Taylor-Lei Series에 의한 지연이 있는 비선형 시스템의 시간 이산화

Time-Discretization of Nonlinear control systems with State-delay via Taylor-Lie Series

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Abstract - In this paper, we propose a new scheme for the discretization of nonlinear systems using Taylor series expansion and the zero-order hold assumption. This scheme is applied to the sample-data representation of a nonlinear system with constant state time-delay. The mathematical expressions of the discretization scheme are presented and the effect of the time-discretization method on key properties of nonlinear control system with state time-delay, such as equilibrium properties and asymptotic ability, is examined. The proposed scheme provides a finite-dimensional representation for nonlinear systems with state time-delay enabling existing controller design techniques to be applied to them. The performance of the proposed discretization procedure is evaluated using a nonlinear system. For this nonlinear system, various sampling rates and time-delay values are considered..

Key Words :NonlinearControl System, State Time-Delay, Taylor-Series, Time-Discretization

1. Introduction

As is well known, state time delays are inherent features of many physical process, eg., chemical process, electrical heater and long transmission lines in pneumatic, hydraulic and rolling mill systems, etc. Many of these processes are also significantly nonlinear which motivates the research in control of nonlinear systems with time delay. The presence of time delays in a practical system may induce instability, oscillation and poor performance and complicates the analysis and design of feedback controllers. Control systems with time-delays exhibit complex behavior because of their infinite dimensionality, even in the case of linear time-invariant systems that have constant time-delays in the input or states have infinite dimensionality when expressed in the continuous-time domain. For these reasons, during the last few decades it was not possible to apply a controller design technique having any time-delays in the variables. Thus it is necessary to develop a control system design scheme that resolves these time-delays.

For the original continuous-time systems with time free case (Franklin et al,. 1998) the traditional numerical techniques such as the Euler and Runge-Kutta methods have been used for getting the sampled-data representations. But these methods need a small sampling time interval. Because it is necessary to meet the desired accuracy and they can not be applied to the large sampling period case. But due to the physical and technical limitation slow sampling is becoming inevitable. A time-discretization method which expands the well-known time-discretization of the linear time-delay system (Franklin et al,. 1998; Vaccaro, 1995) to nonlinear continuous-time control system with time-delay (Kazantzis, et al., 2003) can solve this problem. The effect of this approach on system-theoretic properties of nonlinear systems, such as equilibrium properties, relative order, stability, zero dynamics and minimum-phase characteristics revealing the natural and transparent way in which Taylor methods permeate the relevant theoretical aspects is also studied (Kazantzis, et al., 1994).

This paper proposed a time discretization method of nonlinear control systems with statetime-delay in control. The proposed discretization scheme applies the Taylor Series expansion and the zero order hold (ZOH) assumption according to the mathematical structure developed for the delay-free nonlinear system (Kazantzis and Kravaris, 1997; 1999). The effect of sampling on system-theoretic properties of nonlinear systems with state time-delay of this discrete-time scheme, such as

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equilibrium properties and stability is examined.

2.Time-Discretization of Nonlinear Control Systems with State-Delay

The nonlinear continuous system with state- delay can be represented by the following state-space form:

$$\frac{dx(t)}{dt} = f(x(t)) + f_1(x(t-D)) + g(x(t))u(t) + g_1(x(t-D))u(t) \quad (5)$$

We assume that in the time interval $t \in [kT, kT+T)$:

$$x(t-D) \approx \frac{1}{2}[x(kT+T-D) + x(kT-D)] = constant \quad (6)$$

Case 1. $D \geq T$

Assume that in the time interval $t \in [kT, kT+T)$, $D \approx mT$, ($m \geq 1, m$ is a integer).

$$\begin{aligned} x(t-D) &\approx \frac{1}{2}[x(kT+T-D) + x(kT-D)] \\ &= \frac{1}{2}[x(kT+T-mT) + x(kT-mT)] \\ &= \frac{1}{2}[x(k+1-m) + x(k-m)] = constant \end{aligned} \quad (7)$$

Under the ZOH assumption and within the sampling interval, the solution described in Eq. (5) is expanded in a uniformly convergent Taylor series and the resulting coefficients can be easily computed by taking successive partial derivatives of the right-hand-side of Eq. (5):

$$x(k+1) = x(k) + \sum_{l=1}^{\infty} A^{[l]}(x(k), u(k)) \frac{T^l}{l!} \quad (8)$$

where $x(k)$ is the value of the state vector x at time $t = t_k = kT$ and $A^{[l]}(x,u)$ are determined recursively by:

$$\begin{aligned} A^{[1]}(x,u) &= f(x(t)) + f_1(x(t-D)) \\ &\quad + u(t)g(x(t)) + u(t)g_1(x(t-D)) \\ A^{[l+1]}(x,u) &= \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x(t)) + f_1(x(t-D)) \\ &\quad + u(t)g(x(t)) + u(t)g_1(x(t-D))) \end{aligned} \quad (9)$$

with $l = 1, 2, 3, \ldots$.

Case 2. $D < T$

Assume that in the time interval $t \in [kT, kT+T)$, $T \approx nD, D \approx \frac{1}{n}T$, ($n > 1, n$ is a integer).

In the time interval $t \in [kT, kT + \frac{1}{n}T)$:

$$x(t-D) \approx \frac{1}{2}\left[x(kT + \frac{1}{n}T - D) + x(kT-D)\right] = constant \quad (10)$$

$$x(kT + \frac{1}{n}T) = x(kT) + \sum_{l=1}^{\infty} A^{[l]}(x(kT), u(kT)) \frac{(\frac{T}{n})^l}{l!} \quad (11)$$

In the time interval $t \in [kT + \frac{1}{n}T, kT + \frac{2}{n}T)$:

$$x(t-D) \approx \frac{1}{2}\left[x(kT + \frac{2}{n}T - D) + x(kT + \frac{1}{n}T - D)\right] = constant \quad (12)$$

$$x(kT + \frac{2}{n}T) = x(kT + \frac{1}{n}T) + \sum_{l=1}^{\infty} A^{[l]}(x(kT + \frac{1}{n}T), u(kT + \frac{1}{n}T)) \frac{(\frac{T}{n})^l}{l!} \quad (13)$$

$\vdots$

In the time interval $t \in [kT + \frac{n-1}{n}T, kT+T)$:

$$x(t-D) \approx \frac{1}{2}\left[x(kT+T-D) + x(kT + \frac{n-1}{n}T - D)\right] = constant \quad (14)$$

$$x(kT+T) = x(kT + \frac{n-1}{n}T) + \sum_{l=1}^{\infty} A^{[l]}(x(kT + \frac{n-1}{n}T), u(kT + \frac{n-1}{n}T)) \frac{(\frac{T}{n})^l}{l!} \quad (15)$$

where $x(k)$ is the value of the state vector x at time $t = t_k = kT$ and $A^{[l]}(x,u)$ are determined recursively by:

$$\begin{aligned} A^{[1]}(x,u) &= f(x(t)) + f_1(x(t-D)) \\ &\quad + u(t)g(x(t)) + u(t)g_1(x(t-D)) \\ A^{[l+1]}(x,u) &= \frac{\partial A^{[l]}(x,u)}{\partial x}(f(x(t)) + f_1(x(t-D)) \\ &\quad + u(t)g(x(t)) + u(t)g_1(x(t-D))) \end{aligned} \quad (16)$$

with $l = 1, 2, 3, \ldots$.

3. Simulation

The performance of the proposed time-discretization of nonlinear continuous control systems with state time-delay using the Taylor series expansion method is evaluated by applying it to a nonlinear system. The system considered exhibit nonlinear behavior and it is studied for a broad range of values of the sampling period and state-delays. Reference solutions for the system are required to validate the proposed time-discretization method. In this paper the Matlab ODE solver is used to obtain reference solutions. The discrete values obtained at every time step using the proposed time-discretization method are compared to the values obtained using the Matlab ODE solver at the corresponding time steps. The partial derivative terms involved in the Taylor series expansion are determined recursively. For the case study considered these partial derivative terms are calculated using Maple.

The system considered in this paper is assumed to be a nonlinear control system (Zidong Wang, 2003).

$$\dot{x}_1 = -2.8x_1 + 0.1x_2 + 0.2\sin(x_1 + x_2) + 0.05x_1(t-D) - 0.02x_2(t-D)$$
$$\dot{x}_2 = 0.4x_1 - 3.2x_2 + 0.3\cos x_2 + 0.01x_1(t-D) - 0.03x_2(t-D) \quad (17)$$

we choose the initial conditions $x_1(0) = 0.6$, $x_2(0) = 0.6$, $D=0$ and the sampling period $T = 0.008s$. The results obtained by the Matlab ODE solver and the proposed time-discretization method are shown in Table 1. Figure 1 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the Matlab Solution.

Table 1 The results of delay-free case

T=0.008s, D=0, x1=0.6, x2=0.6				
Step	Matlab(x1)	Maple(x1)	Matlab(x2)	Maple(x2)
50	0.2386	0.2386	0.2601	0.2602
100	0.0988	0.0988	0.1509	0.1509
150	0.0444	0.0444	0.1140	0.1140
200	0.0234	0.0234	0.1011	0.1011
250	0.0153	0.0153	0.0965	0.0965
300	0.0122	0.0122	0.0948	0.0948
350	0.0110	0.0110	0.0941	0.0941
400	0.0106	0.0106	0.0939	0.0939
450	0.0104	0.0104	0.0938	0.0938
500	0.0103	0.0103	0.0938	0.0938
550	0.0103	0.0103	0.0938	0.0938
600	0.0103	0.0103	0.0938	0.0938

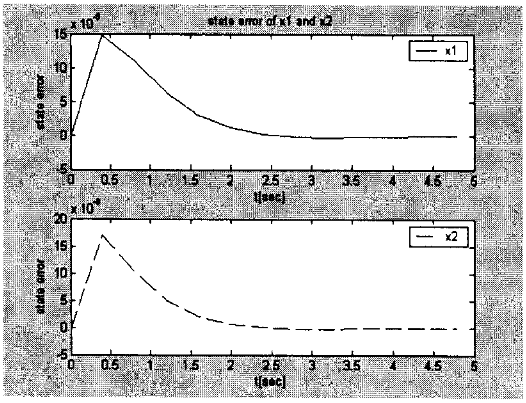

Fig. 1. State error responses of the system for the delay-free case

4. Conclusion

This paper has presented the approach for the discrete-time representation of a nonlinear control system with state time-delay in control. It is based on the ZOH assumption and the Taylor-Series expansion, which is obtained as a solution of continuous-time systems. The mathematical structure of the new discretization scheme is explored and characterized as useful for establishing concrete connections between numerical and system-theoretic properties. In particular, the effect of the time-discretization method on key properties of nonlinear control system with state time-delay, such as equilibrium properties and asymptotic stability, is examined. The proposed scheme provides a finite-dimensional representation for nonlinear systems with state time-delay enabling existing controller design techniques to be applied to them. Extension of the proposed approach to nonlinear control system with output delay is feasible, and it will be the subject of future publication.

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