Neo Fuzzy Set-based Polynomial Neural Networks involving Information Granules and Genetic Optimization

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Astract - In this paper, we introduce a new structure of fuzzy-neural networks Fuzzy Set-based Polynomial Neural Networks (FSPNN). The two underlying design mechanisms of such networks involve genetic optimization and information granulation. The resulting constructs are Fuzzy Polynomial Neural Networks (FPNN) with fuzzy set-based polynomial neurons (FSPNs) regarded as their generic processing elements. First, we introduce a comprehensive design methodology (viz. a genetic optimization using Genetic Algorithms) to determine the optimal structure of the FSPNNs. This methodology hinges on the extended Group Method of Data Handling (GMDH) and fuzzy set-based rules. It concerns FSPNN-related parameters such as the number of input variables, the order of the polynomial, the number of membership functions, and a collection of a specific subset of input variables realized through the mechanism of genetic optimization. Second, the fuzzy rules used in the networks exploit the notion of information granules defined over systems variables and formed through the process of information granulation. This granulation is realized with the aid of the hard C- Means clustering (HCM). The performance of the network is quantified through experimentation in which we use a number of modeling benchmarks already experimented with in the realm of fuzzy or neurofuzzy modeling.

Key Words : FSPNN(Fuzzy Set-based Polynomial Neural Networks), FPN(Fuzzy Polynomial Neuron), GMDH(Group Method of Data Handling), GAs(Genetic Algorithms)

1. Introduction

A lot of researchers on system modeling have been interested in the multitude of challenging and conflicting objectives such as compactness, approximation ability, generalization capability and so on which they wish to satisfy. Fuzzy sets emphasize the aspect of linguistic transparency of models and a role of a model designer whose prior knowledge about the system may be very helpful in facilitating all identification pursuits. In addition, to build models with substantial approximation capabilities, there should be a need for advanced tools.

As one of the representative advanced design approaches comes a family of self-organizingnetworks with fuzzy polynomial neuron (FPN) (called "FPNN" as a new category of neuro-fuzzy networks) [1, 4, 8]. The design procedure of the FPNNs exhibits some tendency to produce overly complex networks as well as comes with a repetitive computation load caused by the trial and error method being a part of the development process.

In this paper, in considering the above problems coming with the conventional FPNN [1, 4, 8], we introduce a new structure of fuzzy rules as well as a new genetic design approach. The new structure of fuzzy rules based on the fuzzy set-based approach changes the viewpoint of input space division. In other hand, from a point of view of a new understandingof fuzzy rules, information granules seem to melt into the fuzzy rules respectively. The determination of the optimal values of the parameters available within an individual FSPN leads to a structurally and parametrically optimized network through the genetic approach.

2. Fuzzy Set-based Polynomial Neural Networks

The FSPN encapsulates a family of nonlinear "if-then" rules. When put together, FSPNs results in a self-organizing Fuzzy Set-based Polynomial Neural Networks (FSPNN). As visualizedin Fig. 1, the FSPN consists of two basic functional modules. The first one, labeled by F, is a collection of fuzzy sets (here denoted by $\{A_k\}$ and $\{B_k\}$) that form an interface between the input numeric variables and the processing part realized by the neuron. The second module (denoted here by **P**) refers to the function based nonlinear (polynomial) processing that involves some input variables This nonlinear processing involves some input variables (x_i and x_j), which are capable of being the input variables (Here, x_p and x_q), or entire system input variables. Each rule reads in the form

$$\text{if } x_p \text{ is } A_k \text{ then } z \text{ is } P_{pk}(x_i, x_j, \mathbf{a}_{pk})$$
$$\text{if } x_q \text{ is } A_k \text{ then } z \text{ is } P_{qk}(x_i, x_j, \mathbf{a}_{qk}) \qquad (1)$$

where $\mathbf{a}_{qk}$ is a vector of the parameters of the

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conclusion part of the rule. while $P(x_i, x_j, \mathbf{a})$denoted the regression polynomial forming the consequence part of the fuzzy rule. The activation levels of the rules contribute to the output of the FSPN being computed as a weighted average of the individual condition parts (functional transformations) P_K (note that the index of the rule, namely "K" is a shorthand notation for the two indexes of fuzzy sets used in the rule (1), that is $K = (l, k)$).

$$z=\sum_{l=1}^{m}\sum_{k=1}^{l}\mu_{(l,k)}P_{(l,k)}(x_i,x_j,\mathbf{a}_{(i,k)})/\sum_{k=1}^{l}\mu_{(l,k)} = \sum_{l=1}^{m}\sum_{k=1}^{l}\widetilde{\mu_{(l,k)}}P_{(l,k)}(x_i,x_j,\mathbf{a}_{(i,k)}) \quad (2)$$

When developing an FSPN, we use genetic algorithms to produce the optimized network. This is realizedby selecting such parameters as the number of input variables, the order of polynomial, and choosing a specific subset of input variables. Based on the genetically optimized number of the nodes (input variables) and the polynomial order, refer to Table 1, we construct the optimized self-organizing network architectures of the FSPNNs.

Table 1. Different forms of the regression polynomials forming the consequence part of the fuzzy rules.

No. of inputs / polynomial	1	2	3
0 (Type 1)	Constant	Constant	Constant
1 (Type 2)	linear	Bilinear	Trilinear
2 (Type 3)	Quadratic	Biquadratic-1	Triquadratic-1
2 (Type 4)		Biquadratic-2	Triquadratic-2

1: Basic type, 2: Modified type

3. Information Granulation through Hard C-Means clustering algorithm

Information granules are defined informally as linked collections of objects (data points, in particular) drawn together by the criteria of indistinguishability, similarity or functionality [9].

3.1 Definition of the premise and consequent part of fuzzy rules using Information Granulation

We assume that given aset of data X={x1,x2,,xn} related to a certain application, there are some clusters which are capable of being found through HCM. The center point and the membership elements represent each cluster. The set of membership elements is crisp. To construct afuzzy mode, we should transform the crisp set into the fuzzy set. The center point means the apex of the membership functionof the fuzzy set. Let us consider building the consequent part of fuzzy rule. We can think of each cluster as a sub-model composing the overall system. The fuzzy rules of Information Granulation-based FSPN are as followings.

if x_p is $\mathbf{A}^*_k$ then $z-m_{pk} = P_{pk}((x_i-v^i_{pk}),(x_j- v^j_{pk}),a_{pk})$

if x_q is $\mathbf{B}^*_k$ then $z-m_{qk} = P_{qk}((x_i-v^i_{qk}),(x_j- v^j_{qk}),a_{qk})$ *(2)*

Where, $\mathbf{A}^*_k$ and $\mathbf{B}^*_k$ mean the fuzzy set, the apex of which is defined as the center point of information granule (cluster) and m_{pk} is the center point related to the output variable on cluster_{pk}, v^i_{pk} is the center point related to the i-th input variable on cluster_{pk} and $\mathbf{a}_{qk}$ is a vector of the parameters of the conclusion part of the rule while $P((x_i-v_i),(x_j- v_j),a)$ denoted the regression polynomial forming the consequence part of the fuzzy rule which uses several types of high-order polynomials (linear, quadratic, and modified quadratic) besides the constant function forming the simplest version of the consequence; refer to Table 1. If we are given m inputs and one output system and the consequent part of fuzzy rules is linear, the overall procedure of modification of the generic fuzzy rules is as followings.

The given inputs are $\mathbf{X}=[\mathbf{x}_1\ \mathbf{x}_2\cdots\mathbf{x}_m]$ related to a certain application, where $\mathbf{x}_k=[x_{k1}\cdots x_{kn}]^T$, n is the number of data and m is the number of variables and the output is $\mathbf{Y}=[y_1\ y_2\ \cdots\ y_n]^T$.

Step 1) build the universe set

Step 2) build m reference data pairs composed of $[\mathbf{x}_1;Y]$, $[\mathbf{x}_2;Y]$, and $[\mathbf{x}_m;Y]$.

Step 3) classify the universe set U into $\boldsymbol{l}$ clusters such as c_{i1}, c_{i2}, , c_{il} (subsets) by using HCM according to the reference data pair $[\mathbf{x}_i;Y]$. Where c_{ij} means the j-th cluster (subset) according to the reference data pair $[\mathbf{x}_i;Y]$.

Step 4) construct the premise part of the fuzzy rules related to the i-th input variable (x_i) using the directly obtained center points from HCM.

Step 5) construct the consequent part of the fuzzy rules related to the i-th input variable (x_i). On this step, we need the center points related to all input variables.

Sub-step1) make a matrix as (5) according to the clustered subsets

$$A^i_j=\begin{vmatrix} x_{21} & x_{22} & \cdots & x_{2m} & y_2 \\ x_{51} & x_{52} & \cdots & x_{5m} & y_5 \\ \vdots & \vdots & & \vdots & \vdots \\ x_{k1} & x_{k2} & \cdots & x_{km} & y_k \\ \vdots & \vdots & & \vdots & \vdots \end{vmatrix} \quad (5)$$

Where, $\{x_{k1}, x_{k2}, , x_{km}, y_k\}\in c_{ij}$ and A_{ij} means the membership matrix of $\boldsymbol{j}$-$\boldsymbol{th}$ subset related to the $\boldsymbol{i}$-$\boldsymbol{th}$ input variable.

Sub-step2) take an arithmetic mean of each column on Aij. The mean of each column is the additional center point of subset c_{ij}. The arithmetic means of column is (6)

$$centerpoints=[\,v^1_{ij}v^2_{ij}\cdots v^m_{ij}m_{ij}] \quad (6)$$

4. Genetic Optimization of FPNN

GAs are aimed at the global exploration of a solution space. The main features of genetic algorithms concern individuals viewed as strings, population-based optimization and stochastic search mechanism (selection and crossover). GAs use serial method of binary type, roulette-wheel as the selection operator, one-point crossover, and an invert operation in the mutation operator [2]. The framework of the design procedure of the genetically optimized FSPNN comprises the following steps

[Step 1] Determine systems input variables

[Step 2] Form training and testing data

[Step 3] specify initial design parameters

[Step 4] Decide FSPN structure using genetic design

[Step 5] Carry out fuzzy-set based fuzzy inference and coefficient parameters estimation for fuzzy identification in the selected node (FSPN)

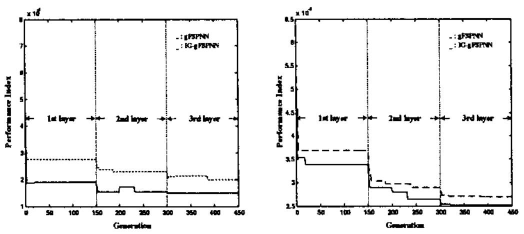
[Step 7] Check the termination criterion

[Step 8] Determine new input variables for the next layer

5. Simulation

We demonstrate how the IG-gFSPNN can be utilized to predict future values of a chaotic Mackey-Glass time series. This time series is used as a benchmark in fuzzy and neurofuzzy modeling. The performance of the network is also contrasted with some other models existing in the literature [5-7]. The time series is generated by the chaotic Mackey-Glass differential delay equation. To come up with a quantitative evaluation of the network, we use the standard RMSE performance index.

Fig. 1 illustrates the different optimization process between gFSPNN and the proposed IG-gFSPNN by visualizing the values of the performance index obtained in successive generations of GA when using Type T^*.



(a) Training error (b) Testing error

Fig. 1. The optimization process quantified by the values of the performance index (in case of using Gaussian MF with Max=5 and Type T*)

Table 2 summarizes a comparative analysis of the performance of the network with other models.

Model				Performance Index		
				PI	PI_s	EPI_s
Wang's model[5]				0.044		
				0.013		
				0.010		
ANFIS[6]					0.0016	0.0015
FNN model[7]					0.014	0.009
Proposed IG-gFSPNN	Type T^*	Triangular	Max=5		1.72e-4	3.30e-4
			Max=5		1.48e-4	2.61e-4

6. Concluding remarks

In this study, we have surveyed the new structure and meaning of fuzzy rules and investigated the GA-based design procedure of Fuzzy Polynomial Neural Networks (FPNN) along with its architectural considerations. The whole system is divided into some sub-systems that are classified according to the characteristics named information granules. Each information granule seems to be a representative of the related sub-systems. A new fuzzy rule with information granule describes a sub-system as a stand-alone system. A fuzzy system with some new fuzzy rules depicts the whole system as a combination of some stand-alone sub-system.

The GA-based design procedure applied at each stage (layer) of the FSPNN leads to the selection of the preferred nodes (or FSPNs) with optimal local characteristics (such as the number of input variables, the order of the consequent polynomial of fuzzy rules, and input variables) available within FSPNN.

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