2005 Proceedings of the Autumn Conference, Korea Statistical Society

Cost optimization for periodic PM policy

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Abstract

This paper considers a preventive maintenance policy following the expiration of renewing warranty. Most preventive maintenance models assume that each PM costs a fixed predetermined amount regardless of the effectiveness of each PM. However, it seems more reasonable to assume that the PM cost depends on the degree of effectiveness of the PM activity. In this paper we consider a periodic preventive maintenance policy following the expiration of renewing warranty when the PM cost is an increasing function of the PM effect. The optimal number and period for the periodic PM policy with effect dependent cost that minimize the expected cost rate per unit time over an infinite time span are obtained.

Keywords : Preventive maintenance, minimal repair, expected cost rate, PM effect, hazard rate

1. Introduction

A maintenance policy of a repairable system attracts a great deal of interests among engineers and reliability analysts and one of the most important and practical areas in reliability theory. Also, a manufacturer usually provides a certain type of warranty policy to the user. Optimal PM policy not only reduces the maintenance cost of a system, but also improves the productivity of the system. The term "optimal" is synonymous to "minimizing the expected cost rate per unit time over finite or infinite time span" throughout this paper.

Extensive researches have been carried out to propose and investigate several maintenance policies due to its wide applicability in real situations. The PM policies are, in general, designed to slow the degradation process of a repairable system while in operation and its optimality is typically defined as minimizing the average cost over a finite or infinite time span. Barlow and Hunter(1960) consider a PM policy of periodic replacement with minimal repair at any intervening failures. Murthy and Nguyen(1981) study an optimal age replacement policy with imperfect preventive maintenance. The preventive is imperfect in the sense that it can cause failure of a non-failed system. Canfield(1986) discusses a periodic PM model for which the PM slows the degradation process of the system, while the hazard rate keeps increasing monotonically. Nakagawa(1986) considers both periodic and sequential preventive maintenance policies for the system with minimal repair upon failure.

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Jung, Lee and Park(2000) discuss an optimal periodic PM policy following the expiration of warranty utilizing Canfield's PM model. Park, Jung and Yum(2000) also consider the situation where each PM relieves stress temporarily and hence slows the rate of system degradation, while the hazard rate of the system remains monotonically increasing. The optimal number and period for the periodic PM that minimize the expected cost rate per unit time over an infinite time span are obtained. Jung and Park(2003) develope the optimal periodic PM policies following the expiration of warranty.

Although many PM policies following the expiration of warranty are proposed in the literature and its optimality is discussed, most of them assume that the cost of preventive maintenance is constant, not depending on the level of improving effect following each PM. However, it is more practical to assume that the PM cost may increase as the PM effect becomes greater. Dagpunar and Jack(1994) determine the optimal number of imperfect PMs needed during a warranty period, under the assumption that the cost of preventive maintenance is an increasing function of age reduction due to each PM. Park and Jung(2002) propose a PM policy for a repairable system. They consider the situation where each PM cost is an increasing function of the PM effect.

In this paper, we consider the PM policy following the expiration of renewing warranty when the PM cost is an increasing function of the PM effect. Explicit formulas to compute the expected cost rate per unit time is derived. We also obtain the optimal period x^* and the optimal number N^* for the periodic PM, which minimize the expected cost rate per unit time for an infinite time span. Section 2 describes the periodic PM model following the expiration of warranty and its assumptions. We also present the formulas to compute the expected cost rate for our periodic PM. In Section 3, we determine the optimal PM policy by finding the optimal period and number for the periodic PM policies simultaneously.

2. PM policy with effect dependent cost after renewing warranty

2.1. Model and assumptions

To derive the optimal periodic PM policy, we assume the following situations. In addition, we consider a repairable system, of which the hazard rate keeps increasing monotonically although the degradation process of the system becomes slower with PM than it would be without PM. The assumptions are :

1. A manufacturer provides a certain type of warranty policy to the user.
2. The system is maintained preventively at periodic times $kx, k=1,2,\cdots,N$, and is replaced by a new system at the *Nth* PM.
3. If the system fails between PMs, it undergoes only minimal repair.
4. The PM cost is an increasing function of the level of PM effect.
5. Each PM slows the rate of system degradation.
6. The times to conduct PM, minimal repair and replacement are negligible.

2.2. Expected Cost Rate

Under a renewing warranty, the system which fails during its warranty period is replaced by a new one and the warranty is renewed. Such a renewing warranty can be further classified into two types of policies according to the responsibility of the user on failure during the warranty period. Under the renewing free-replacement warranty(RFRW) policy, the manufacturer maintains the system during the warranty period at no charge to the user. The renewing pro-rata warranty(RPRW) charges the user a proportion of maintenance cost that is prorated to the age of the system when the failure occurs during the warranty period. Both renewing warranties renew the warranty when the replacement takes place. Let T be the time to failure of a system and let $F(t)$ and $f(t)$ denote the lifetime distribution and its corresponding density function of T, respectively. Then, the hazard rate of F is defined as

$$h(t) = f(t)/\overline{F}(t)$$

for t such that $\overline{F}(t)>0$, where $\overline{F}(t)=1-F(t)$. If $h(t)$ is nondecreasing in t, then F is said to be in IFR.

We first consider the expected cycle length of the system when the periodic PM policy is applied following the expiration of the renewing warranty. A cycle of the system begins with the installation of a new system. If the system fails during its warranty period it is replaced by a new one under the same warranty terms and the cycle ends. Under the renewing warranty, the cycle length is defined as the life length of the new system installed initially. If the system survives to age then the cycle is extended by a fixed number of periodic PM's with each PM having a period of fixed length . Thus, if the system is replaced at the Nth PM, the cycle is extended by Nx. Consequently, the expected cycle length can be represented as

$$\begin{aligned} E[L(x,N)] &= E[T \mid T< w]F(w) + E[w+Nx \mid T> w]\overline{F}(w) \\ &= I(w)+(w+Nx)\overline{F}(w) \end{aligned} \tag{1}$$

where $I(w)=\int_0^w tf(t)dt$.

This section obtains the expressions for the expected maintenance cost under the periodic PM model following the expiration of renewing warranty. To do so, we assume that Canfield's (1986) periodic PM model is utilized to maintain the system after the renewing warranty is expired. Under Canfield's model, each PM reduces operational stress to that existing time units previous to the PM intervention, where is a restoration interval and is less than or equal to the PM intervention interval. Thus, each PM restores the failure rate of the system at time to the one at while its shape remains unchanged. That is, the failure rate keeps monotonically increasing, although the rate of degradation is reduced after each PM. For Canfield's model, the failure rate function after the expiration of warranty can be written as

$$h_{pm}(t)=\begin{cases} h(t), & \text{for } w\le t\le w+x \\ \sum_{i=1}^{k}\{h((i-1)(x-\tau)+(x+w))-h(i(x-\tau)+w)\}+h(t-k\tau), & \\ \qquad \text{for } w+kx< t\le w+(k+1)x,\ k=1,2,\cdots. & \end{cases} \tag{2}$$

where is the failure rate function without PM and Note that implies no PM effect. For more detailed discussions for the hazard rate of (2.2), see Park, Jung and Yum(2000).

To obtain an optimal PM policy with effect dependent cost, we first utilize the formula (2) to derive an expression to compute the expected cost rate per unit time. Let $C_{pm}(x,\alpha)$ denote the cost of preventive maintenance, depending on the effect of PM for given x and

τ. Note that τ is the magnitude of age reduction due to PM effect and thus, τ may be interpreted as a measure of the PM effect. If τ increases, it indicates that the PM effect becomes greater. We assume that $C_{pm}(x, \tau)$ is an increasing function of $\tau \leq x$ for fixed x and thus, $C_{pm}(x, \tau)$ is a decreasing function of $(x - \tau)$. Hence, as the PM becomes more effective, the PM cost may increase.

Applying the results of Park and Jung(2002) and Jung and Park(2003), we obtain the expected cost rate per unit time for running periodic PM policy following the expiration of renewing warranty when the periodic PM with effect dependent cost is performed at $kx, k=1,2,\cdots,N-1$, and the system is replaced by a new one at the Nth PM as follows.

$$C(x,N) = \frac{c_1 + \overline{F}(w)(N-1)C_{pm}(x,\tau) + (c_m + c_{fm})\overline{F}(w)c_2}{I(w) + (w+Nx)\overline{F}(w)}, \tag{3}$$

where

$$c_1 = \begin{cases} \frac{c_r}{w} I(w) + c_r \overline{F}(w) + c_{fw} F(w), & \text{under } RPRW \\ c_r \overline{F}(w) + c_{fw} F(w), & \text{under } RFRW. \end{cases}$$

$$c_2 = \sum_{k=1}^{N-1} \sum_{i=1}^{k} \{h((i-1)(x-\tau) + (x+w)) - h(i(x-\tau)+w)\}x + \sum_{k=1}^{N-1} \int_{kx+w}^{(k+1)x+w} h(t-k\tau)dt$$

and c_m is the unit cost of minimal repair, c_r is the cost of replacement, c_{pm} is the cost of preventive maintenance, c_{fm} is the cost of failure during the maintenance period, c_{fw} is the cost of failure during the warranty period and $h(t)$ is the hazard rate function until the first PM is performed.

If the cost of preventive maintenance is a constant, i.e., $C_{pm}(x, \tau) = c_{pm}$, then Eq. (3) is reduced to the following expected cost rate per unit time, which is equivalent to the result of Jung and Park(2003).

$$C(x,N) = \frac{c_1 + \overline{F}(w)(N-1)c_{pm} + (c_m + c_{fm})\overline{F}(w)c_2}{I(w) + (w+Nx)\overline{F}(w)}.$$

Also, if $w = 0$, then the expected cost rate pet unit time given in (3) is reduced to the result of Park and Jung(2002).

In Section 3, we determine the values of x and N which minimize the expected cost rate per unit time over an infinite time span when both are unknown. The existence and uniqueness of these values can also be established under some mild conditions.

3. Optimal PM policy

This section determines an optimal periodic PM policy when the PM cost varies depending on the effect of each PM. We first fix N and find an optimal period x^* for our periodic PM policy. If such an x^* exists, then $C(x,N)$, given in (3), is minimized at x^* for a fixed N. Differentiating $C(x,N)$ with respect to x and setting it equal to 0, we obtain

$$\begin{aligned} &(I(w) + w\overline{F}(w))((N-1)C'_{pm}(x,\tau) + (c_m + c_{fm})(a_1 + xa_2 + a_3) \\ &\quad + N\overline{F}(w)((N-1)(xC'_{pm}(x,\tau) - C_{pm}(x,\mathrm{a})) + (c_m + c_{fm})(x^2 a_2 + xa_3 - a_4) = Nc_1 \end{aligned} \tag{4}$$

where

$$a_1=\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\tau)+(x+w))-h(i(x-\tau)+w)\}$$

$$a_2=\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h'((i-1)(x-\tau)+(x+w))i-h'(i(x-\tau)+w)i\}$$

$$a_3=\sum_{k=0}^{N-1}\{(k+1)h((k+1)x+w-k\tau)-kh(kx+w-k\tau))\}$$

$$a_4=\sum_{k=0}^{N-1}\int_{kx+w}^{(k+1)x+w}h(t-k\tau)\ dt$$

Sahin and Polatoglu(1996) utilize the concept of pseudo-convexity of the cost rate function to determine the optimal maintenance period. If the cost rate function is known to be pseudo-convex, it has exactly one local minimum and thus that becomes a global minimum. The sufficient conditions for a function to have the pseudo-convexity is discussed in Avriel (1976).

Lemma 3.1. If F is an IFR distribution with strictly increasing and convex failure rate function, then $C(x,N)$ given in (3), is pseudo-convex in $x\geqq 0$ for a fixed $N\geqq 1$.

Using the property of pseudo-convexity of $C(x,N)$ and (4), it is straightforward to prove Theorem 3.2.

Theorem 3.2. Suppose that F is an IFR distribution with strictly increasing and convex failure rate function and $C_{pm}(x,\tau)$ is a decreasing convex function in $x\geqq\tau$, then there exists a x^* which satisfies (4) for a given N and it is unique.

Next, we consider the problem of finding the optimal period, x^* and the optimal number of PM, N^* prior to the replacement of the system, assuming that neither x nor N is fixed. To solve the problem, we first determine x_N as a function of N satisfying (4). If the conditions of Theorem 3.2 is satisfied, then x_N exists and is uniquely determined. Thus, the expected maintenance cost rate per unit time during one cycle can be expressed as a function of N as follows.

$$C(x_N,N)=\frac{c_1+\overline{F}(w)(N-1)C_{pm}(x_N,\tau)+(c_m+c_{fm})\overline{F}(w)c_4}{I(w)+(w+Nx_N)\overline{F}(w)} \tag{5}$$

where

$$c_4=\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x_N-\tau)+(x_N+w))-h(i(x_N-\tau)+w)\}x_N + \sum_{k=1}^{N-1}\int_{kx_N+w}^{(k+1)x_N+w}h(t-k\tau)dt.$$

Since the formula (5) is a function of N alone, N^* can be determined by

$$N^*=\min_N C(x_N,N),\quad N=1,2,3,\cdots. \tag{6}$$

Theorem 3.3. Assume that F is an IFR distribution with strictly increasing and convex hazard rate function and $C_{pm}(x, \tau)$ is a decreasing convex function in $x \geq \alpha$. For a given N there exists a finite and unique x_N which satisfies (4). Thus, the value of N^* which satisfies (6) is the optimal number of PM which minimizes the expected cost rate per unit time, given in (3).

Once the value of N^* is determined, the optimal period x^* can be obtained by using the result of Theorem 3.1. Thus, the optimal period and the optimal number of PMs which minimize the expected cost rate per unit time, given in (3), are x^* and N^*, respectively.

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