Statistical Estimation of Optimal Portfolios for non-Gaussian Dependent Returns of Assets+

Masanobu Taniguchi and Hiroshi Shiraishi
Waseda University

Abstract

This paper discusses the asymptotic efficiency of estimators for optimal portfolios when returns are vector-valued non-Gaussian stationary processes. We give the asymptotic distribution of portfolio estimators $\hat{g}$ for non-Gaussian dependent return processes. Next we address the problem of asymptotic efficiency for the class of estimators $\hat{g}$ First, it is shown that there are some cases when the asymptotic variance of $\hat{g}$ under non-Gaussianity can be smaller than that under Gaussianity. The result shows that non-Gaussianity of X(t) does not always affect worse. Second, we give a necessary and sufficient condition for $\hat{g}$ to be asymptotically efficient when the return process is Gaussian, which shows that $\hat{g}$ is not asymptotically efficient generally. From this point of view we propose to use maximum likelihood type estimators for g, which are asymptotically efficient. We examine our approach numerically.

1. Introduction

In the theory of portfolio analysis, optimal portfolios are determined by the mean μ and variance Σ of the portfolio return. Several authors proposed estimators of optimal portfolios as functions of the sample mean $\hat{\mu}$ and the sample variance $\hat{\Sigma}$ for independent returns of assets (e.g.Jobson and Korkie,1980 and 1989;Lauprete,Samarov and Welsch,2002). However, empirical studies show that financial return processes are often dependent and non-Gaussian. The following figure is East Japan Railway Company's stock return $\{X_t\}$ from 1993/10/27 to 2005/01/28.

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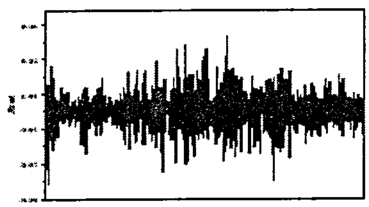

Figure 1: X_t

The sample autocorrelation function $(acf_{X_t}(l))$of $\{X_t\}$ is given in Figure 2.

Figure 2: $acf_{X_t}(l)$

From this we can observe that $\{X_t\}$ is almost uncorrelated

i.e. $acf_{X_t}(l) \approx 0 \ (l \neq 0)$.

Next we plot the autocorrelation function $(acf_{X_t^2}(l))$ of $\{X_t^2\}$ in Figure 3.

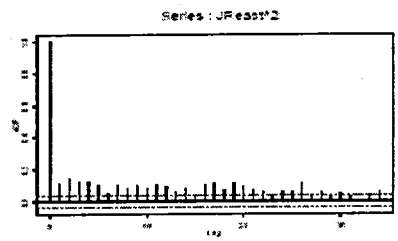

Figure 3: $acf_{X_t^2}(l)$

From this we can see that the squared process $\{X_{it}^2\}$is correlated. This symptom leads us to the assumption that financial return processes are dependent and non-Gaussian. From this point of view, Basak, Jagannathan and Sun (2002) showed the consistency of optimal portfolio estimators when portfolio returns are stationary processes. However, in the literature there has been no study on the asymptotic efficiency of estimators for optimal portfolios. Therefore, in this paper, denoting optimal portfolios by a function $g=g(\mu,\Sigma)$ of μ and Σ,we discuss the asymptotic efficiency of estimators $\hat{g}=g(\hat{\mu},\hat{\Sigma})$ when returns are vector-valued non-Gaussian stationary processes. Then it is shown that $\hat{g}$is not asymptotically efficient generally even if $\{X_t\}$ is Gaussian, which gives a strong warning for use of the usual estimator $\hat{g}$ We also show that there are some cases when the asymptotic variance $V_{NG}(\hat{g})$ of $\hat{g}$under non-Gaussianity can be smaller than that under Gaussianity $V_G(\hat{g})$. Numerical studies are given to illuminate the results above. For non-Gaussian dependent return processes, we propose to use maximum likelihood type estimators for g, which are asymptotically efficient. Numerical examples for an actual financial data are provided. As a conclusion it seems very important to make the consideration for non-Gaussianity and dependence of return processes.

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