A Test for Autocorrelation in Dynamic Panel Data Models

Hosung Jung *

Department of Economics, Hitotsubashi University, Japan

Abstract

This paper presents an autocorrelation test that is applicable to dynamic panel data models with serially correlated errors. The residual-based GMM t-test is a significance test that is applied after estimating a dynamic model by using the instrumental variable (IV) method and is directly applicable to any other consistently estimated residuals. Monte Carlo simulations show that the t-test has considerably more power than the m_2 test or the Sargan test under both forms of serial correlation (i.e., AR(1) and MA(1)).

Keywords: Dynamic panel data; t-test; $m2$ and Sargan tests

1 Introduction

The main purpose of this paper is to propose a test of serial correlation for dynamic panel models and to compare this test with the m_2 and Sargan tests proposed by Arellano and Bond (1991) (hereafter, AB). If the disturbance has an AR(1) structure, the usual approach of using lagged values of the dependent variables as instruments in the differenced equations, applied by, for example, Anderson and Hsiao (1981, 1982) and Arellano and Bover (1995), is no longer valid. Furthermore, an estimator that uses lags as instruments under the assumption of white noise errors is inconsistent if the disturbances are autocorrelated. Thus, the m_2 and Sargan tests are inapplicable because they use inconsistently estimated residuals based on standard first-difference GMM estimation (hereafter GMM), which also uses invalid instruments. To solve this problem, the t-test utilizes consistently estimated residuals based on IV estimation that uses lags of exogenous variables as instruments for the lagged dependent variables.

The remainder of this paper is organized as follows. In the next section, we present the model and describe the performance of the m_2 and Sargan tests when the disturbances follow an AR(1) process. In Section 3, we propose a t-test for first-order serial

*Corresponding author. Address: 308 1-19-10 Naka Kunitachi-si Tokyou Japan. E-mail: ed031007@srv.cc.hit-u.ac.jp

correlation and show that the t-test is applicable to both forms of serial correlation (i.e., AR(1) or MA(1)). In Section 4, we present the simulation results.

2 Models of the Two Autocorrelation Tests Proposed by AB (1991)

A simple dynamic panel model with strictly exogenous variables and with unobserved individual-specific effects is an autoregressive specification of the following form (e.g., Nerlove, 1971a; Baltagi and Li, 1995):

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + x'_{it}\beta + u_{it}, \qquad |\delta| < 1. \\ u_{it} &= \mu_i + v_{it} \qquad \mu_i \sim \text{NID}(0, \sigma^2_\mu) \end{aligned} \tag{1}$$

where for $i = 1, \cdots, N$ and $t = 2, \cdots, T$. We assume that μ_i and v_{it} have the familiar one-way error component structure in which

$$E(\mu_i) = E(v_{it}) = E(\mu_i v_{it}) = 0 \qquad \forall \quad i, t \tag{2}$$

Adopting the standard assumption that the classical error term, vit, is a white noise error process, AB (1991) noted the validity of the following $p = (T-1)(T-2)/2$ linear moment restrictions for the dynamic model (1) given by

$$E\left[(\Delta y_{it} - \delta \Delta y_{i,t-1}) y_{i,t-j}\right] = 0 \quad \text{for} \quad (j = 2, \cdots, t-1; \quad t = 3, \cdots, T) \tag{3}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$. However, if the standard assumption of a white noise error for v_{it} is violated, these orthogonality conditions no longer hold. Hence, values of y lagged two periods or more cannot be used as instruments for $\Delta y_{i,t-1}$. Consider two alternative cases of serially correlated disturbances. First, consider the case of AR(1) stationary disturbances in the classical error term, v_{it}:

$$v_{it} = \rho v_{i,t-1} + \epsilon_{it} \qquad 0 < \rho < 1 \tag{4}$$

Second, consider an invertible MA(1) disturbance:

$$v_{it} = \epsilon_{it} + \theta \epsilon_{i,t-1} \qquad 0 < \theta < 1 \tag{5}$$

where the innovations are independent over time and are homoskedastic; i.e., $\epsilon_{it} \sim$ i.i.d $N(0, 1)$. Since violation of the orthogonality conditions is expected to affect the $m2$ and Sargan tests, it is worth considering how these statistics behave when there is an AR(1) error process. The consistency of the GMM estimator relies on $E[\Delta u_{it} \Delta u_{i,t-2}] = 0$. Therefore, a test of the hypothesis that there is no second-order serial correlation in the disturbances of the first-differenced equation takes the following form [1]

$$m_2 = \frac{\Delta \hat{v}'_{-2} \Delta \hat{v}_*}{\tilde{v}^{1/2}} \sim \quad \text{N}(0, 1) \tag{6}$$

[1]See AB (1991) for details.

To focus on the effect of the AR(1) serial correlation on the numerator of (6) under $H_1 : 0 < \rho < 1$, we obtain

$$\begin{aligned} E[\Delta u_{it}\Delta u_{i,t-2}] &= E[(v_{it} - v_{i,t-1})(v_{i,t-2} - v_{i,t-3})] \\ &= 2\gamma_2 - \gamma_1 - \gamma_3 = \frac{\sigma_\epsilon^2}{\rho^2 - 1}\left[\rho^2 - 2\rho + 1\right]\rho \\ &= \frac{\rho(\rho-1)}{\rho+1}\sigma_\epsilon^2 \neq 0 \end{aligned} \tag{7}$$

where γ_h is an autocovariance function of (v_{it}) for a fixed i.[2] Equation (7) shows that the usual standard-normal asymptotic result in (6) cannot be used and that the power of the test depends on ρ if the error follows an AR(1) process. The invalidity of the orthogonality condition also affects the power of the Sargan test, as follows:

$$S = \Delta\hat{v}'Z\left(\sum_{i-1}^{N} Z_i'\Delta\hat{v}_i\Delta\hat{v}_i'Z_i\right)^{-1} Z'\Delta\hat{v} \sim \chi^2_{p-1} \tag{8}$$

The effect of the autocorrelated errors on $\Delta\hat{v}'Z$ in S can be succinctly expressed as

$$E[\Delta\hat{v}_{it}'y_{i,t-s}]^2 \cong [\rho(\rho-1)]^2, \cdots, [\rho(\rho-1)]^{2(t-1)} \cdots \quad \text{for} \quad s = 2, \cdots, t-1 \tag{9}$$

Since the $m2$ and Sargan test statistics are functions of $[\rho(\rho-1)]$ and $[\rho(\rho-1)]^2$ respectively, the power of these tests is expected to decrease as ρ approaches unity under the AR(1) alternative.

Proposition 2.1 *The power of the m_2 and Sargan tests is maximized at $\rho = 0.5$ and approximately $\rho = 0.7$, respectively, under the AR(1) alternative. Consequently, for these tests, the probabilities of Type II errors increase as ρ approaches unity, which suggests misspecification.*

3 A Residual-based GMM *t*-test

3.1 The AR(1) Case

The poor performance of the two standard tests, the m_2 and the Sargan test, in the presence of AR(1) disturbances motivates the discussion in this section. We consider using GMM estimation to estimate the residuals, $\hat{u}_{it}$, to test whether the coefficient ρ is significantly different from 0. Whether the unobserved disturbances, v_{it}, follow an AR(1) or an MA(1) process, the first-differenced disturbances in levels are

$$\Delta u_{it} = u_{it} - u_{i,t-1} = v_{it} - v_{i,t-1} = \Delta v_{it}$$

This simple relationship between u_{it} and v_{it} in first differences is useful for deriving our t-test. In the case of an AR(1) disturbance, as in (4), the first-differenced equation is

$$\Delta v_{it} = \rho\Delta v_{i,t-1} + \Delta\epsilon_{it} \tag{10}$$

[2]This equation equals 0 if the errors in the model in levels are not autocorrelated or if they follow a random walk.

When Δv_{it} in the above equation is replaced with the first-differenced Δu_{it}, equation (10) is the same as AB's (1991) AR(1) dynamic random-effects specification. Consequently, we want to test $H_0 : \rho = 0$ after obtaining the GMM estimator, $\hat{\rho}$, and its t-value, $t_{\hat{\rho}}$. Thus, the significance test for ρ in (10) is an autocorrelation test on the classical error term in (1). To perform this test, Δv_{it} in (10) is replaced with the estimated differenced residual, $\Delta\hat{u}_{it}$, which is obtained from the first-step IV estimation. If we use $\Delta\hat{u}_{it} = \Delta u_{it} - \Delta X_{it}(\hat{\delta} - \delta)$, we obtain

$$\begin{aligned} \Delta\hat{u}_{it} &= \rho\Delta\hat{u}_{i,t-1} + \Delta\epsilon_{it} - (\Delta X_{it} - \rho\Delta X_{i,t-1})(\hat{\delta} - \delta) \\ &= \rho\Delta\hat{u}_{i,t-1} + \Delta\eta_{it} \qquad \text{say} \end{aligned} \tag{11}$$

where $X_{it} = (y_{i,t-1}, x_{it})$ and $\hat{\delta} = (\hat{\delta}_{IV}, \hat{\beta}_{IV})'$. For $T \geq 3$, this new derived AR(1) dynamic model (11) implies that the linear moment restrictions in vector form, $E[W'_{ui}\Delta\eta_i] = \mathbf{0}$, are satisfied, where $\eta_i = (\eta_{i3} \cdots \eta_{iT})'$ and W_{ui} is a block diagonal matrix with an sth block of $(\hat{u}_{i1} \cdots \hat{u}_{is})$. The GMM estimator, $\hat{\rho}$, is based on the sample moments $N^{-1}\sum_{i=1}^{N} W_i'\Delta\eta_i$ and is given by

$$\hat{\rho} = \text{argmin}_{\rho}(\Delta\eta' W_u)V_N(W_u'\Delta\eta) \tag{12}$$

where $\Delta\eta = (\Delta\eta_1', \cdots, \Delta\eta_N')$ and $W_u = (W_{u1}', \cdots, W_{uN}')$. The one-step GMM estimator, $\hat{\rho}$, is obtained by setting $V_N = (N^{-1}\sum_{i=1}^{N} W_{ui}'GW_{ui})^{-1}$, where G is a $(T-2)$-dimensional square matrix with 2s on the main diagonal, -1s in the first subdiagonals and 0s elsewhere.

Proposition 3.1 *Under the null of* $H_0 : \rho = 0$

$$t_{\hat{\rho}} = \hat{\sigma}_{\eta}([\Delta\hat{u}_{-1}]'W_u\hat{V}_N^{-1}W_u'[\Delta\hat{u}_{-1}])^{-\frac{1}{2}}([\Delta\hat{u}_{-1}]'W_u\hat{V}_N^{-1}W_u'\Delta\hat{u}) \sim \mathrm{N}(0,1) \tag{13}$$

The proof of asymptotic normality is quite straightforward and is therefore not presented. However, it is worth noting that, unlike the m_2 test, [3] the t-test does not rely on the efficiency of the first-step estimator; i.e., $\hat{\delta}$. Although '$\widehat{\text{avar}}(\hat{\delta} - \delta)$' appears in the estimator of σ_η in (13), it disappears as $N \to \infty$ because $\sqrt{N}(\hat{\delta} - \delta) = O_p(1)$.

3.2 The MA(1) Case

In the previous section, we derived a t-test based on the residuals from IV estimation. In this section, we show that the t-test is valid even if the classical error term in the true disturbances follows an MA(1) process; i.e., $v_{it} = e_{it} + \theta e_{i,t-1}$. As is conventional, we use the m_2 test or the Sargan test to detect any serial correlation in the error term. However, the MA(1) error can be converted to an AR(1) error to apply our t-test, as follows:

$$\begin{aligned} \Delta v_{it} &= \Delta e_{it} + \theta\Delta e_{i,t-1} \\ &= \theta\Delta v_{i,t-1} - \theta^2\Delta v_{i,t-2}, \cdots, +\Delta e_{it} \\ &= \theta\Delta v_{i,t-1} + \sum_{j=2}^{\infty} -(-\theta)^j\Delta v_{i,t-j} + \Delta e_{it} \qquad (14) \\ &= \theta\Delta v_{i,t-1} + \Delta\zeta_{it} \qquad (15) \end{aligned}$$

[3]See Appendix A. in AB (1991) for details.

where $\Delta\zeta_{it} = \sum_{j=2}^{\infty} -(-\theta)^j \Delta v_{i,t-j} + \Delta e_{it}$. This equation is similar to the first-differenced AR(1) specification in (10).[4] In the MA(1) case, the autocorrelation test case is again a significance test on θ. Hence, the t-test can be applied after (15) has been estimated by GMM to test whether θ is significantly different from 0.

Proposition 3.2 *The residual-based GMM t-test is applicable to both forms of serial correlation, AR(1) and MA(1). Hence, under the null of* $H_0 : \theta = 0$, $t_{\hat{\theta}} \sim \mathrm{N}(0.1)$.

However, a shortcoming of the test is that it may not be possible to distinguish between the AR(1) and MA(1) structures if the null hypothesis that $\rho = 0$ is rejected. In this case, we suggest a different testing strategy. First, use the t-test to determine whether serial correlation is present. If it is, apply the m_2 or the Sargan test to determine whether the error follows an MA(1) process. If it does not, conclude that the error term has an AR(1) structure. This two-step testing procedure can detect any first-order serial correlation structure in the error term of a dynamic panel data model.

4 Monte Carlo Experiments

To investigate how the three tests, the m_2 test, the Sargan test and the t-test, perform in practice, Monte Carlo simulations were conducted under the null hypotheses, $\rho = 0$ and $\theta = 0$. Following Nerlove (1971a) and Sevestre and Tronogon (1991), we assume the following data-generating process:

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + \beta x_{it} + u_{it} \\ x_{it} &= \alpha x_{i,t-1} + \omega_{it} \qquad \omega_{it} \sim \mathrm{U}(-1/2, 1/2) \\ u_{it} &= \mu_i + v_{it} \qquad \mu_i \sim N(0,1) \end{aligned}$$

The classical error term, v_{it}, is generated either by the AR(1) process (4) or by the MA(1) process in (5). For x_{i1}, we used ω_{i1}, and for y_{i1}, we generate $\frac{\beta x_{i1}}{1-\delta} + \frac{\mu_i}{(1-\delta)} + \frac{v_{i1}}{\sqrt{1-\delta^2}}$. The testing procedures were repeated five thousand times for each set of parameter values. The parameter δ takes the values $0.3, 0.5, 0.7$ and 0.9 while $\beta = 2, \alpha = 0.4$ remain fixed. We choose the error process parameters, ρ and θ, so that $\rho = 0, 0.1, \cdots 0.9$.

First, the three tests were applied to AR(1) errors. Table 1 shows the size and power of the three test statistics when there is an AR(1) error process . The empirical sizes of the m_2 test, the Sargan test and the t-test are reported in the first row for $\rho = 0$. The tests have reasonable size properties except that the Sargan test rarely rejects the null. Theoretically, the m_2 test and the Sargan test are maximized at around $\rho = 0.5$ and $\rho = 0.7$, respectively. This makes the conventional autocorrelation test difficult to apply as ρ approaches unity because of the increased likelihood of a Type II error. The bias in these two tests implies that the presence of serially correlated errors invalidates the use of lagged values of y as instruments. Consequently, using lagged ys as instruments biases not only standard GMM estimation but also the two serial-correlation tests. However, the t-test is unbiased and consistent because it uses consistently estimated residuals from the IV estimation, which does not use lagged ys as instruments.

[4] The correlation between $v_{i,t-j}$ and ζ_{it} becomes negligible as j increases.

Table 1: Size and Power of the Three Tests (AR(1) error)

	$\delta = 0.3$			0.5			0.7			0.9		
ρ	$m2$	S	t	$m2$	S	t	$m2$	S	t	$m2$	S	t
0.0	0.07	0.01	0.06	0.02	0.01	0.05	0.02	0.01	0.06	0.02	0.03	0.04
0.1	0.19	0.05	0.22	0.14	0.05	0.30	0.13	0.05	0.46	0.18	0.03	0.17
0.2	0.31	0.06	0.67	0.28	0.11	0.77	0.28	0.12	0.84	0.25	0.09	0.68
0.3	0.42	0.31	0.95	0.36	0.16	0.95	0.35	0.18	0.93	0.31	0.20	0.84
0.4	0.43	0.31	0.99	0.53	0.36	0.99	0.52	0.36	0.99	0.46	0.39	0.99
0.5	0.52	0.48	1.00	0.59	0.47	1.00	0.58	0.48	1.00	0.61	0.49	1.00
0.6	0.50	0.54	1.00	0.54	0.61	1.00	0.53	0.62	1.00	0.44	0.44	1.00
0.7	0.48	0.65	1.00	0.38	0.60	1.00	0.37	0.60	1.00	0.36	0.32	1.00
0.8	0.39	0.57	1.00	0.34	0.56	1.00	0.35	0.57	1.00	0.21	0.13	1.00
0.9	0.11	0.40	1.00	0.12	0.35	1.00	0.12	0.29	1.00	0.07	0.02	1.00

Notes: $T = 7, N = 100$. S and t denote the Sargan test and the t-test, respectively. Size-corrected powers in the AR(1) case for $T = 7$ and $T = 11$ are available from the author on request. The results remained unchanged.

Table 2: Size and Power of the Three Tests (MA(1) error)

	$\delta = 0.3$			0.5			0.7			0.9		
θ	$m2$	S	t	$m2$	S	t	$m2$	S	t	$m2$	S	t
0.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.1	0.13	0.06	0.26	0.15	0.08	0.29	0.17	0.06	0.35	0.19	0.05	0.39
0.2	0.35	0.11	0.67	0.42	0.16	0.78	0.53	0.14	0.85	0.48	0.05	0.87
0.3	0.73	0.29	0.94	0.83	0.49	0.98	0.88	0.30	0.99	0.76	0.06	0.99
0.4	0.97	0.63	0.99	0.99	0.84	1.00	0.98	0.43	1.00	0.88	0.06	1.00
0.5	1.00	0.91	1.00	1.00	0.99	1.00	0.99	0.52	1.00	0.90	0.07	1.00
0.6	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.59	1.00	0.87	0.06	1.00
0.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	0.82	0.07	0.99
0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	0.77	0.07	0.99
0.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78	1.00	0.73	0.07	0.99

Note: Sizes were corrected previously.

We also applied the same three tests to MA(1) errors. To apply the t-test, the MA(1) error process is approximated by an AR(1) process. Table 2 shows the size and power of the three test statistics. Although there is no maximum value of the power, unlike in the case of the AR(1) alternative, the $m2$ and Sargan tests have lower power than the t-test. Note also that the size of the Sargan test becomes distorted as T increases. The use of too many moment conditions dramatically reduces the size and power of the Sargan test. This result confirms previous work by Bowsher (2002). Consequently, the t-test is a useful alternative to the standard m_2 and Sargan tests because of its size and power and its performance when T is large.

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