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# 전기철도 시스템의 불평형 해석

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# Analysis of Voltage Unbalance on Electric Railway System

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**Abstract** - The railway characteristic, which is concerned, as most utilities is unbalance produced by the large single-phase loads. Here are two theoretical concerns associated with unbalanced loads. First, generator rotor heating resulting from unbalanced current flow. Second, there is the possibility of motor overheating in industrial plants, due to the unbalanced voltage. Therefore, the exact assessment of the voltage unbalance must be carried out preferentially as well as load forecast at stages of designing and planning for the electric railway system. This paper proposes a new analysis model to more effectively estimate voltage unbalance. Numerous distributed circuits in the electric railway system are composed by components. The entire system can be easily modeled by the combination of four-port representation of each component in parallel and/or series. Simulation results using the model are compared with field data, and it verifies the accuracy of the proposed model.

## 1. Introduction

The railway characteristic, which is concerned, as most utilities is unbalance produced by the large single-phase loads. Here are two theoretical concerns associated with unbalanced loads. First, generator rotor heating resulting from unbalanced current flow. Second, there is the possibility of motor overheating in industrial plants, due to the unbalanced voltage. The effects on motors are addressed by the voltage unbalance criteria applying to distribution voltage levels. The effects on generators are addressed by limits on the permissible level of negative sequence currents. The generator is allowed to absorb. The unbalanced single-phase load causes negative sequence currents to flow in the generators and transmission lines. The negative sequence currents produce in the alternator stator a field rotating in a direction opposite to the rotation of the rotor. This field produces rotor core losses and it also induces second harmonics in the rotor. In a salient pole generator, the induced rotor currents cause heating of the amortisseur windings. In a non-salient pole generatorthat does not have amortisseur windings, the induced currents cause heating of the rotor surface particularly in the vicinity of slot wedges at the ends of the rotor [1].

The conventional researches on the voltage unbalance have dealt with connection schemes of the transformers used in AC AT-fed electric railway system and induced formulas to briefly evaluate the voltage unbalance in the system [2, 3]. These formulas are still being used widely due to their easy applicability on the voltage unbalance evaluation. Meanwhile, they do not take into account detailed characteristics of AC AT-fed electric railway system, being founded on some assumptions. The evaluation of voltage unbalance using two-port network model in areas, such as electric railwaydepots was carried out [4]. System of the depot was multi track composed by only contact wire and rails. Hence, two-port network model is fit in electric railway depot. However, four-port network model is fit in general electric railway system composed by contact wires, rails and feeders. Four-port network model has two input ports, two output portsand a basic port.

The proposed model is carried out to forecast power qualities at stages of designing and planning for the electric railway system. At this moment, the paper deal with the exact assessment of the voltage unbalance in lots of power qualities

The paper proposes a new analysis model to more effectively estimate the voltage unbalance. Numerous distributed circuits in the electric railway system are composed by components. The entire system can be easily modeled by the combination of four-port representation of each component in parallel and/or series. Simulation results using the model are compared with field data, and it verifies the accuracy of the proposed model.

## 2. Conventional Researches

The evaluation method induced by Chen has been used widely to estimate the voltage unbalance occurred by electric trains. The catenary system and the scott-transformer are assumed as an ideal to simplify inducing process to evaluate voltage unbalance. Voltage drops and losses are not occurred in scott-transformer and the catenary system by this assumption. Hence, it does not authenticate the accuracy. Under assuming that the primary side of scott-transformer is balanced, equations are developed. Therefore, theses equations have contradiction evaluating the unbalance regarding this voltage.

In the past, it was carried out that the evaluation of the voltage unbalance using two-port network model in electric railway depots. The electric railway depot is the system that has single track composed by only the contact wire and rails. This two-port network model can not be used in the general system composed by contact wires, rails and feeders. Therefore, analysis model using four-port representation is developed.

## 3. Formulation by Four-port Network model

This paper proposes a new model for the voltage unbalance in the power supply system including feeders, contact lines, rails. The system model is based on four-port representation that is an extension of two-port network theory.

Four-port representation for each element in the AC electric railway system can be derived as follows:

### *3.1 Autotransformer*

The autotransformer is placed on between the catenary and the adjacent feeder with the rails connected to the center point on the winding. The AC electric railway system supplies 55 kV between the contact wire and the feeder with the autotransformer of ratio 1 : 1 (feeder-rail : rail-contact wire) to step down the high voltage 55 kV to 27.5 kV. Autotransformers are installed approximately every 10 kilometers along the railroad.

### *3.1.1 Autotransformer in substation*

For the substation, the model of the autotransformer is expressed in Fig. 1.

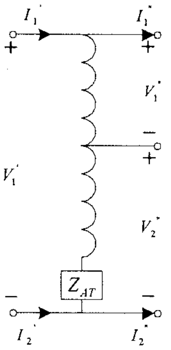

Fig. 1. Equivalent circuit of autotransformer for substation

Eq. (1) expresses voltages and currents relations to model the autotransformer for the substation.

$$
\begin{aligned}
&(a)\ \begin{cases} V_1' = V_1'' + V_2'' \\ I_1' - I_1'' = -I_1' - I_2'' \Rightarrow I_1' = \frac{1}{2}I_1'' - \frac{1}{2}I_2'' \end{cases} \\
&(b)\ \begin{cases} V_1' = 2V_1'' + \frac{Z_{AT}}{2}I_1'' + \frac{Z_{AT}}{2}I_2'' \\ I_1' = \frac{1}{2}I_1'' - \frac{1}{2}I_2'' \end{cases}
\end{aligned}
\tag{1}
$$

$$
\begin{bmatrix} V_1' \\ I_1' \\ V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 2 & 0 & \frac{Z_{AT}}{2} & \frac{Z_{AT}}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_{SSAT} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix}
$$

### *3.1.2 Autotransformer in sub-sectioning post*

For sub-sectioning post, the model is shown in Fig. 2.

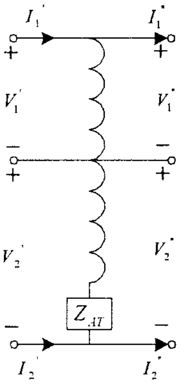

Fig. 2. Equivalent circuit for sub-sectioning post

The relations between voltages and currents can be expressed as Eq. (2).

$$
\begin{aligned}
&V_1' = V_1'' \\
&V_2' = V_2'' \\
&I_1' - I_1'' = I_2' - I_2'' \qquad \Rightarrow I_1' = \frac{1}{Z_{AT}}V_1'' - \frac{1}{Z_{AT}}V_2'' + I_1'' - 2I_2'' \\
&V_2'' = V_1'' + \left(I_2'' - I_2'\right)Z_{AT} \Rightarrow I_2' = \frac{1}{Z_{AT}}V_1'' - \frac{1}{Z_{AT}}V_2'' - I_2''
\end{aligned}
\tag{2}
$$

$$
\begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{Z_{AT}} & -\frac{1}{Z_{AT}} & 1 & -2 \\ \frac{1}{Z_{AT}} & -\frac{1}{Z_{AT}} & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_{MidAT} \cdot \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix}
$$

### *3.1.3 Autotransformer in sectioning post*

The autotransformer in the sectioning post is also installed as the sub-sectioning post and the substation. The model is shown in Fig. 3.

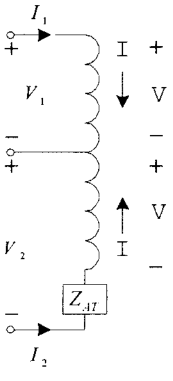

Fig. 3. Equivalent circuit for autotransformer in sectioning post

Eq. (3) is expressed for the autotransformer in the sectioning post.

$$
\left.\begin{aligned} &V_1' = V \\ &V_2' = V - Z_{AT} \cdot I \\ &I_1' = I \\ &I_2' = I \end{aligned}\right\} \Rightarrow \begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -Z_{AT} \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} = M_{sp} \cdot \begin{bmatrix} V \\ I \end{bmatrix}
\tag{3}
$$

### *3.2 Catenary System*

The catenary system has several conductors with a complex geometry. The system consists of contact wire (3), messenger wire (2), feeder (1), rails (4, 5), protection wire (6), and buried earth wire (7) as shown in Fig. 4. Droppers connect two conductors such as the contact wire and the messenger wire. Those conductors are electrically regarded as one conductor. This simplification is made possible by the auto aforementioned continuous parallel connection of some conductors. Finally, we can reduce the overall conductors to equivalent 3 conductors [5, 6].

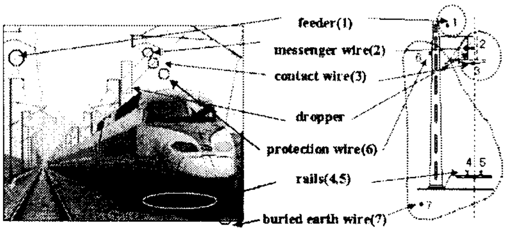

Fig. 4. Configuration of catenary system

Catenary system has not only its self and mutual impedances but also shunt admittances. Equivalent T-type model for the catenary system can be represented with these parameters as shown in Fig 5.

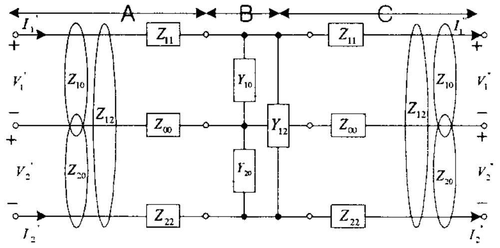

Fig. 5. Equivalent T-type model for catenary system

The voltages and currents of the sending end for each block(Z) can be expressed in terms of the receiving end quantities by:

$$\begin{cases} V_1' = V_1'' + (Z_{11} + Z_{00} - 2Z_{10})I_1'' + (Z_{00} + Z_{12} - Z_{10} - Z_{20})I_2'' \\ V_2' = V_2'' + (Z_{10} - Z_{00} - Z_{12} + Z_{20})I_1'' + (2Z_{20} - Z_{00} - Z_{22})I_2'' \\ I_1' = I_1'' \\ I_2' = I_2'' \end{cases} \tag{4}$$

$$\Rightarrow \begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & Z_{11}+Z_{00}-2Z_{10} & Z_{00}+Z_{12}-Z_{10}-Z_{20} \\ 0 & 1 & Z_{10}-Z_{00}-Z_{12}+Z_{20} & 2Z_{20}-Z_{00}-Z_{22} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_Z \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix}$$

For the middle block (Y), the voltages and currents of the sending end are obtained as follows :

$$\begin{cases} V_1' = V_1'' \\ V_2' = V_2'' \\ I_1' = I_1'' + Y_{10}V_1'' + Y_{12}(V_1'' + V_2'') = (Y_{10} + Y_{12})V_1'' + Y_{12}V_2'' + I_1'' \\ I_2' = I_2'' - Y_{20}V_2'' - Y_{12}(V_1'' + V_2'') = -Y_{12}V_1'' - (Y_{20} + Y_{12})V_2'' + I_2'' \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Y_{10}+Y_{12} & Y_{12} & 1 & 0 \\ -Y_{12} & -Y_{20}-Y_{12} & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_Y \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} \tag{5}$$

Therefore, the relations for voltages and currents are rearranged as Eq. (6)

$$\begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = M_Z \cdot M_Y \cdot M_Z \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_{LINE} \cdot \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} \tag{6}$$

### *3.3 Electric Train*

The railway load is a constant MVA load in the normal range of operation. The load is represented as Eq. (7).

$$P_{train} = V_{train} \cdot I_{train}^{*} = V_{train} \cdot \frac{V_{train}^{*}}{Z_T^{*}}$$

$$Z_T = V_{train}^{*} \cdot \frac{V_{train}}{P_{train}^{*}} \tag{7}$$

For the electric train, the model is expressed as Fig. 6.

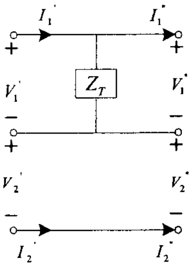

Fig. 6. A model for railroad electric train

The relations for voltages and currents are represented in Eq. (8).

$$\begin{bmatrix} V_1' \\ V_2' \\ I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{Z_T} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} = M_{Train} \cdot \begin{bmatrix} V_1'' \\ V_2'' \\ I_1'' \\ I_2'' \end{bmatrix} \tag{8}$$

### *3.4 Modeling of Substation*

The power utility supplies 154kV with the AC electric railway system through transmission lines. The scott-transformer in the substation steps down from 154kV to 55kV.

Zs implies the impedance of scott-transformer and power utility.

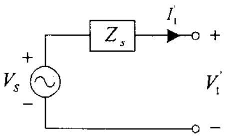

Fig. 7. Equivalent model for power utility and scott- transformer at substation.

The relations between voltages and currents in Fig.7 can be expressed as Eq. (9).

$$\begin{bmatrix} V_S \\ V_S \end{bmatrix} = \begin{bmatrix} 1 & Z_S & 0 & 0 \\ 0 & 0 & 1 & Z_S \end{bmatrix} \begin{bmatrix} V_1' \\ I_1' \\ V_1' \\ I_1' \end{bmatrix} = M_{SS} \begin{bmatrix} V_1' \\ I_1' \\ V_1' \\ I_1' \end{bmatrix} \tag{9}$$

*3.5 Combining Fundamental Element Models*

Now, the entire system can be easily modeled by the combination of four-port representation of each component in parallel and/or series. Therefore, following equation make a result as follows.

$$\begin{bmatrix} V_S \\ V_S \end{bmatrix} = M_{ZS} \cdot M_{LINE} \cdot M_{AT} \cdots\cdots M_{LINE} \cdot M_{TRAIN} \cdot M_{LINE} \cdot M_{SP} \cdot \begin{bmatrix} V \\ I \end{bmatrix} \tag{10}$$

Voltage and current at sectioning post can be obtained. In like manner, voltage and current at any point of the line can be calculated straightforwardly.

$$\begin{bmatrix} V \\ I \end{bmatrix} = M^{-1} \cdot \begin{bmatrix} V_S \\ V_S \end{bmatrix} \tag{11}$$

## 4. Calculation of Voltage Unbalance

Fig. 8 represents the equivalent circuit model for power supply system in electric railway. Its corresponding $I_T$, $I_M$ are calculated by voltage and current relations of the scott-transformer.

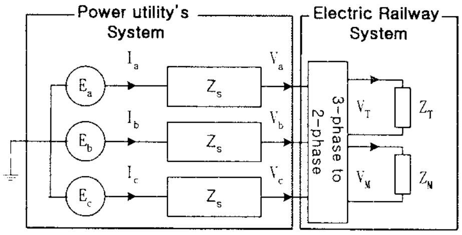

Fig. 8. Equivalent circuit model for power supply system in electric railway

Voltages and currents of the primary side of the scott-transformer are calculated as shown in Eq. (12).

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{N_2}{N_1} \begin{bmatrix} \frac{2}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{3}} & 1 \\ -\frac{1}{\sqrt{3}} & -1 \end{bmatrix} \begin{bmatrix} I_T \\ I_M \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} Z_S & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{12}$$

By definition,

$$V_{012} = T_S^{-1} V_{abc}$$
$$I_{012} = T_S^{-1} I_{abc} \tag{13}$$

And

$$T_S^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{14}$$

In which $a = e^{j\frac{2\pi}{3}}, a^2 = e^{-j\frac{2\pi}{3}}$

By definition, the amount of the voltage unbalance is expressed in the ratio of the negative sequence voltage to the positive sequence voltage:

$$Voltage\_unbalance\_factor(d_2) = \frac{Negative\_sequence\_voltage(|V_2|)}{Positive\_sequence\_voltage(|V_1|)} \tag{15}$$

Maximum voltage unbalance as the mean value for 10 minutes should be under 1%, the limitation by Korean Railway Facility Regulation in the section of KTX operation.

## 5. Case Studies

In the power feeding section of Miryang substation, the voltage unbalance is measured to verify the feasibility of the proposed model. Field data are compared with simulation result. The configuration of this section is shown in Fig. 9.

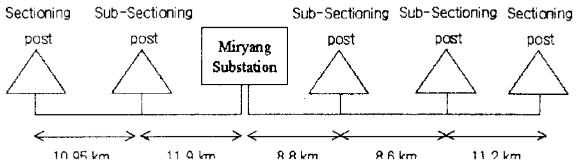

Fig. 9. The configuration of the power feeding section of Miryang substation

Details for the power feeding section in Miryang substation are shown in Table 1.

**Table 1. Details for power feeding system in Miryang substation**

| | Categories | Values |
|---|---|---|
| Power supply | 3-phase voltage [kV] | 154 |
| | Short impedance [%] | 0.6640+j3.3280 |
| | Transmission line impedance [%] | 0.6369+j2.2599 |
| Scott-Tr. | Rated capacity [MVA] | 45 |
| | Rated voltage [kV] | 154/55 |
| | Impedance [’ ] | 0.5840+j13.4318 |
| Auto-Tr. | Rated capacity [MVA] | 7.5 (Sectioning post, Sub-sectioning post)<br>10 (Substation) |
| | Rated voltage [kV] | 55/27.5 |
| | Impedance [%] | 6 |
| Catenary system | Feeder [’ /km] | 0.1776+j0.4409 |
| | Contact wire [’ /km] | 0.1410+j0.2898 |
| | Rail [ /km] | 0.0554+j0.1033 |
| | Contact wire-rail [ ] | 0.5252+j9.8289 |
| | Contact wire-feeder [ ] | 0.5033+j5.2127 |
| | Rail-feeder [ ] | 0.5004+j5.0983 |

Korea Railroad offered vehicle operation condition in 2004. The operation of electric train was simulated by

KTX with 11 minutes headways. The diagrams of electric train (KTX) in Dongdaegu-Pusan section are shown in Fig. 10. The condition of electric train is represented in Table 2.

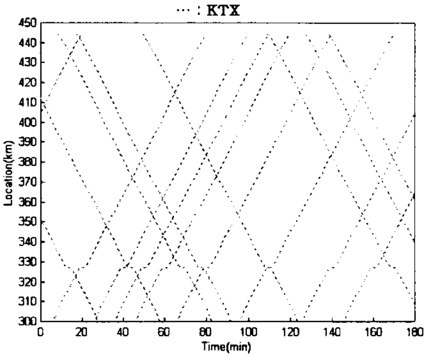

Fig. 10. The diagram of electric train (KTX) in 2004

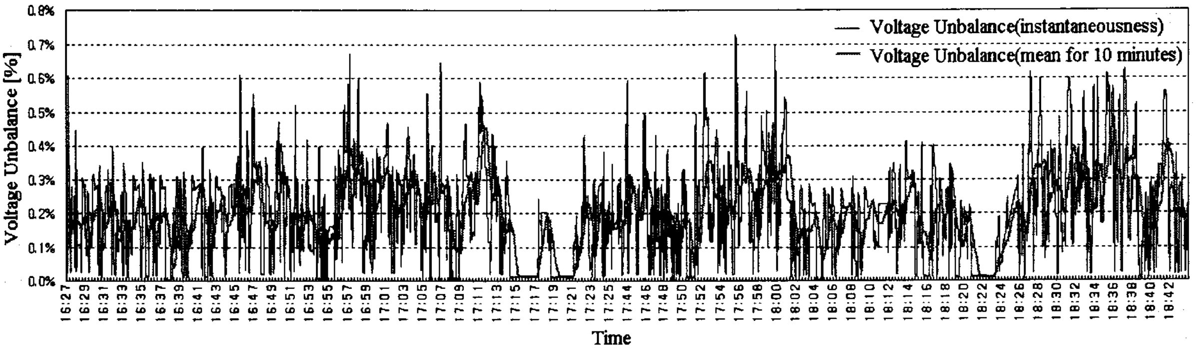

Fig. 11. Measured voltage unbalance in the power feeding section of Miryang substation

Table 2. Electric train dimensions

| | | KTX |
|---|---|---|
| supply voltage | maximum | 27.5kV |
| | normal | 25kV |
| | minimum | 1kV |
| Output power | maximum | 15,485kW |
| | traction | 13,560kW |
| | SIV | 1,925kW |
| speed | maximum | 300km/h |

Recently, measured voltage unbalance in this section is 0.49% as the mean value for 10 minutes. Field data are represented in Fig. 11.

Simulated voltage unbalance is 0.4942% as the mean value for 10 minutes. Simulation results for the voltage unbalance are shown in Fig. 12.

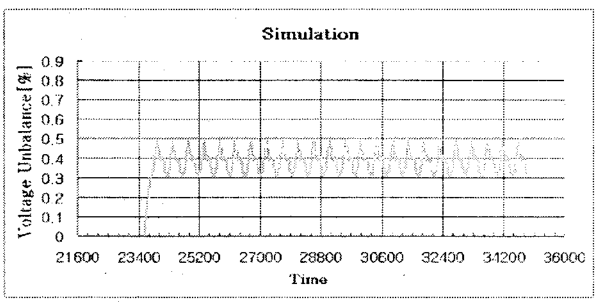

Fig. 12. Simulated voltage unbalance in the power feeding section of Miryang substation

The limitation for the voltage unbalance in Korea should be under 3% as the mean value for 2 hours. However, this limitation needs to be strengthened. Therefore, the limitation of voltage unbalance is under 1% as the mean value for 10 minutes in the section of KTX operation.

Moreover, we measured and simulated the voltage unbalance on Kyongbu high-speed railway system. The result as the mean value for 10 minutes is shown in Table 3 and Fig. 13.

Table 2. Comparing measurement with simulation of voltage unbalance on Kyongbu high-speed railway system

| Substation | Measurement [%] | Simulation [%] |
|---|---|---|
| AnSan | 0.22 | 0.33 |
| PyongTaek | 0.47 | 0.42 |
| SinChungJu | 0.15 | 0.19 |
| OkChun | 0.35 | 0.36 |
| KimChun | 0.69 | 0.77 |

From results, the voltage unbalance is satisfied in the limit value. It is confirmed that there is no problem for voltage unbalance. Also, The electric train is a moving load. The voltage unbalance is instantaneously varying. Therefore, it is understandable that measured and simulated results are almost identical.

## 6. Conclusion

The proposed model is carried out to forecast power qualities at stages of designing and planning for the electric railway system. This paper proposed a new analysis model to more effectively estimate voltage unbalance. Four-port network model is fit in general electric railway system composed by contact wires, rails and feeders. The entire system could be easily modeled by the combination of the four-port representation of each component in parallel and/or series.

Maximum voltage unbalance as the mean value for 10 minutes should be under 1%, the limitation by Korean Railway Facility Regulation in the section of KTX operation.

Voltage unbalance on Kyongbu high-speed railway section was under 1% as the mean value for 10. The voltage unbalance was satisfied in the limit value. It is confirmed that there is no problem for the voltage unbalance.

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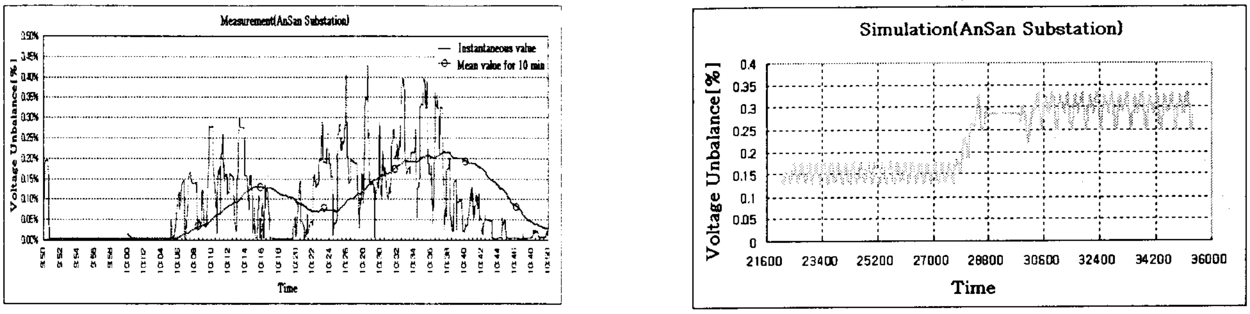
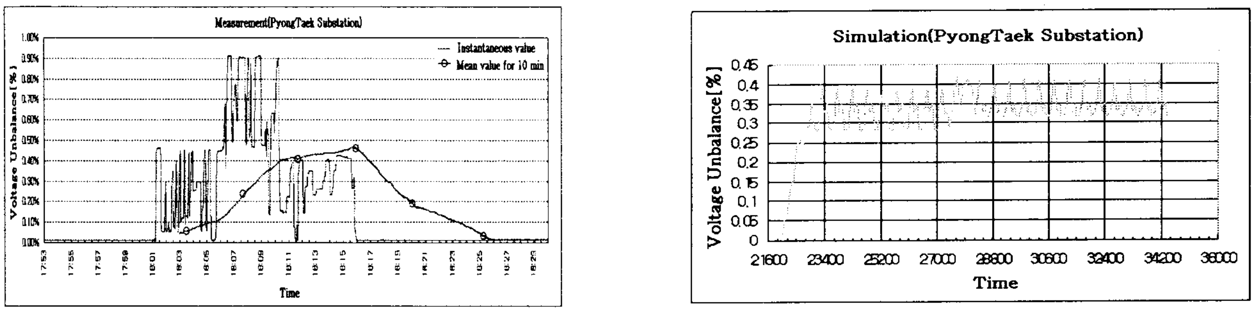


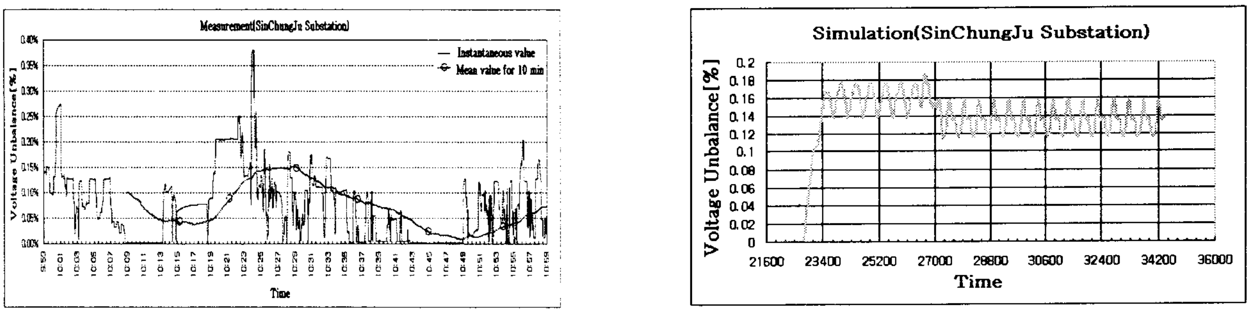
(a) Measurement(Ansan Substation)

(b)Simulation(Ansan Substation)



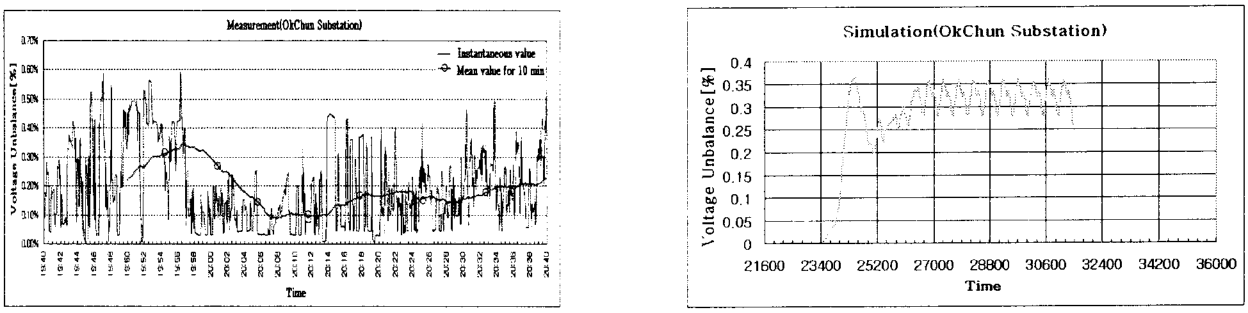
c) Measurement(PyongTaek Substation)

(d)Simulation(PyongTaek Substation)



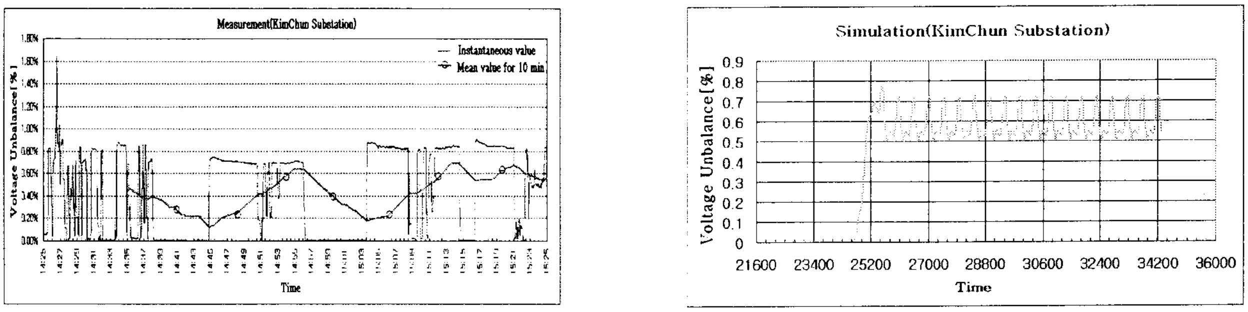
(e) Measurement(SinChungJu Substation)

(f)Simulation(SinChungJu Substation)



(g) Measurement(OkChun Substation)

(h)Simulation(OkChun Substation)



(i) Measurement(KimChun Substation)

(j)Simulation(KimChun Substation)

Fig. 13 Comparing measurement with simulation of voltage unbalance on Kyongbu high-speed railway system(mean value of 10 minutes)