Short-term Distributed Rainfall Prediction using Stochastic Error Field Modeling

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Abstract

이류모형을 이용한 단기예측 레이더 강우자료와 관측 레이더자료의 비교를 통하여 얻어진 예측오차를 분석하였다. 임의 시점까지의 예측오차 장에 나타나는 확률분포 형태와 공간적 상관성을 분석하여 이들 특성을 반영하는 추후의 예측오차 장을 모의할 수 있었다. 모의된 예측오차 장과 합성된 단기예측 강우 장은 이류모형을 이용한 예측에 따른 불확실성을 추계학적으로 반영한 예측강우를 제공한다.

Key words: Radar rainfall data, Translation model, Real-time prediction, Error structure

1. Introduction

Currently, there are many hydrological models. Whether these are distributed or lumped models, these can properly simulate basin characteristic if parameters of the models are optimized to a subject basin. In the case, the most effective element on simulation results is its input data. The accuracy of the simulation results which usually represent as a discharge hydrograph is mostly affected by accuracy of rainfall data. This accuracy effectiveness appears more severely when we forecast discharge with predicted rainfall. Considering an uncertainty of predicted rainfall, it is important to provide prediction accuracy of rainfall for real-time rainfall-runoff simulation.

During short-term rainfall prediction with any prediction model, if an error structure of predicted rainfall is properly analyzed with past prediction results and effective error factors are found, the error structure would be very useful to a present rainfall prediction with giving uncertainty of the prediction values. Considering points to analyze the error structure would be amount and distribution of error bias and its spatial correlation.

In this research, an error structure of a real-time rainfall prediction by a translation model is analyzed to obtain its probability distribution and spatial correlation coefficient. Then prediction error fields are simulated using a spatially correlated random field according to the characteristics of the prediction error structure. The simulated error fields are successfully reflect the characteristics of the past prediction error.

2. Prediction Error Structure Analysis

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2.1 Translation Model

Translation model (Shiiba *et al.*, 1984) is used for short-term rainfall prediction. In the translation model, the horizontal rainfall distribution $z(x,y,t)$ with the spatial coordinate (x,y) at time t is modeled as:

$$\frac{\partial z}{\partial t}+u\frac{\partial z}{\partial x}+v\frac{\partial z}{\partial y}=w, \quad u=\frac{dx}{dt}, \quad v=\frac{dy}{dt}, \quad w=\frac{dz}{dt} \qquad (1)$$

where, u and v are advection velocity along x and y, and w is rainfall growth-decay rate. The translation model assumes u, v, and w are having the forms:

$$u=c_1x+c_2y+c_3, \quad v=c_4x+c_5y+c_6, \quad w=c_7x+c_8y+c_9 \qquad (2)$$

The values of parameters c_1, ~ , c_9 are identified by the least square method using observed radar rainfall data and they are updated on a real-time basis. In this research, three consecutive spatial rainfall distribution data, which have 3km and 5minutes resolution, are used to determine u and v. When forecasting rainfall fields, the growth decay rate w is always assumed to be zero. Radar rainfall event used here is observed at Miyama radar station of Kinki area, Japan from 12th to 14th September 1990.

2.2 Error Distribution and Spatial Correlation

Absolute prediction errors E_a at every grid are calculated from differences between predicted rainfalls R_p, and observed rainfalls R_o for each prediction. Here, predictions are done at 13:00pm for 30minutes lead time with 5minutes interval,

$$E_a = R_o - R_p \qquad (3)$$

Frequency distributions of E_a show a normal distribution pattern while it is various as time passes (Fig 1). Standard deviations of the E_a gradually increase as prediction lead time increase (Table 1), and spatial correlation coefficients also gradually increase (Fig 2). These error characteristics can be admitted as a reasonable result when we consider longer prediction lead time.

Table.1 Statistics of the *Ea* (unit : mm/hr)

Lead time	5min	10min	15min	20min	25min	30min
Average	-0.15	-0.02	0.19	-0.09	-0.32	-0.41
St. Dev.	4.18	5.56	6.61	7.03	7.56	7.78

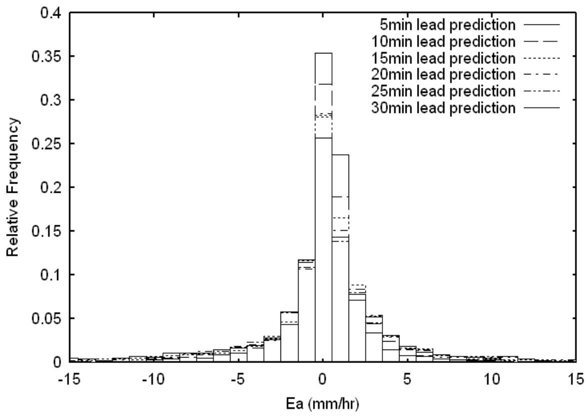


Fig. 1 Frequency distribution of the E_a

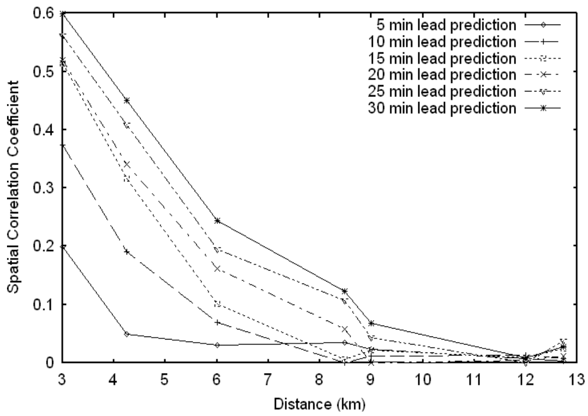


Fig. 2 Spatial correlation coefficient of the E_a

The spatial correlation coefficients which present in Figure 2 are calculated by using one error field of each lead prediction. The spatial correlation coefficients for 10 minutes lead prediction, for example, are calculated from the error field at 13:10pm (=13:00pm+ 10min lead prediction) by grouping each pairs of error which apart one grid for 3km, two grid for 6km, etc. It is assumed that error fields have ergodicity and so, each error field can reflect time-average spatial correlation characteristic. When spatial correlation coefficients from time-average and ones from specific error field are compared as shown in Figure 3, the coefficients from specific error field are within error range of the time-average coefficients.

Spatial correlation coefficient could be calculated from several error fields to give much generality as time-average ones. More research on spatial correlation should be carried out.

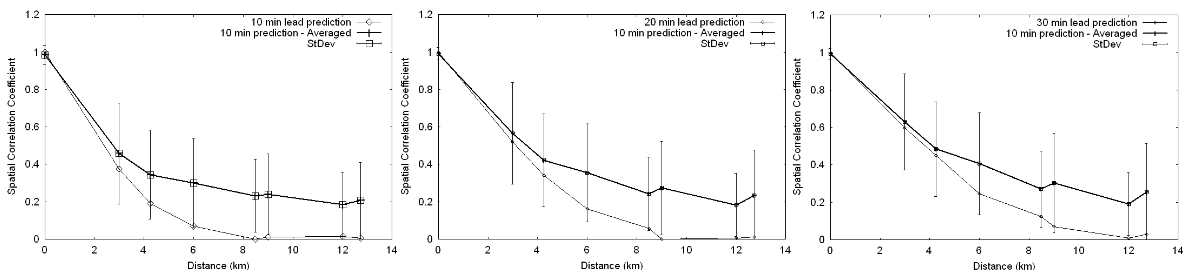


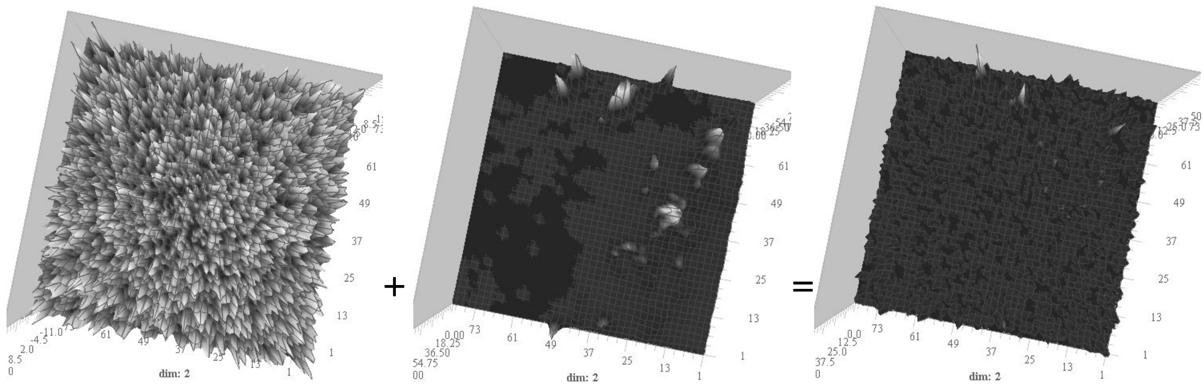
Fig. 3 Frequency distribution of the E_a

3. Prediction Error Fields Simulation

Let's assume that a predicted error field vector Y can be simulated using a matrix B which has a spatial correlation characteristic and a random value vector x ($Y=Bx$). Here, the random values x are uncorrelated each other but follows a given probability distribution. Then a covariance of the vector Y is represented as (Tachikawa *et al.*, 2003);

$$\boldsymbol{E}[YY^{\mathrm{T}}] = \boldsymbol{E}[Bxx^{\mathrm{T}}B^{\mathrm{T}}] = B\boldsymbol{E}[xx^{\mathrm{T}}]B^{\mathrm{T}} = BB^{\mathrm{T}} \tag{3}$$

The matrix B is obtained from the covariance matrix BB^{T} using the *Cholesky* decomposition method, then the error field vector Y can be simulated with the matrix B and the random value vector x. Because the error field vector Y can be generated unlimited, we can also generate prediction fields by combining the error fields and prediction rainfall field by the translation model. Figure 4 shows one example of simulated error field and generated prediction rainfall field.



Error field Predicted rainfall by One of the realization of Translation model rainfall prediction field

Fig. 4 Example of simulated error field and generated prediction field

To check out the effectiveness of error field simulation, probability distribution and spatial correlation coefficients are calculated and compared with input values. In Figure 5, the simulated error field (dash line) shows same distribution pattern with the distribution of random values x, but the generated prediction error field (prediction + error field, dot line) shows different pattern to the distribution of original error field. It is caused by 'minus rainfall handling' when the prediction fields are generated. If predicted rainfall on any grid becomes minus value after joining with the simulated error field, those minus rainfalls are assumed be zero rainfall because the minus rainfall is not acceptable physically. When distributions are checked again without minus rainfall handling, the distribution (thick solid line) is matched with the distribution of the original error field.

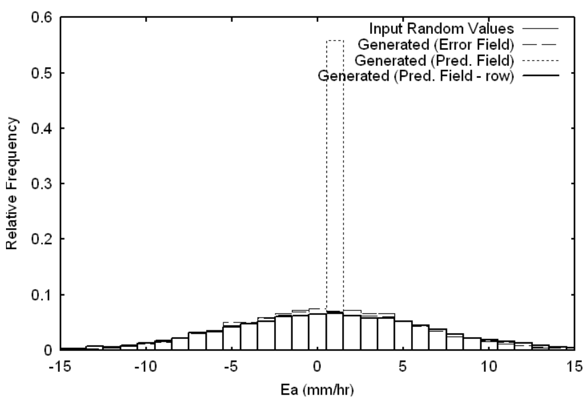


Fig. 5 Distributions compare

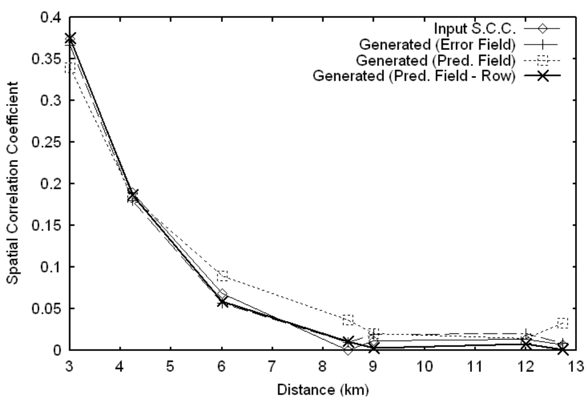


Fig. 6 Spatial correlation coefficient compare

The same phenomenon is found at the spatial correlation coefficient as shown in Figure 6. From the both results, we can get a conclusion that the simulated error field successfully reflects the error structure of the past predicted rainfall fields.

4. Further Research

There are several problems to be solved and considered. First, Cholesky decomposition method sometimes fail to get the matrix $\boldsymbol{B}$. Second, the minus rainfall handling affects to the probability distribution and spatial correlation coefficient of the original simulated error field. Last, the error structure only with absolute error E_a can not properly simulate variant error amounts according to rainfall intensity. So, a relative prediction error E_r such as;

$$E_r = (R_o - R_p)/R_p \tag{4}$$

can be considered. Distribution of the E_r in Figure 7 shows that it also has a typical pattern. Properly simulated prediction error fields would be very useful to figure out uncertainties in a rainfall prediction and also in a runoff prediction.

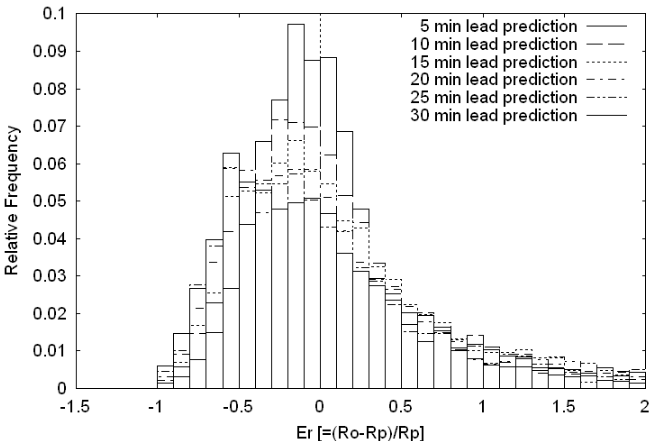


Fig. 7 Frequency distribution of the E_r

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