Seasonal cointegration for daily data

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Abstract

In this paper, we propose an extension of the maximum likelihood seasonal cointegration procedure developed by Johansen and Schaumburg (1999) for daily time series. We presented the finite sample distribution of the associated rank test statistics for daily data.

Keywords: Seasonal cointegration, Seasonal Error Correction model, Finite sample distribution, Asymptotic distribution

1. Introduction

A number of papers on seasonal cointegration have been developed by Hylleberg et al(1990), Lee(1992), Johansen and Schaumburg(1999) and Cubadda(2001). Expecially, Johansen and Schaumburg (1999) analyzed the error correction model for seasonal cointegration and showed that asymptotic distribution of the cointegration rank statistics are asymptotically mixed Gaussian.

It is well known that the finite sample distribution of cointegration rank statistics is very different from the associated asymptotic distribution, especially, in small sample size. Lee (1992), Lee and Siklos(1995) and Darne (2004) discussed this issue and compute the finite sample critical values of the rank statistics for quarterly and monthly.

Contrary to macroeconomic data, financial data such as stock price and exchange rate have a period with five days of the week. Darne (2003) first computed the finite sample distribution of the rank statistics for daily data. But he used the model developed by Lee(1992) which impose a particular parameter restriction on the seasonal error correction model.

In this paper we also deal with seasonal cointegration for daily data. But we extend the Johansen-Schaumburg (1999) 's seasonal cointegration procedure because this approach does not impose a parameter restriction on seasonal error correction model. In section 2 we extend the seasonal cointegration developed by Johansen-Schaumburg (1999) for daily time series. We provide the finite sample distribution of the seasonal cointegration rank statistics and compare with the associated asymptotic distribution in section 3.

2. Seasonal cointegration for daily data

Consider a n-dimensional autoregressive process X_t defined by the equation,

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$$X_t = \sum_{i=1}^{k} \Pi_i X_{t-i} + \Phi D_t + \epsilon_t \tag{1}$$

where ϵ_t is i.i.d $N_n(0,\Sigma)$ and initial values $X_0, X_1, \cdots, X_{-k+1}$ are fixed. If we define $n \times n$ matrix polynomial $A(z) = I_n - \sum_{i=1}^{k} \Pi_i z^i$, the properties of the process X_t completely depend on roots of characteristic equation written by $|A(z)| = 0$. We consider the process of which the characteristic function has five roots on the unit circle $(z_1 = 1, z_2 = e^{2\pi i/5}, z_3 = e^{-2\pi i/5}, z_4 = e^{4\pi i/5}, z_5 = e^{-4\pi i/5})$, i.e., the each roots of the characteristic function exist on zero, $2\pi/5, -2\pi/5$, $4\pi/5$ and $-4\pi/5$ frequency.

Under the proper regularity condition, equation (1) has the following seasonal error correction model proposed by Johansen and Schaumburg (1999),

$$p(L)X_t = \sum_{m=1}^{5} \alpha_m \beta_m{}^* X_t^{(m)} + \sum_{j=1}^{k-5} \Gamma_j p(L) X_{t-j} + \Phi D_t + \epsilon_t, \tag{2}$$

where α_m, β_m are $(n \times r_m)$ matrices with full rank r_m and $\beta_m^* = \beta_R{}' - i\beta_I{}'$ describe the long-run behavior of the series at each of the five frequencies,

$$X_t^{(m)} = \frac{p_m(L)L}{p_m(z_m)z_m} X_t$$

$$p_j(z) = \prod_{m \neq j}^{5} (1 - \bar{z}_m z) = \frac{p(z)}{1 - z_j z}$$

$$p(z) = \prod_{m=1}^{5} (1 - \bar{z}_m z) = (1 - z^5) = (1 - z)(1 + z + z^2 + z^3 + z^4)$$

and the z_m are the non seasonal and seasonal unit roots for the daily data and $\bar{z}_m$ is the complex conjugate.

The estimation of $\alpha_1 \beta_1^*$ is obtained from canonical correlations in analogy to Johansen(1998). However, the estimation of parameters at the other seasonal frequencies is nonstandard, and is obtained iteratively by using the switching algorithm. See Dahl Pedersen (1996), Johansen and Schaumburg(1999), and Löf and Lyhagen (2002) for detailed description of this algorithm.

In addition to equation (2) we also consider the following model suggested by Johansen and Schaumburg (1999) in which we give a particular restriction in deterministic term D_t. If we allow for a linear trend, we do not restrict at zero frequency then seasonal error correction model (2) is written in

$$p(L)X_t = \alpha_1 \beta_1{}' X_t^{(1)} + \sum_{m=2}^{5} \alpha_m \begin{pmatrix} \beta_m^* \\ \rho_m \end{pmatrix} \begin{pmatrix} X_t^{(m)} \\ \bar{z}_m^t \end{pmatrix} + \sum_{j=1}^{k-5} \Gamma_j p(L) X_{t-j} + \Phi_1 + \sum_{m=6}^{\bar{s}} \Phi_m \bar{z}^t + \epsilon_t. \tag{3}$$

3. Finite sample distribution of the cointegration rank statistics

In this section we consider a simulation study of the finite sample behavior of the cointegration rank test statistics at four seasonal frequencies except zero frequency because finite sample critical values at zero frequency are equivalent to result from Lee and Sikos (1995). Data generating process is the k dimensional seasonal integrated process given by $\Delta_5 X_t = \epsilon_t$ $(t = 1, 2, ..., T)$ with $\epsilon_t \sim i.i.d.\ N(0, I_k)$, k = 1, 2 and 3 for $(n-r)$=1, 2 and 3. Four sample sizes are considered throughout, T=50, 100, 200 and 400 over 100,000 replications. As compared with the asymptotic distribution of the rank test statistics, empirical critical values are in generally shifted to the right than asymptotic one. That is, when we use the asymptotic critical values for rank test there is a tendency to reject the null hypothesis more often. The difference between finite sample critical value and the asymptotic one increases when the sample size gets smaller and the number of the common trends, $(n-r)$ gets larger. When considering the deterministic trend we observe the gap of two critical value gets larger. Therefore, we must be careful to use the asymptotic critical value in small samples because we may lead a wrong inference at cointegration test. Johansen and Schaumburg (1999) proved that seasonal cointegration rank test statistics at other seasonal frequencies except zero and π frequency have the same distribution. Actually we obtain the equivalent simulation results to Johansen and Schaumburg (1999).

4. Conclusion

In this paper we propose an extension of the seasonal cointegration procedure developed by Johansen and Schaumburg(1999) for daily time series. The finite sample distribution of the rank statistics is quite different from its asymptotic distribution and relatively shifted to the right. The difference between the finite sample critical value and the asymptotic one gets larger as the sample size gets smaller and the number of the common trends, $(n-r)$, gets larger. Therefore we have to careful in using the finite sample critical values for the cointegrating rank test to do appropriate inference in small sample.

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