비대칭 출력부하에 대한 포화함수를 이용한 자동동조 알고리듬

Autotuning algorithm for asymmetric output using saturation function

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Abstract- An unknown linear time invariant plant with asymmetric oscillation in the output such as a static load disturbance. A saturation function nonlinear element is used to find the one point information in the frequency domain. An asymmetric self-oscillation caused by such as a static load disturbance saturation function feedback is analyzed. a new method to tune a PID controller based on the analysis is proposed in the presence of asymmetric oscillation. The proposed method does not require the knowledge of plant d.c. gain with an asymmetric oscillation in the plant output.

Key Words : PID controller, asymmetric self-oscillation, saturation function feedback

I. Introduction

PID controller has been used in the industry, even though many advanced control algorithm has been developed, since PID controller is simple and its parameters well matched to physical parameters. To tune the PID controller parameter, it heavily depend on the experienced engineers. Autotuning of PID parameters needed to reduce time and expenses. Many methods were proposed and commercialized[1]. The excellent review of the different autotuning methods can be found in [1]. One of simple methods is the use of relay feedback to identify the unknown plant[2]. The relay feedback generates self-sustained oscillation in the plant output. One point frequency domain information can be obtained by the observation of self-sustained oscillation. one point frequency domain information is correspond to the intersection point with negative real axis in the Nyquist plot of the unknown plant. Some of PID controller design methods can be applied with phase crossover information[3,4]. The work[5] used the saturation function instead of relay as a test signal to reduce the error due to the high frequency components. The accuracy of phase crossover estimation in the Nyquist plot of the unknown plant was improved with the use of saturation function. All of works[2,5] assumed that the plant output has a symmetric oscillation. The assumption of the symmetric oscillation enabled to the describing function approximation to find a phase crossover information. We consider that there is an asymmetric oscillation in the plant output. The harmonic valance approximation is used to identify the one point information in the frequency domain. Based on the anaysis, a new method to tune the PID controller is proposed.

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II. The autotuning of PID controller with an asymmetric oscillation

We need to know some information of unknown plant to design the PID controller. A relay as a nonlinear element was used to identify a phase crossover point during tuning of the PID controller[2]. After the tuning of PID controller, the PID controller is connected to the closed-loop system.

In more detail for the identification of unknown plant, when the following harmonic equation has a non-trivial solution

$$1+G(jw)\cdot N(a)=0 \tag{1}$$

there is a symmetric self-oscillation in the plant output, where $G(jw)$ is the frequency response of the plant, a is the amplitude of fundamental frequency component of the plant output, and $N(a)=\frac{\pi a}{4d}$ is a describing function of the relay where d is magnitude of the relay. A phase crossover information of the unknown plant can be calculated using the equation (1), since a and w can be obtained from the observation of the plant output. A saturation function defined by

$$sat(x) = \begin{cases} -d & x < -\frac{d}{s} \\ sx & -\frac{d}{s} \le x \le \frac{d}{s} \\ d & x > \frac{d}{s} \end{cases} \quad (2)$$

We can calculate the ultimate gain defined by $\frac{1}{|G(jw)|}$ from the equation (1) and the ultimate period from the observation of magnitude and period of the plant output. When there is an asymmetric oscillation caused by static load disturbance in the plant output, the output of relay and the saturation function nonlinear element is shown with $x_1 - x_0 \neq x_2 - x_1$ and $t_2 - t_1 \neq t_5 - t_4$ in the Figure 1. Note that when $x_1 - x_0 = x_2 - x_1$ and $t_1 - t_2 = t_4 - t_5$, the plant output is a symmetric one.

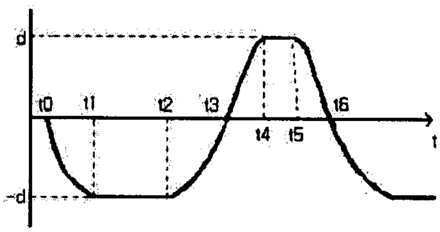

Figure 1. The output of relay and saturation function with asymmetric plant output

We suppose that there is an asymmetric oscillation caused by an unknown static load disturbance in the plant output. Since plant output and the output of nonlinear element are periodic signal, the plant output $y(t)$ and the output of nonlinear element $\psi(-y)$ can be represented by the following Fourier series.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkwt} \quad (3)$$

$$\psi(-y) = \sum_{k=-\infty}^{\infty} c_k e^{jkwt} \quad (4)$$

where a_k and c_k are Fourier series coefficients for $y(t)$ and $\psi(-y)$, respectively. Since $y(t)$ is the output of the plant, $G(s)$, and $\psi(-y) + l$ is the input of plant, the following equation should hold

$$d(p)y(t) - n(p)(\psi(-y) + l) = 0$$

(5)

where $p = \frac{d()}{dt}$, n(s) and d(s) are the numerator and denominators polynomials of G(s). Using the following relation, The equation (5) can be rewritten as

$$\sum_{k=-\infty}^{\infty} [d(jkw)a_k - n(jkw)(c_k + l_k)]e^{jkwt} = 0$$

(6)

where $l_k = l$ for $k = 0$, and $l_k = 0$ for $k \geq 1$.

Since e^{jwkt} are orthogonal for each k, the equation (6) is equivalent to

$$G(jkw)(c_k + l_k) - a_k = 0 \quad (7)$$

Then the first order harmonic balance approximation is

$$G(0)(\hat{c}_0 + l) - a_0 = 0$$

(8)

$$G(jw)\,\hat{c}_1 - \frac{a}{2j} = 0$$

(9)

where a_0 is the d.c. value of the plant output, $\hat{c}_o = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a\sin wt))dt$, $\hat{c}_1 = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a\sin wt)e^{-jwt})dt$, a and w are the amplitude and frequency of the first harmonic of the plant output, respectively. Note that c_k is a Fourier coefficient for infinite dimensional case, while $\hat{c}_k$ is Fourier coefficient for finite dimensional case which is an approximation of infinite dimensional case. We use the saturation function to alleviate the effect of high harmonics. The Fourier coefficient of the first harmonic is given by

$$\begin{aligned} \hat{c}_1 = w/2\pi\{ & \int_{t_0}^{t_1} -sl(a_o + a\sin wt)e^{-jwt})dt \\ & + \int_{t_1}^{t_2} -d \times e^{-jwt}dt + \int_{t_2}^{t_4} -sl(a_o + a\sin wt)e^{-jwt}dt \\ & + \int_{t_4}^{t_5} d \times e^{-jwt}dt + \int_{t_5}^{t_6} -sl(a_o + a\sin wt)e^{-jwt}dt\} \end{aligned} \quad (10)$$

and the Fourier coefficient of the second harmonic is given by

$$\begin{aligned} \hat{c}_2 = w/2\pi\{ & \int_{t_0}^{t_1} -sl(a_o + a\sin wt)e^{-j2wt})dt \\ & + \int_{t_1}^{t_2} -d \times e^{-j2wt}dt + \int_{t_2}^{t_4} -sl(a_o + a\sin wt)e^{-j2wt}dt \\ & + \int_{t_4}^{t_5} d \times e^{-j2wt}dt + \int_{t_5}^{t_6} -sl(a_o + a\sin wt)e^{-j2wt}dt\} \end{aligned}$$

for the saturation function output where t_i is defined in Figure 1, sl and d are the slope and magnitude of the saturation function, respectively. Since saturation function can reduce the high harmonic element, we will use the saturation function feedback as a test signal to identify one point information of the plant in the Nyquist plot. We can calculate $\hat{c}_1$ using the equation (9) when the saturation function is given. Suppose that $\hat{c}_1 = \alpha + j\beta$ in the equation (8). After substituting the $\hat{c}_1$ into the equation (8),

$$Re[G(jw)]\alpha - Im[G(jw)]\beta + j(Im[G(jw)]\alpha + Re[G(jw)]\beta + a/2) = 0$$

$$\Rightarrow Re[G(jw)]\alpha - Im[G(jw)]\beta = 0 \text{ and}$$

$$Im[G(jw)]\alpha + Re[G(jw)]\beta + a/2 = 0$$

Thus

$$\begin{aligned} Re[G(jw)] &= (\beta/\alpha)(a/2)\frac{1}{\alpha + \beta^2/\alpha} \\ Im[G(jw)] &= -\frac{a}{2(\alpha + \beta^2/\alpha)} \end{aligned} \tag{10}$$

The equation (10) does not require the plant d.c. gain to find $G(jw)$. Once we know one point information in the Nyquist plot of the unknown plant, we can design the PID controller[1] using frequency domain design scheme. Suppose that we get $G(jw) = r_p e^{j(\pi + \phi_p)}$ from the equation (10). Suppose that the desired phase margin is ϕ_m for the closed-loop system and the structure of PID controller is given by $G_c(s) = k(1 + sT_d + \frac{1}{sT_i})$ where k, T_d, and T_i are proportional gain, differential time, and integral time, respectively. It can be shown that k, T_d, and T_i satisfied the phase margin specification is given by

$$\begin{aligned} k &= \frac{\cos(\phi_m - \phi_p)}{r_p} \\ T_d &= (1/2w_1)\{\tan(\phi_m - \phi_p) + \sqrt{4\alpha + \tan^2(\phi_m - \phi_p)}\} \\ T_i &= \frac{T_d}{\alpha} \end{aligned}$$

where the typical value of $\alpha = 0.25$ and w_1 is the period of the plant output. Note that it is possible that the static load disturbance disappear or change the its magnitude by the nature of disturbance after the tuning phase. The static load disturbance might result in the presence of a steady state offset. However this will not cause the problem, since our controller contains an integrator.

III. Conclusion

Asymmetric oscillation caused by a static load disturbance in the plant output has been considered. We analyzed the asymmetric oscillation with a saturation nonlinear element using a first order harmonic balance equation. Based on the analysis, we propose a new method which does not require a knowledge of the plant d.c. gain to find one point information on the Nyquist plot of unknown plant. The proposed method can improved the accuracy of the estimation of the unknown plant in the Nyquist plot. As a result of accurate estimation of unknown plant, we proposed that PID controller design algorithm.

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