단일추진제 추진시스템의 과도기유체 해석

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A fluid transient analysis for the propellant flow in a monopropellant propulsion system

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A fluid transient analysis for the propellant flow in a monopropellant propulsion system is conducted using the method of characteristics (MOC). Algebraic simultaneous equations method and Cramer's rule method utilized to drive the compatible and characteristic equations are reviewed to understand MOC more extensively. The identification of fluid transient phenomena of propulsion system of Koreasat 1 is carried out through parametric studies. Also this work describes the reason that the propulsion system of Koreasat 1 has no orifice to control flow transients or to limit the initial hydrazine flow rate for the first-pulse firing.

Key Words: 과도기유체(Transient Flow), 특성곡선법(Method of Characteristics), 단일추진제(Monopropellant)

1. Introduction

Fluid transient analysis essentially consists of solving the governing equations for a wide variety of boundary and initial conditions, and system topologies. The equations cannot be analytically solved, so many methods have been developed over the years. The method of characteristics is usually adopted for transient phenomena. Its strong point is to transform the two partial differential equations (PDEs) of continuity and momentum conservations into four ordinary differential equations (ODEs) that are solved numerically using finite difference techniques [1, 2, 3].

A fluid transient analysis for the propellant flow in a satellite propulsion system is needed to verify the design of propulsion system. This work is not to verify the design of propulsion system of Koreasat 1 but to identify it through parametric studies. Main parameter is the thruster valve operation time, and given conditions are pipeline materials and number of pressure drop devices (i.e. filters, latching isolation valves, etc.) and pipelines and their lengths. They are fully considered in a model to analyze fluid transients.

Algebraic simultaneous equations method and Cramer's rule method utilized to drive the compatible and characteristic equations are reviewed to understand the MOC extensively

2. Derivations

2.1 Basic differential equations for transient flow

The one-dimensional unsteady pressure flow equations are given by

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$$gH_x + V_t + \frac{f}{2D}V|V| = 0 \quad (1)$$

$$H_t + \frac{a^2}{g}V_x = 0 \quad (2)$$

The momentum equation (1) and continuity equation (2) form a pair of quasilinear hyperbolic partial differential equations in terms of two dependent variables, velocity (V) and hydraulic-grad-line elevation (H), and two independent variables, distance along the pipe (x) and time (t). The equations are transformed into four ordinary differential equations by using the method of characteristics. The subscripts x and t denote partial differentiation (i.e., $H_x = \partial H/\partial x$). In the following section the derivation of four ordinary differential equations are reviewed and based on [1] and [4] in order to understand the method of characteristics more extensively.

2.1 Algebraic simultaneous equations and calculus

The simplified equations of motion and continuity are identified as L_1 and L_2 [from Eqs. (1) and (2)]:

$$L_1 = gH_x + V_t + \frac{f}{2D}V|V| = 0 \quad (3)$$

$$L_2 = H_t + \frac{a^2}{g}V_x = 0 \quad (4)$$

These equations are combined linearly using an unknown multiplier λ:

$$\begin{aligned} L &= L_1 + \lambda L_2 \\ &= gH_x + V_t + \frac{f}{2D}V|V| + \lambda\left(H_t + \frac{a^2}{g}V_x\right) \\ &= \lambda\left(H_x\frac{g}{\lambda} + H_t\right) + \left(V_x\lambda\frac{a^2}{g} + V_t\right) + \frac{f}{2D}V|V| \\ &= 0 \end{aligned} \quad (5)$$

Any two real, distinct values of λ will again yield two equations in terms of the two dependent variables H and V that are in every way the equivalent of Eqs. (1) and (2). Appropriate selection of two particular values of λ leads to simplification of Eq. (5). In general, both variables H and V are functions of x and t. If the independent variable x is permitted to be a function of t, then, from total differential of calculus,

$$\frac{dH}{dt} = H_x\frac{dx}{dt} + H_t \quad \frac{dV}{dt} = V_x\frac{dx}{dt} + V_t \quad (6)$$

Now, by examination of Eq. (5) with Eqs. (6) in mind, it can be noted that if

$$\frac{dx}{dt} = \frac{g}{\lambda} = \frac{\lambda a^2}{g} \quad (7)$$

Eq. (5) becomes the ordinary differential equation

$$\lambda\frac{dH}{dt} + \frac{dV}{dt} + \frac{f}{2D}V|V| = 0 \quad (8)$$

The solution of Eq. (7) yields the two particular values of λ,

$$\lambda = \pm\frac{g}{a} \quad (9)$$

By substituting these values of λ back into Eq. (7), the particular manner in which x and t are related is given:

$$\frac{dx}{dt} = \pm a \quad (10)$$

This shows the change in position of a wave related to the change in time by the wave propagation velocity a. When the positive value of λ is used in Eq. (7), the positive value of λ must be used in Eq. (8). A similar parallelism exists for the negative λ. The substitution of these values of λ into Eq. (8) leads to two pairs of equations which are grouped and identified as C+ and C− equations.

$$C^+ : \begin{cases} \dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{f}{2D}V|V| = 0 & (11) \\ \dfrac{dx}{dt} = +a & (12) \end{cases}$$

$$C^- : \begin{cases} -\dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{f}{2D}V|V| = 0 & (13) \\ \dfrac{dx}{dt} = -a & (14) \end{cases}$$

Thus two real values of λ have been used to convert the original two partial differential equations to two total (ordinary) differential equations, Eqs. (11) and (13), each with the restriction that it is valid only when the respective Eqs. (12) and (14) are valid.

It is convenient to visualize the solution as it develops on the independent variable plane (i.e., the xt plane). Inasmuch as a is generally constant for a given pipe. Eq. (12) plots as a straight line on the xt plane; and similarly, Eq. (14) plots as a different straight line as shown in Fig. 1. These lines on the xt plane are the "characteristic" lines along which Eqs. (11) and (14) are valid. The latter equations are referred to as compatibility equations, each one being valid only on the appropriate characteristic line. No mathematical approximations have been made in this transformation of the original partial differential equations. Thus every solution of this set will be a solution of the original system given by Eqs. (3) and (4).

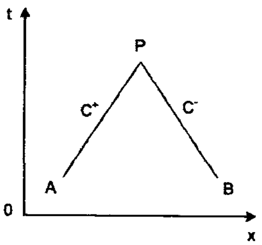

Fig.1 Characteristic lines in the xt plane

2.3 A method based on Cramer's rule

Rearrange the system of quasi-linear equations given Eqs. (1), (2) and (6) as

$$gH_x + V_t = -\frac{f}{2D}V|V| \tag{15}$$

$$H_t + \frac{a^2}{g}V_x = 0 \tag{16}$$

$$dH = H_x dx + H_t dt \tag{17}$$

$$dV = V_x dx + V_t dt \tag{18}$$

Eqs. (15) through (18) constitute a system of four linear equations with four unknowns (H_x, H_t, V_x, and V_t). These equations can be written in matrix form as

$$\begin{bmatrix} g & 0 & 0 & 1 \\ 0 & 1 & \dfrac{a^2}{g} & 0 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix} \begin{bmatrix} H_x \\ H_t \\ V_x \\ V_t \end{bmatrix} = \begin{bmatrix} -\dfrac{f}{2D}V|V| \\ 0 \\ dH \\ dV \end{bmatrix} \tag{19}$$

Let [A] denote the coefficient matrix.

$$[A] = \begin{bmatrix} g & 0 & 0 & 1 \\ 0 & 1 & \dfrac{a^2}{g} & 0 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix} \tag{20}$$

Let us solve Eq. (19) for the unknown H_x, using Cramer's rule. To do this, we define the matrix [B] as the matrix [A] with its first column replaced by the column vector on the right-hand side of Eq. (19), i.e.,

$$[B] = \begin{bmatrix} -\dfrac{f}{2D}V|V| & 0 & 0 & 1 \\ 0 & 1 & \dfrac{a^2}{g} & 0 \\ dH & dt & 0 & 0 \\ dV & 0 & dx & dt \end{bmatrix} \tag{21}$$

Denoting the determinants of [A] and [B] by |A| and |B|, respectively, Cramer's rule gives the solution for H_x as

$$H_x = \frac{|B|}{|A|} \tag{22}$$

Recall from linear algebra that, if a square coefficient matrix for a set of n linear equations has a vanishing determinant, then a necessary condition for finite solutions to exist is that when the RHS is substituted for any column of the coefficient matrix, the resulting determinant must also vanish (cf., the Cramer's rule for solving linear systems of equations).

If only |A| were zero, then H_x would be infinite. However, the definition of a characteristic line states that H_x be indeterminant along the characteristic, not infinite. Thus, for H_x to be indeterminant, |B| in Eq. (22) must also be zero. Then, H_x is of the form

$$H_x = \frac{|B|}{|A|} = \frac{0}{0} \tag{23}$$

namely, an indeterminant form which can have a finite value. Hence, from Eqs. (20) and (21)

$$|A| = \begin{vmatrix} g & 0 & 0 & 1 \\ 0 & 1 & \frac{a^2}{g} & 0 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{vmatrix} = 0 \tag{24}$$

$$|B| = \begin{vmatrix} -\frac{f}{2D}V|V| & 0 & 0 & 1 \\ 0 & 1 & \frac{a^2}{g} & 0 \\ dH & dt & 0 & 0 \\ dV & 0 & dx & dt \end{vmatrix} = 0 \tag{25}$$

Expansion of the determinant in Eq. (24) yields a characteristic line as Eq. (10). Expansion of the determinant in Eq. (25) yields an ordinary differential equation in term of dH and dV, where dx and dt are restricted to hold along a characteristic line. The equation for the dependent variables H and V which comes from Eq. (25) is called the compatibility equation as Eqs (11) and (13). It is an equation involving the unknown dependent variables which holds only along the characteristic line; the advantage of this compatibility equation is that it is in one less dimension than the original partial differential equations. Since the governing equations given in this paper are partial differential equations in two dimensions, then the compatibility equation is in one dimension-hence it is an ordinary differential equation-and the "one dimension" is along the characteristic direction.

2.3 A method based on eigenvalues of the system

The eigenvalue method is based on a display of the system of partial differential equations, Eqs. (1) and (2) written in column vector form.

Defining W as the column vector

$$W = \begin{bmatrix} H \\ V \end{bmatrix} \tag{26}$$

the system of equation given by Eqs. (1) and (2) can be written as

$$\begin{bmatrix} g & 0 \\ 0 & \frac{a^2}{g} \end{bmatrix} \frac{\partial W}{\partial x} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{\partial W}{\partial t} = \begin{bmatrix} -\frac{f}{2D}V|V| \\ 0 \end{bmatrix} \tag{27}$$

or

$$[K]\frac{\partial W}{\partial x} + [M]\frac{\partial W}{\partial t} = \begin{bmatrix} -\frac{f}{2D}V|V| \\ 0 \end{bmatrix} \tag{28}$$

where [K] and [M] are the appropriate 2 × 2 matrices in Eq. (27). Multiplying Eq. (27) by the inverse of [K], we have

$$\frac{\partial W}{\partial x} + [K]^{-1}[M]\frac{\partial W}{\partial t} = [K]^{-1}\begin{bmatrix} -\frac{f}{2D}V|V| \\ 0 \end{bmatrix} \tag{29}$$

or

$$\frac{\partial W}{\partial x} + [N]\frac{\partial W}{\partial t} = [K]^{-1}\begin{bmatrix} -\frac{f}{2D}V|V| \\ 0 \end{bmatrix} \tag{30}$$

where by definition $[N]=[K]^{-1}[M]$. The eigenvalues of [N] are precisely the slopes of the characteristic lines and determine the classification of the system. If the eigenvalues are all real, the equations are hyperbolic. If the eigenvalues are all complex, the equations are elliptic.

The eigenvalues of [N] is examined as

$$[N]=[K]^{-1}[M]$$
$$=\begin{bmatrix} g & 0 \\ 0 & \frac{a^2}{g} \end{bmatrix}^{-1}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 0 & \frac{1}{g} \\ \frac{g}{a^2} & 0 \end{bmatrix} \tag{31}$$

$$\left|[N]-\lambda[I]\right|=0 \tag{32}$$

where [I] is the identity matrix. Hence

$$\begin{vmatrix} -\lambda & \frac{1}{g} \\ \frac{g}{a^2} & -\lambda \end{vmatrix}=0 \tag{33}$$

Expanding the determinant, we have

$$\lambda^2-\frac{1}{a^2}=0 \tag{34}$$

or

$$\lambda=\pm\frac{1}{a}$$
$$=\frac{dt}{dx} \tag{35}$$

Rearrange Eq. (30), Eqs. (17) and (18) to obtain the compatibility equation as results

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{g} \\ 0 & \frac{g}{a^2} & 1 & 0 \\ dx & dt & 0 & 0 \\ 0 & 0 & dx & dt \end{bmatrix}\begin{bmatrix} H_x \\ H_t \\ V_x \\ V_t \end{bmatrix}=\begin{bmatrix} -\frac{f}{2Dg}V|V| \\ 0 \\ dH \\ dV \end{bmatrix} \tag{36}$$

but this equation is equivalent to Eq. (3-17) through little manipulation so that it follows the same procedure to get the compatibility equations.

2.4 Finite Difference Equations

So far the different methods are reviewed and the two PDEs of continuity and momentum conservations are transformed into four ODEs solved numerically using finite difference techniques. It refers parts of FDM to literatures [1, 4].

3. Fluid transient analysis

3.1 Propulsion system of Koreasat 1

It shows schematic of propulsion system of Koreasat 1 in Fig. 2. It is a hydrazine monopropellant blowdown subsystem. It is divided two independent half-systems each capable of performing all thrusting maneuvers and each half-system consists of two 0.5 m diameter tanks supplying fuel for six 0.9 N catalytic rocket engine assemblies (REAs) and two 0.4 N electrothermal hydrazine thrusters (EHTs). A Ti all-welded *manifold network include latching* valves, *fill* and drain valves, filters, pressure and temperature instrumentation, and thermal control equipment. Each half-system will have a normally-open latch valve (LV1 and LV2) which can be closed in the event of leak to isolate the tanks of half-system from its thrusters. The cross-over lines are normally closed off by the latching valves (LV3 and LV4), which can be opened if one thruster set must be isolated due to leakage [5].

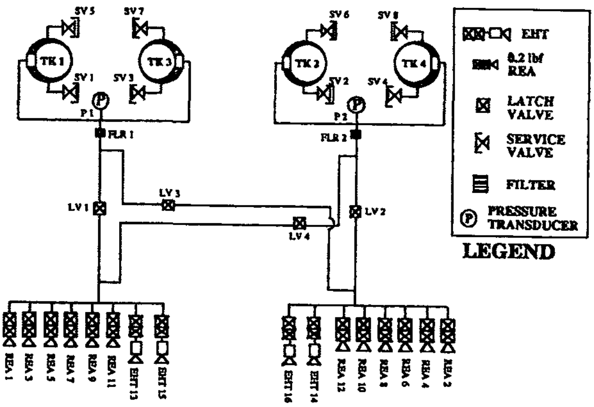

Fig. 2 Schematic of propulsion system

3.2 Thruster valve operations

The thruster inlet pressure range of the REAs is from 400 psia to 80 psia and EHTs, 350 psia to 85 psia as shown in the Table 1. The response times of thruster valve are usually less than 20 ms, in this work 1 ms, 10 ms and 20 ms are considered as parameters shown in the Table 2. Other conditions are the pipeline materials, Ti-3Al-2.5V and 304L SS, an on/off duty cycle of 250 ms on/750 ms off.

Table 1 Operation and allowable inlet pressure ranges

Thruster	Operation inlet pressure range (psia)	Allowable inlet pressure range (psia)
REA	400 - 80	420 - 70
EHT	350 - 85	350 - 80

In the Table 2 the specific impulse requirements is shown.

Table 2 Specific Impulse Requirements

Feed Pressure Range (psia)	Duty Cycle (s) On	Duty Cycle (s) Off	Isp (minimum)
350 - 300	0.25	0.75	192
350 - 300	3.00	9.00	216
350 - 100	All others		113
350 - 100	1.0	83.2	137

Parametric studies are conducted according to the Table 3. In Fig. 3 the thruster valve opening/closing times are shown with 250 ms On/750 ms Off.

Table 3 Case Studies

Pressure (psia)	Thruster valve opening/closing time (ms)		
400	20	10	1
80	20	10	1

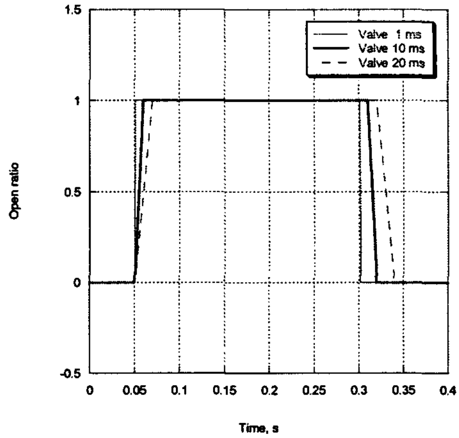

Fig. 3 Thruster Valve Opening/Closing Operations

3.3 Results

The calculations are conducted for the opening and closing thruster valves of REA 5, 6, 7, and 8. But this section shows only the results of REA 8 because of similarities of the results.

3.3.1 Inlet Pressure at 400 psia

Because of thruster valve opening/closing the fluid flow rate at the Latching Isolation Valve, LV2 oscillates and the inlet pressure of thruster valve oscillates as shown Figs. 4 and 5, respectively.

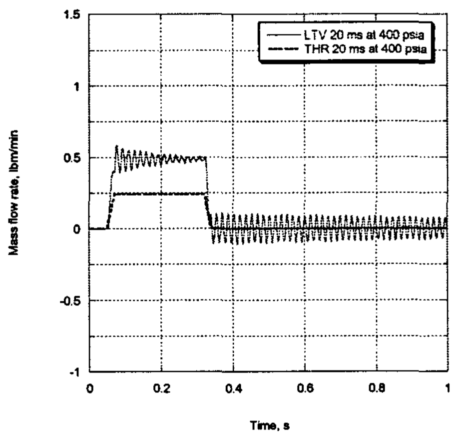

Fig. 4 Mass flow rate of 20 ms at 400 psia

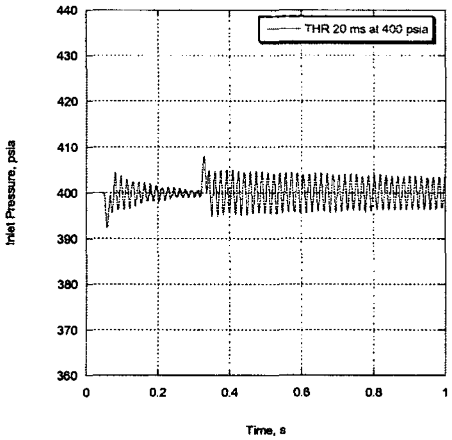

Fig. 5 Inlet Pressure of 20 ms at 400 psia

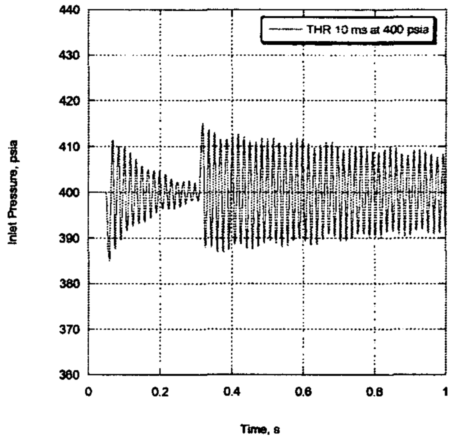

Fig. 7 Inlet Pressure of 10 ms at 400 psia

Because thruster valve opening/closing time is getting shorter, 10 ms, the oscillation amplitudes of fluid flow rate and the inlet pressure of thruster valve are enlarged as shown Figs. 6 and 7, respectively. The inlet pressure difference is more than 20 psid so it may have negative effects on the thruster performance.

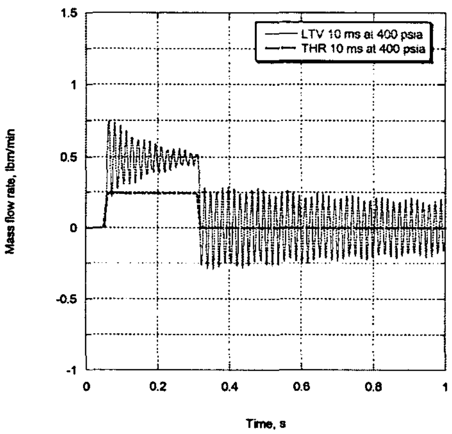

Fig. 6 Mass flow rate of 10 ms at 400 psia

In the worst case the thruster valve opening/closing time is 1 ms, the oscillation amplitudes of fluid flow rate and the inlet pressure of thruster valve are so increased as shown in Figs. 8 and 9, respectively, that the inlet pressure exceeds the allowable limit in Table 1. The inlet pressure difference is of more than 60 psid, unless an proper orifice being installed, it may not run at this condition.

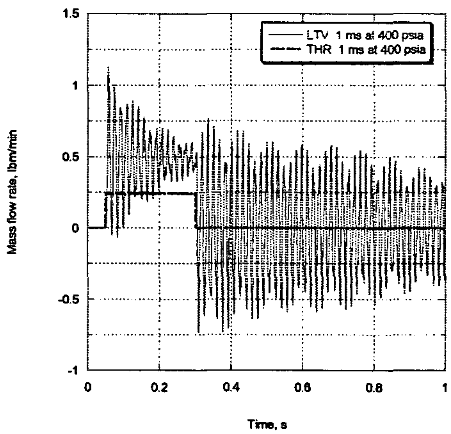

Fig. 8 Mass flow rate of 1 ms at 400 psia

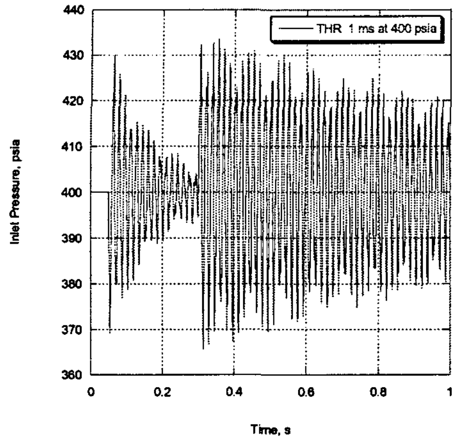

Fig. 9 Inlet Pressure of 1 ms at 400 psia

3.3.2 Inlet Pressure at 80 psia

The inlet pressure of thruster valve is reduced to 80 psia. But the fluctuation of the fluid flow rate and the inlet pressure of thruster valve at 80 psia case are similar to the results of 400 psia as shown in Figs. 10 to 15, respectively.

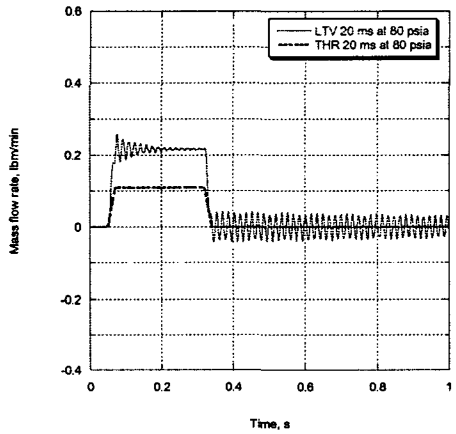

Fig. 10 Mass flow rate of 20 ms at 80 psia

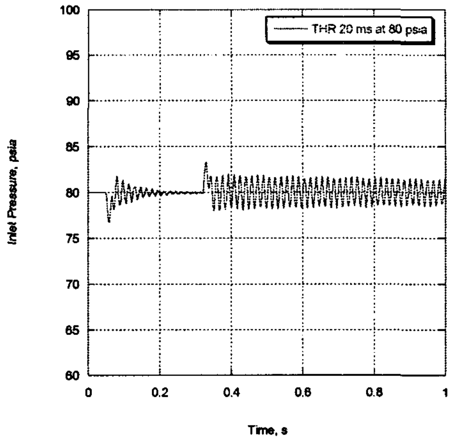

Fig. 11 Inlet Pressure of 20 ms at 80 psia

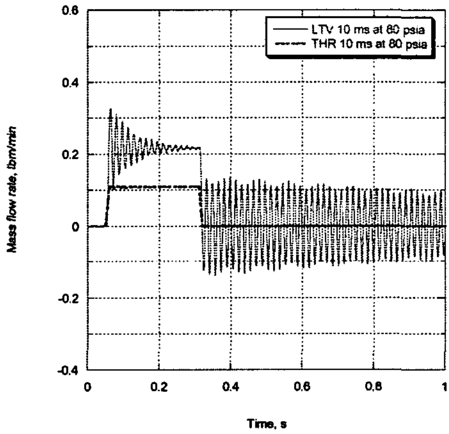

Fig. 12 Mass flow rate of 10 ms at 80 psia

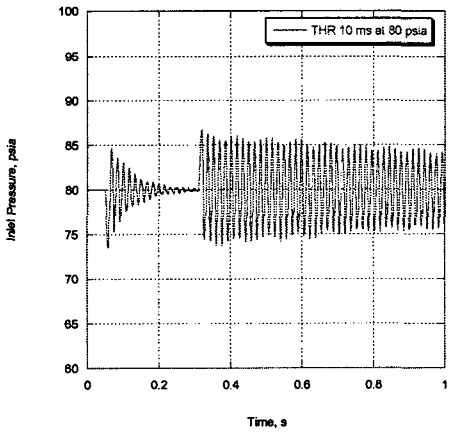

Fig. 13 Inlet Pressure of 10 ms at 80 psia

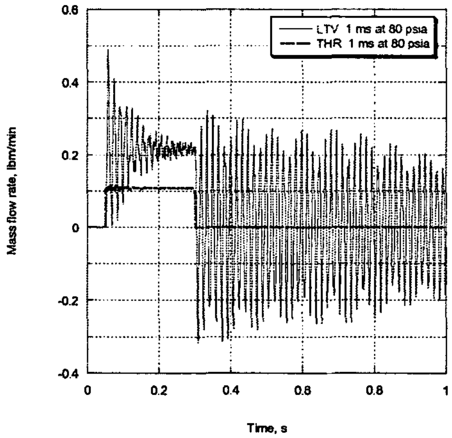

Fig. 14 Mass flow rate of 1 ms at 80 psia

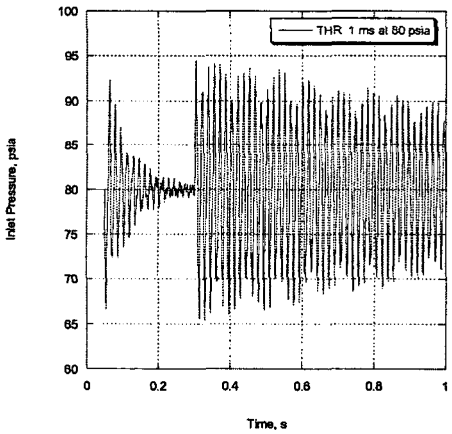

Fig. 15 Inlet Pressure of 1 ms at 80 psia

4. Conclusion

This work is to review the different derivation methods that result in the same compatible and characteristic equations in order to understand the method of characteristics extensively.

This work is done for the identification of fluid transient phenomena of Koreasat 1 through parametric studies. The valve response time is one of the dominant parameters governing the fluid transient phenomena. The results show that shorter closing time induces the greater pressure response amplitude.

Naturally Koreasat 1 has no orifice, as shown in Fig. 2, to control flow transients or to limit the initial hydrazine flow rate for the first-pulse firing because the propulsion system is designed to endure flow transients to some extent and it has launched with the hydrazine filled to the thruster valves. If a propulsion system in transfer orbit operation fills out the lines, which being made in highly vacuum, with hydrazine, the hydrazine moves at very high velocity due to hugh pressure difference. When the hydrazine reaches the thruster valves, the deceleration shock can be energetic enough to decompose the hydrazine. To avoid this problem, orifices are required in the propellant lines to limit the initial propellant flow rate [6].

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