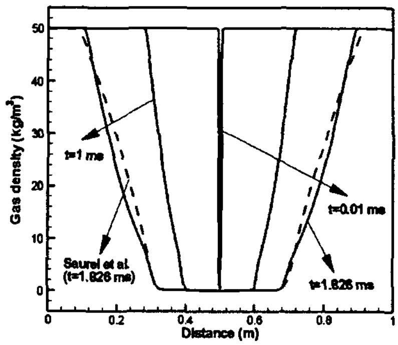

Fig. 6 Gas density

extrapolation is used for the end wall boundary conditions.

Figs. 2-7 show numerical results at three different times: 0.01 ms, 1 ms, and 1.826 ms. At the last time instant, t=1.826 ms, the result of Saurel and Lemetayer is also presented. The outputs are very similar each other except for the liquid density. Comparison is not made for the gas velocity since in Saurel and Lemetayer's work the gas velocity plot is missing. The void fraction rapidly increases to a unit in the middle part as the rarefaction waves run apart toward the opposite directions. The cavitation zone is expanded with the traveling waves.

5. Conclusions

A compressible two-fluid two-phase model with the stiffened-gas equation of state has been proposed in this paper. We here proposed a new method to estimate the fastest characteristic wave speeds using an approximate Jacobian matrix of a reduced system, eliminating the interfacial transfer terms. The analytic eigenvalues obtained this way are compact as well as independent of the artificial stability terms, making the numerical scheme efficient and robust. We solved the cavitation problem by strong rarefaction waves and compared with Saurel and Lemetayer's data. The result shows more detailed wave structure in the cavitating process than the previous one.

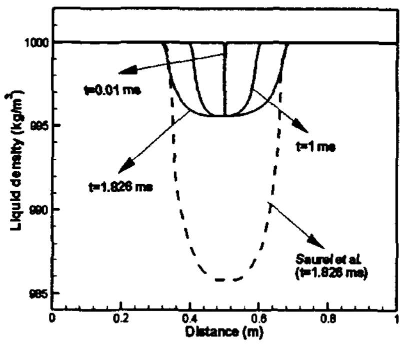

Fig. 7 Liquid density

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