Efficient Maximum-Likelihood and Sub-optimal Decoders for V-BLAST

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ABSTRACT

In this paper, a low-complexity ML decoder based on QR decomposition, called LCMLDec decoder, is proposed for the V-BLAST with 2 transmit antennas. Then, it is combined with other suboptimal interference nulling and cancelling decoders originated form QR decomposition such as sorted QR or MMSE-SQRD decoder to generate efficient decoders that significantly improve the performance of the V-BLAST with more than 2 transmit antennas, at the expense of a small increase in complexity. Simulation results are provided to demonstrate the performances and complexities of the proposed decoders.

Keywords
LAST, MIMO systems, diversity, wireless communication, Zero-Forcing.

I. INTRODUCTION

The deployment of multipletransmit and receive antennas, resulting in the so-called multiple input multiple output (MIMO) systems, in rich-scattering environments is theoretically shown to be capable of enormously increasing spectral efficiencies [1]-[2]. A MIMO system, which has been implemented in real time and demonstrated its performance in an indoor slow-fading environment for confirming the theoretical results, is the Vertical Bell Labs Layered Space-Time (V-BLAST) architecture [3]. Initial results of the V-BLAST showed that spectral efficiencies ranging from 20 to 40bits/s/Hz can be obtained. Undoubtedly, those spectral efficiencies are unattainable using traditional techniques.

In order to decode the V-BLAST architecture, various decoding algorithms can be used. Obviously, brute-force maximum likelihood (ML) decoder is the optimal one for the V-BLAST. Nonetheless, it is not a preferable detection method due to its complexity, which is exponential with the number of transmit antennas. To avoid the complexity problem of brute-force ML detection, suboptimal detection schemes have been developed. Possible suboptimal methods include linear decoders, namely, zero forcing (ZF) or minimum mean square error (MMSE) [4], and decoders using interference nulling and successive interferencecancellation such as ZF-BLAST [3], QR-decomposition (QRD) [4], sorted QR-decomposition (SQRD) [5], MMSE-BLAST [6], or MMSE-SQRD [7]-[8]. Nevertheless, for the V-BLAST employing equal transmit and receive antennas, a new problem arises from the fact that the use of interference suppression, either by ZF, by MMSE, or by QR decomposition, has reduced the diversity order of the first detected symbol to one, leading to a very poor system performance. As shown in [7]-[8], although MMSE-BLAST and MMSE-SQRD decoders remarkably improve system performance compared to other suboptimal decoders such as ZF-BLAST or SQRD, the slopes of the bit-error-rate (BER) curves of the two decoders indicate that they are able to improve diversity of the system only in the low signal-to-noise power ratio (SNR) region. As the SNR moves toward high values, the diversity orders provided by MMSE-BLAST and MMSE-SQRD decoders tend to reduce to one again.

In this paper, based on QR decomposition, we first propose a low-complexity ML decoder for a V-BLAST scheme with $n_T = 2$ transmit antennas, called LCMLDec decoder. We then combine the proposed LCMLDec decoder with the conventional QRD decoder to create a new high-performance suboptimal decoder, called HPQRD decoder, for V-BLAST schemes with more than 2 transmit antennas. In the HPQRD decoding algorithm, the LCMLDec decoder is used to jointly detect the first two layers, while the QRD decoder is used to detect the rest layers. As a consequence, HPQRD decoder is able to provide V-BLAST schemes with a diversity order of 2. We also propose two other suboptimal decoders, namely, HPSQRD and HPMMSESQRD decoders, which are respectively the combinations of LCMLDec decoder with the SQRD and the MMSE-SQRD decoders, so as to further enhance system performance. Performances and complexities of the proposed decoders are investigated via computer simulation.

II. SYSTEM MODEL

We consider an uncoded V-BLAST configuration with n_T transmit and $n_R \geq n_T$ as shown in **Fig. 1**.

At the transmitter, the input data sequence is partitioned into n_T sub-streams (layers), each of which is then modulated by an M-level modulation scheme and transmitted from a different transmit antenna. For the sake of simplicity, we investigate one-time-slot complex baseband signal model, where at each symbol period a $n_T \times 1$ transmit signal vector $\mathbf{s}$ consisting of n_T symbols, s_i, $i = 1, \ldots, n_T$, is sent through n_T transmit antennas.
Under the assumptions that the signals are narrowband and the channel is quasi-static, *i.e.*, it remains constant during some block of arbitrary length and changes from one block to another, the relationship between transmitted

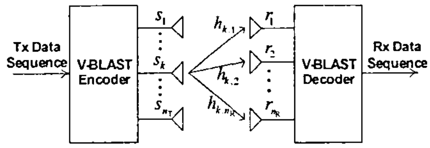

Fig. 1 Model of a V-BLAST scheme with n_T transmit and n_R receive antennas.

and received signals can be expressed in the following form:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \qquad (1)$$

where $\mathbf{r} = [r_1, \ldots, r_{n_R}]^T$ is the $n_R \times 1$ received signal vector, T denotes the transpose of a matrix, $\mathbf{w} = [w_1, \ldots, w_{n_R}]^T$ represents the noise samples at n_R receive antennas, which are modeled as independent samples of a zero-mean complex Gaussian random variable with noise variance σ^2, H is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas *modeledas the samples of a zero-mean complex* Gaussian random variable with equal variance of 0.5 per complex dimension. In the paper, we assume that the signals transmitted from individual antenna have equal powers of P/n_T, *i.e.*, $E\{\mathbf{s}\mathbf{s}^H\} = \frac{P}{n_T}\mathbf{I}_{n_T}$, H denotes the Hermitian transpose of a matrix, $\mathbf{I}_{n_T}$ indicates the $n_T \times n_T$ *identity matrix.*

III. PROPOSED LCMLDEC DECODER

Assuming the channel gains are perfectly known, the ML decoder at the receiver is described by the following rule:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in C} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \qquad (2)$$

where C is the transmission constellation, and $\|\mathbf{a}\|^2 = \mathbf{a}^H\mathbf{a}$.

The computational load of the ML decoder given by (2) is of order M^{n_T}, which is intractable for large number of transmit antennas and/or high-level modulation schemes. Therefore, in this and the following subsections,

```
[ŝ] = LCMLDec(v, R, x, N)
  for k = 2:-1:1
  if k == 2
  RX(k,:) = R(k,k) * x
  t = |v(k) - RX(k,:)|^2
  [d, i] = sort(t)
  else
  RX(k,:) = R(k,k) * x
  t = |v(k) - R(k,2) * x(i(1)) - RX(k,:)|^2
  [d1, i1] = min(t + d(1))
   end
  end
  Dmin = d1   ŝ = [x(i1); x(i(1))]   j = 2
  while j ≤ N and d(j) < Dmin
  t = |v(1) - R(1,2) * x(i(j)) - RX(1,:)|^2
  [d1, i1] = min(t + d(j))
  if Dmin > d1
       Dmin = d1   ŝ = [x(i1); x(i(j))]
  End
  j = j + 1
  end
```

Note: sort() and min() are MATLAB functions. $\mathbf{B}(k,:)$ denotes the *kth*row of matrix $\mathbf{B}$.

Fig. 2 Proposed LCMLDec decoding algorithm

we present a low complexity ML decoder for $n_T = 2$ based on QR decomposition and different high-performance suboptimal decoders for $n_T > 2$.

When $n_T = 2$, the Modified Gram-Schmidt (MGS) algorithm [9] enable the $(n_R \times 2)$ channel matrix $\mathbf{H}$ to be factorized as:

$$\mathbf{H} = \mathbf{QR} \tag{3}$$

where $\mathbf{Q}$ is a $(n_R \times 2)$ unitary matrix, *i.e.*, $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_2$, and $\mathbf{R}$ is a (2×2) upper triangular matrix .

Pre-multiplying both sides of (1) with $\mathbf{Q}^H$ yields:

$$\mathbf{v} = \mathbf{Rs} + \mathbf{n} \tag{4}$$

where $\mathbf{v} = [v_1; v_2]^T = \mathbf{Q}^H\mathbf{r}$ and $\mathbf{n} = [n_1; n_2]^T = \mathbf{Q}^H\mathbf{w}$ are respectively the received signal vector and the noise vector after QR decomposition.

Using (3) and the unitary property of $\mathbf{Q}$, we can easily show that:

$$\|\mathbf{r} - \mathbf{Hs}\|^2 = \|\mathbf{v} - \mathbf{Rs}\|^2 + \mathbf{r}^H\mathbf{r} - \mathbf{r}^H\mathbf{QQ}^H\mathbf{r} \tag{5}$$

Since the last two terms in the right-hand side of (5) are not functions of $\mathbf{s}$, finding $\mathbf{s}$ to minimize $\|\mathbf{r} - \mathbf{Hs}\|^2$ amounts to finding $\mathbf{s}$ to minimize $\|\mathbf{v} - \mathbf{Rs}\|^2$. Consequently, the proposed decoder chooses the ML solution, $\hat{\mathbf{s}} = (\hat{s}_1; \hat{s}_2)$, from the constellation C that satisfies:

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s} \in C} \|\mathbf{v} - \mathbf{Rs}\|^2 \tag{6}$$

Let us suppose that $\mathbf{x}$ is an array containing all N signal points (*e.g.*, for 16-QAM, $N = 16$) within the transmission constellation C. The proposed LCMLDec algorithm for the detection of the transmitted signal vector $\mathbf{s}$ based on (6) is described in **Fig. 2**.

By employing the proposed algorithm, the transmitted symbolsare detected as follows:

$$[\hat{\mathbf{s}}] = \mathbf{LCMLDec}(\mathbf{v}, \mathbf{R}, \mathbf{x}, N)$$

IV. PROPOSED HPQRD, HPSQRD, AND PMMSESQRD DECODERS

The biggest drawbacks associated with decoders using interference nulling and cancellation, namely, ZF-BLAST, QRD, SQRD, and MMSE-SQRD, are perhaps the reduction in the diversity order of the first detected layer and the error propagation. Especially, in V-BLAST schemes with $n_R = n_T$, the diversity order of the first detected layer reduces to one, thereby causing severe degradation in the system performance [3]-[4], [8]. In order to

HPQRD DECODER
INPUT: $\mathbf{r};\mathbf{H};\mathbf{x};N$
OUTPUT: $\hat{\mathbf{s}}=(\hat{s}(1),\hat{s}(2),\ldots,\hat{s}(n_T))$
1. Preprocessing $[\mathbf{Q},\mathbf{R}]=\mathrm{QRD}(\mathbf{H})$ %QR decomposition of $\mathbf{H}$ $\mathbf{v}=\mathbf{Q}^H\mathbf{r}$ *2. Searching* $\mathbf{v1}=\mathbf{v}(n_T-1:n_T)$ $\mathbf{R1}=\mathbf{R}(n_T-1:n_T,n_T-1:n_T)$ $[\hat{s}(n_T-1),\hat{s}(n_T)]=\mathbf{LCMLDec}(\mathbf{v1},\mathbf{R1},\mathbf{x},N)$ for $k=n_T-2:-1:1$ $\tilde{s}=\left[\mathbf{v}(k)-\sum_{i=k+1}^{n_T}\mathbf{R}(k,i)\hat{s}(i)\right]/\mathbf{R}(k,k)$ $\hat{s}(k)=q(\tilde{s})$ end
Note: $\mathbf{b}(n:m)$ denotes the elements of $\mathbf{b}$ from row n to row m. $\mathbf{B}(n:m,l:k)$ denotes the submatrix of $\mathbf{B}$ with elements extracted from rows, $n,\ldots,m$ and columns $l,\ldots,k$. $q(\cdot)$ denotes the slicing operation.

Fig. 3 Proposed HPQRD decoding algorithm.

improve the performances V-BLAST schemes, it is of crucial importance to improve the reliability of the first detected layer. To achieve the goal, we propose to combine the LCMLDec decoder with the decoders originated from QR decomposition, say, QRD, SQRD, and MMSE-SQRD. In the new suboptimal detection algorithms, called HPQRD, HPSQRD, and HPMMSESQRD decoders, the LCMLDec algorithm is used to detect the first two layers and the QRD, SQRD, and MMSE-SQRD algorithms are used to detect the remaining ones.

A. HPQRD Decoder

HPSQRD DECODER
INPUT: $\mathbf{r};\mathbf{H};\mathbf{x};N$
OUTPUT: $\hat{\mathbf{s}}=(\hat{s}(1),\hat{s}(2),\ldots,\hat{s}(n_T))$
1. Preprocessing $[\mathbf{Q},\mathbf{R}]=\mathrm{SQRD}(\mathbf{H})$ %Sorted QR decomposition of $\mathbf{H}$ $\mathbf{v}=\mathbf{Q}^H\mathbf{r}$ *2. Searching* $\mathbf{v1}=\mathbf{v}(n_T-1:n_T)$ $\mathbf{R1}=\mathbf{R}(n_T-1:n_T,n_T-1:n_T)$ $[\hat{s}(n_T-1),\hat{s}(n_T)]=\mathbf{LCMLDec}(\mathbf{v1},\mathbf{R1},\mathbf{x},N)$ for $k=n_T-2:-1:1$ $\tilde{s}=\left[\mathbf{v}(k)-\sum_{i=k+1}^{n_T}\mathbf{R}(k,i)\hat{s}(i)\right]/\mathbf{R}(k,k)$ $\hat{s}(k)=q(\tilde{s})$ end

Fig. 4 Proposed HPSQRD decoding algorithm

In this detection algorithm, the channel matrix H is factorized using the MGS algorithm [9]. The description of HPQRD decoder is shown in **Fig. 3**.

B. HPSQRD Decoder

In this detection algorithm, the channel matrix H is decomposed using the sorted QR decomposition [5]. The description of HPSQRD decoder is presented in **Fig. 4**.

C. HPMMSESQRD Decoder

Similar to the MMSE-SQRD decoder proposed in [7]-[8], in the HPMMSESQRD detection algorithm, the $(n_T+n_R)\times n_T$ extended channel matrix $\underline{\mathbf{H}}$ and the $(n_T+n_R)\times 1$ extended receive signal vector $\underline{\mathbf{r}}$ defined by

$$\underline{\mathbf{H}}=\begin{bmatrix}\mathbf{H}\\ \sigma\mathbf{I}_{n_T}\end{bmatrix}\text{ and }\underline{\mathbf{r}}=\begin{bmatrix}\mathbf{r}\\ \mathbf{0}_{n_T,1}\end{bmatrix}\qquad(7)$$

are also employed for detection. The summar

HPMMSESQRD DECODER

INPUT: $\mathbf{r}; \mathbf{H}; \mathbf{x}; N$

OUTPUT: $\hat{\mathbf{s}} = (\hat{s}(1), \hat{s}(2), \ldots, \hat{s}(n_T))$

1. Preprocessing

$[\mathbf{Q}, \mathbf{R}] = \text{MMSESQRD}(\mathbf{H})$ %Factorization of **H**

$\mathbf{v} = \mathbf{Q}^H \mathbf{r}$

2. Searching

$\mathbf{v1} = \mathbf{v}(n_T - 1 : n_T)$

$\mathbf{R1} = \mathbf{R}(n_T - 1 : n_T, n_T - 1 : n_T)$

$[\hat{s}(n_T - 1); \hat{s}(n_T)] = \mathbf{LCMLDec}(\mathbf{v1}, \mathbf{R1}, \mathbf{x}, N)$

for $k = n_T - 2 : -1 : 1$

$$\tilde{s} = \left[\mathbf{v}(k) - \sum_{i=k+1}^{n_T} \mathbf{R}(k,i)\hat{s}(i) \right] \Big/ \mathbf{R}(k,k)$$

$\hat{s}(k) = q(\tilde{s})$

end

Fig. 5 Proposed HPMMSESQRD decoding algorithm

of HPMMSESQRD decoder is shown in **Fig. 5**.

V. SIMULATION RESULTS

To evaluate performances and complexities of our proposed decoders, we apply them to different V-BLAST configurations. For convenience, we denote a V-BLAST system with n_T transmit and n_R receive antennas as the (n_T, n_R) system. In oursimulations, the burst length is set equal to 100 symbol durations. Besides, the channel matrix $\mathbf{H}$ is assumed to remain constant within one burst and changes randomly from one burst to another.

In **Fig. 6**, BER performances versus SNR per receive antenna of the MMSE-BLAST, joint ML (brute-force ML), and the proposed LCMLDec decoders for (2, 2) and (2, 4) systems using 8-PSK modulation are provided. Here,

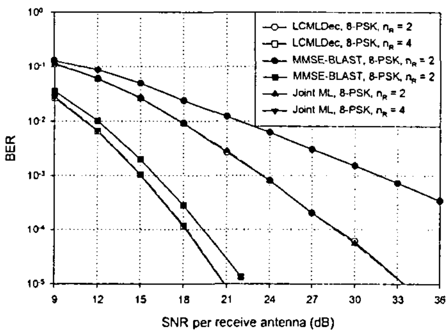

Fig. 6 BER performances of the MMSE-BLAST, joint ML, and LCMLDec decoders for (2, 2) and (2,4) systems at 6 bits/s/Hz.

LCMLDec decoder employs sorted QR decomposition [5] to decompose the channel matrix $\mathbf{H}$. Note that the use of sorted or unsorted QR decomposition does not affect the performance of the LCMLDec decoder. Nonetheless, sorted QR decomposition allows it to have lower complexity. It can be seen from **Fig. 6** that our proposed decoder significantly outperforms the MMSE-BLAST decoder. Furthermore, performances of the LCMLDec decoder are almost identical to those of the joint ML decoder for $n_R = 2$ and $(n_R = 4)$.

In Table I, we compare the average complexities per burst of the LCMLDec decoder with those of the joint ML decoder in terms of numbers of floating point operations (flops). In the simulations, 15000 channel realizations are generated. When computing the complexities of the LCMLDec decoder, we have neglected the complexity of QR decomposition since the numbers of antennas are small and the decomposition is required only at the beginning of each burst. **Fig. 6** and Table I clearly demonstrate that the proposed decoder is able to provide the V-BLAST schemes with not only ML performance but also remarkable reduction in detection complexity as compared to the joint ML decoder.

From **Fig. 7** we can see that the proposed decoders are capable of improving diversity. Consequently, they considerably outperform other well-known suboptimal decoders based on interference nulling and cancelling reported in the literature. For example, at BER of 10^{-4}, HPQRD, HPSQRD and HPMMSESQRD decoders respectively offer around 2dB, 4dB,

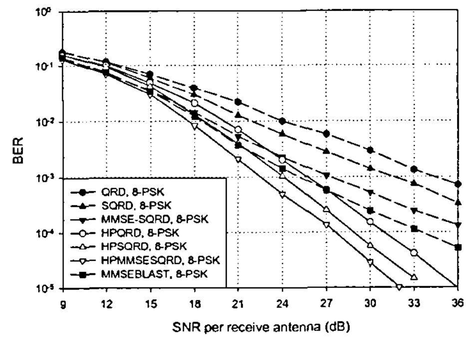

Fig. 7 BER performances of various suboptimal decoders, including the proposed ones, for a (4, 4) system at 12 bit/s/Hz.

and 5.5dB improvement in the system performance over the MMSEBLAST decoder (the highest-performance decoder utilizing interference nulling and cancellation), as illustrated in **Fig. 7**. Among the proposed suboptimal decoders, HPMMSESQRD decoder has highest performance, yet at the cost of highest complexity, as shown in Table II. However, the complexity of HPMMSESQRD decoder is only slightly higher than that of MMSE-SQRD decoder. For instant, at SNR = 9dB, the complexity of HPMMSESQRD decoder is about 112.6% of that needed for performing MMSE-SQRD algorithm, *i.e.*, an increase of approximately 12.6% in the number of flops. Although HPQRD and HPSQRD decoders have higher complexities than does QRD decoder, their complexities are still are still lower than that of the MMSE-SQRD decoder. Regarding both performance and complexity, HPSQRD decoder seems to be the most potential suboptimal decoder, particularly as the number of transmit antennas increases.

TABLE I. AVERAGE COMPLEXITIES PER BURST OF THE JOINT ML AND LCMLDEC DECODERS IN (2,2) AND (2,4) SYSTEMS; BURST OF 100 SYMBOLS.

Flops	LCMLDec $(n_R=2)$	Joint ML $(n_R=2)$	LCMLDec $(n_R=4)$	Joint ML $(n_R=4)$
SNR=9dB	17559	108800	17442	211200
12dB	16238	108800	17065	211200
15dB	15261	108800	16932	211200
18dB	14496	108800	16903	211200

TABLE II. AVERAGE COMPLEXITIES PER BURST OF QRD, MMSE-SQRD, AND THE THREE PROPOSED DECODERS APPLIED TO THE (4, 4) SYSTEM IN FIG. 7; BURST OF 100 SYMBOLS.

Flops	QRD	MMSE-SQRD	HPQRD	HPSQRD	HPMMSE - SQRD
SNR=9d	29600	42400	40311	37889	47752
12dB	29600	42400	38672	36628	47221
15dB	29600	42400	37252	35326	46704
18dB	29600	42400	35966	34467	46392

VI. CONCLUSION

In this paper, we propose a new, low-complexity ML decoder based on QR decomposition, namely, LCMLDec decoder, for a V-BLAST configuration having $n_T=2$ transmit antennas. We also propose suboptimal HPQRD, HPSQRD, and HPMMSESQRD decoders for any V-BLAST scheme with $n_T>2$, which are the combinations of the LCMLDec decoder with the respective QR-decomposition-based decoders including QRD, SQRD, and MMSE-SQRD decoders. At almost the same complexity levels, the proposed decoders noticeably outperform suboptimal decoders based on interference nulling and cancellation such as MMSE-BLAST or MMSE-SQRD. Consequently, they appear to be very promising candidates for the detection of V-BLAST architectures and of other MIMO systems as well.

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